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# Novel Scheduling of A Mixed Batch/Continuous Sugar Milling Plant Using Petri nets

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### Abstract

Scheduling of processes in mixed batch/continuous plants, due to their hybrid nature can become very complex. This paper presents the Timed Hybrid Petri net (THPN) as a suitable tool for modelling and scheduling of hybrid systems. One of the major benefits over traditional methods is a significant reduction in complexity during problem formulation. A sugar milling plant containing both batch and continuous processing units is considered as a case study by which the application of the proposed scheduling algorithm is illustrated.

Keywords: scheduling, mixed batch/continuous, Petri net

#### 1. Introduction

A scheduling method should formulate a problem by considering all the associated constraints to find the best sequence of operations that optimises a specific objective function. Although much research has been done on the development of different methods for scheduling of either continuous or batch chemical plants, there is a distinct absence of research in the scheduling of mixed batch/continuous systems (hybrid systems). In the past, scheduling of these plants has been accomplished by either discretising time or considering a continuous time model (Nott, 1998). Simulation models have often been used to schedule these plants. Djavdan relied on such a model with an emphasis on the importance of the size of intermediate storage (Djavdan, 1993). Many researchers have acknowledged that the scheduling of mixed-batch/continuous plants is a very important issue (Neville et al., 1982; Sicignanoet al., 1984) but still, a concrete method for solving these types of scheduling problems with less computational complexity remains.

The Timed Petri net based formulation has proven to be a promising technique to solve many scheduling problems (Ghaeli et al., 2004). This paper considers a timed Hybrid Petri net (THPN), which is a class of Petri net, as a suitable tool for modelling and scheduling of mixed batch/continuous plants. A substantial reduction in the complexity of the problem formulations has also been achieved. This is because the THPN tool

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allows for effective illustration of the batch and continuous operations and storage between continuous units with the associated capacities and all other constraints. Initially, an introduction to THPN is presented. Then, in section 3 the model of a sugar milling case study, which is an example of a mixed batch/continuous plant is developed and the proposed approach applied. Finally, a brief conclusion is presented.

# 2. Timed Hybrid Petri net

A Petri net is a particular kind of bipartite directed graph populated by three types of objects: places, transitions and directed arcs. The dynamics of the model are represented by the movement of tokens, which are located in places and illustrated by either small dots or a real number. The firing rules of transitions in a Petri net model make the consideration of the resource use, storage policy and product sequence decisions.

The graphical and Mathematical properties of Petri net are also useful when it is used to model hybrid plants. Hybrid Petri net (HPN) is a class of Petri net made up of a "continuous part" (continuous places and transitions) and a "discrete part" (discrete places and transitions). Figure 1 shows the graphical notations for places and transitions in HPN as used in this paper.

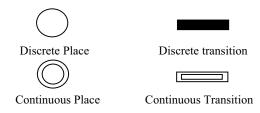


Figure 1. The representation of places and transitions in HPN

Formally, HPN is a six-tuple  $HPN = \{P, T, pre, post, m_o, h\}$  (David and Alla, 2001) where  $P = \{P_1, P_2, ..., P_n\}$  and  $T = \{T_1, T_2, ..., T_n\}$  are finite but not empty set of places and transitions respectively;  $pre : P \times T \rightarrow R^+$  is the input incidence matrix while  $post : P \times T \rightarrow R^+$  is the output incidence matrix. Note that  $P \cap T = \phi$ ;  $m_o : P \rightarrow R^+$  is the initial marking. An incidence matrix is a matrix, which includes the weights of the arcs connecting places to transitions or transitions to places. A node is defined to be either discrete (D) or continuous (C) by the hybrid function:  $h : P \cap T \rightarrow \{D, C\}$ ;  ${}^{\circ}P_i({}^{\circ}T_j)$ and  $P_i^{\circ}(T_j^{\circ})$  depict the set of input transitions (places) and the set of output transitions (places) respectively. The state equation is the same in a hybrid Petri net  $m = m_0 + U.V$ and the difference is that in HPN an integer value in vector V is related to the number of firings of a discrete transition while a nonnegative real number corresponds to a firing quantity of a continuous transition. To consider quantitative properties in a THPN, time is added to HPN. Based on how the places and transitions are defined in a THPN, time can be introduced to either places or transitions.

