European Symposium on Computer Aided Process Engineering – 15
L. Puigjaner and A. Espuña (Editors)

A Multiple Model, State Feedback Strategy for Robust Control of Non-linear Processes

F. Y. Wang [a*], P. Bahri [b], P. L. Lee [c] and I. T. Cameron [a]

[a] Division of Chemical Engineering, The University of Queensland
Brisbane, Qld 4072, Australia
[b] School of Engineering, Murdoch University, Rockingham Campus
Murdoch, WA 6150, Australia
[c] Division of Engineering, Science and Computing, Curtin University of Technology
GPO Box U1987, WA 6845, Australia

Abstract
In order to achieve global stability using well-established linear control theory and techniques, a multiple model approach has attracted increased attention in recent years. In our previous work, a mini-max optimisation strategy was developed within the framework of a multiple model approach, in which a global controller can be designed without the requirement of membership/validity functions used in conventional methods, and the regime division was realised using a gap metric method. The major limitation of the reported methods is that robustness against process/controller disturbances cannot be addressed if the process switches from stable to unstable in operation. Furthermore, the number of local models is still large for highly non-linear processes even though the gap-metric method is incorporated. In this paper, a significantly modified multiple model approach is developed to achieve robust control with global stability. The main new features of the current approach include: (1) stabilization of open-loop unstable plants using a state feedback strategy, (2) incorporation of an adjustable pre-filter to achieve offset-free control, and (3) implementation of a Kalman filter for state estimation where necessary. The improved controller design method is successfully applied to two non-linear processes with different chaotic behaviour, namely a continuous stirred tank reactor and a *Zymomonas mobilis* reactor. Compared with conventional methods without model modifications, the new approach has achieved significant improvement in control performance and robustness with dramatically reduced number of local models.

Keywords: Chaos; Localised models; Mini-max optimisation; Model modification; Robust control

[*] Author to whom correspondence should be address: fuyang@cheque.uq.edu.au

1. Introduction

Most chemical processes are non-linear in nature. However, for effective control and operation, low dimensional, linear models are highly desirable. It is not always possible to represent a non-linear process by a single linear model. Consequently, a multiple model approach has attracted increased attention in recent years (Murray-Smith and Johansen, 1997). In the conventional multiple model approach, a complex, non-linear model is reduced to a set of localised, linear sub-models. The overall model is the weighted combination of the local models (Shorten et al., 1999). However, it is not always easy to determine the so-called membership/validity functions used in the reconstruction of the overall model, and to effectively divide the operational regimes. In an attempt to eliminate the difficulties involved in the determination of blended models, Bartholomaus (2002) developed a mini-max optimisation strategy, in which a global controller can be designed without the requirement of membership/validity functions. However, the method leads to a large number of local models due to the lack of an effective strategy for regime division. In order to reduce the number of local models, we have incorporated the