# 3. Case Study

To show the details of the proposed method, a sugar milling case study (Nott, 1998) is considered. Sugar Milling is a hybrid system, which can be divided into two parts: the high-grade and the low-grade systems. The flowsheet for the sugar mill is given in Figure 2. Boxes with a solid outline represent batch operations, whereas continuous operations are represented with a broken outline. Triangular shapes represent storage facilities. The input and output of each pan are shown in the flowsheet. As the use and production of water does not have any effect on the scheduling, it is not considered in the flowsheet. The processing time of each pan is depicted in the parenthesis next to each pan in the flowsheet. The main assumption is that processing of fugal A and fugal B can not be done simultaneously. The maximum flowrate of super, csystems and dryer are 16.5, 32.5, 22 tonnes per half hour, respectively. The syrup flow into the system is 106 tonnes per hour. The maximum capacity as well as the initial amount for each storage are given in Table 1.

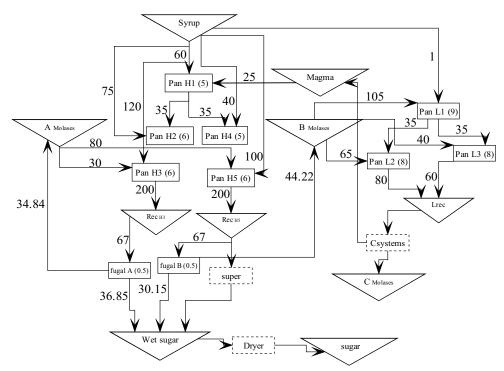


Figure 2. The flowsheet of the sugar milling case study

Table 1. Storage Specifications (tonnes)

	Syrup	Magma	AMolasses	BMolasses	Rec3	Rec5	LRec	Wetsugar
Max	410	50	310	200	330	330	230	250
Initial value	100	20	140	110	200	200	70	25

In order to solve the scheduling problem of this case study, the model should first be created using THPN.

### 3.1 Model of the case Study

The THPN model of the sugar mill system is depicted in Figure 3. In this model, it is assumed that there is a possible change of flowrates for the continuous processing units every half hour. While discrete transitions are used to show the batch operations, continuous units are represented by continuous transitions. In addition, discrete places between the batch operations show the availability of the materials in the previous batch processing unit and the continuous places illustrate the storages. The major benefit of the proposed model is the formulation part of the problem, which is much less complicated than traditional methods. Applying THPN for modelling and scheduling of hybrid systems, the formulation of the important constraints such as assignment constraints, material balance constraints and resource constraints will become much simpler as discussed below:

- Assignment constraints: consideration of places having one token as an input to a discrete transition enforces the constraint that there should not be more than one task processed in a processing unit at any time.
- Material balances constraints: the amount of inputs and outputs of a processing unit are considered as weights located on arcs connecting a transition (place) to a place (transition).
- Resource constraints: for the storages with a limited capacity, a place is connected as an input to a transition (operation unit) producing the products to be stored in the associated storage. The arc connecting this place to the transition has a weight equal to the amount of the resulting products. The same amount is released as soon as the materials leave the associated processing unit.

It should be noted that all the above constraints have been considered in the firing rules of transitions and there is no need to set any time consuming formulations or variables.

The objective function of the system is to find a schedule with the maximum product (profit) or equivalently minimum idle time (A period during which a processing unit is not in use, but is available), within a horizon time of 16 hours. There is also a penalty of 0.1 for changes in the flowrates of continuous units.

#### **3.2 Proposed algorithm**

In the proposed approach, there are two steps. First, based on some initial values for the flowrates of continuous units, the scheduling algorithm will find the optimal schedule with respect to the objective function. Second, the optimal schedule obtained is passed to the CFSQP optimisation algorithm (Lawrence et al., 1997). CFSQP will find the best flowrates for each continuous unit, which maximises the products (sugar and Cmolasses) within the obtained optimal schedule. It should be noted that before applying the proposed algorithm, the horizon time is divided into half hourly periods. This will allow the change in the flowrate of each continuous unit every half hour. The following shows the steps of the scheduling algorithm:

**S1:** Begin with the initial marking and a large value for the lower\_idle\_time; set the flowrates of the continuous units to some initial values.

**S2:** With the initial marking, check if the current transition is not enabled due to the lack of materials or space in the storage, then consider the associated idle time.

**S3:** With the initial marking, check if the current transition is enabled; determine the new marking and the usage time of each processing unit including its idle time and put them all with the old markings into one result matrix.

S4: If there are more enabled transitions with the current initial marking, go to S2.

**S5:** Check the last two rows of the result matrix and perform merging if the related transitions are not in conflict; in case of conflict check if the shared input place has enough markings (weights) to fire these two transitions simultaneously. If so perform the merging and repeat **S5** for the next two rows.

**S6:** Check the time of all the batch processing units and if there is at least one with the time of less than the horizon time go to **S2**.

**S7:** If the latest total idle time of the batch processing units is less than the lower\_idle\_time update lower\_idle\_time to this time.

**S8:** If all the rows of the result matrix have been assessed, the search is complete. Output the lower\_idle\_time and the feasible schedule to this lower\_idle\_time.

**S9:** Start checking from the last row of the result matrix upward and find the row in which the associated transition is enabled and has not been fired yet; set the marking of this row as the initial marking and go to S2.

The scheduling algorithm was implemented in the C programming language and the optimal solution yields the Gantt chart shown in Figure 4. As was mentioned previously, based on the optimal schedule, CFSQP gives the best flowrate per half hour for each continuous processing unit (Table 2).

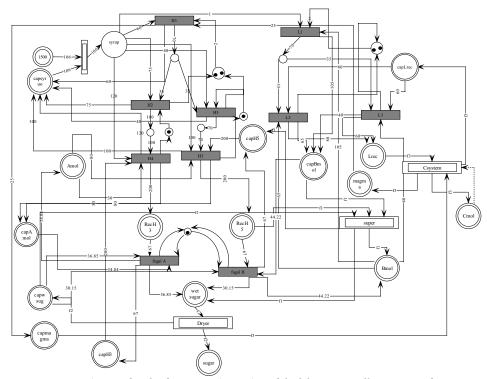


Figure 3. Timed Hybrid Petri net (THPN) model of the sugar milling case study

Applying the proposed method to solve the sugar mill case study reduces the computational time from 140 seconds in the previous method (Nott, 1998), which used a sets implementation technique, to 1 second in the current study. This confirms the power of THPN for modelling and scheduling of hybrid systems.

Units	0-2 hours	2-4 hours	4-6 hours	6-8 hours	8-10 hours	10-12 hours	12-14 hours	14-16 hours
Pan H1	1111		11111	11111			11111	11111
Pan H2								
Pan H4								
Pan H3								
Pan H5							~ ~ ~ ~ ~	~ ~ ~ ~ ~
AFugal								
BFugal								
Pan L1								
Pan L2								
Pan L3								

Figure 4. Gantt chart of sugar milling case study

Table 2. Optimal flowrates (tonnes/half hour)

Time	fl (super)	f2 (dryer)	f3 (Csystem)
0-2 hours	2.04375	7.0125	2.16875
2-14 hours	2.06875	7.0375	2.19375
14-16 hours	2.04375	7.0125	2.16875

### 4. Conclusions

The THPN has been introduced as a suitable tool for systems with both discrete and continuous behaviour in which many of the constraints can be shown graphically. The great potential of this model for handling complicated operations in the mixed batch/continuous processes has been illustrated through a sugar milling case study, which is an example of a hybrid system. An algorithm, which is a mixture of scheduling and optimisation, is proposed to solve the scheduling problem of the case study. A substantial reduction in the computation time is achieved thereof. Division of the horizon time into a smaller period is currently being researched.

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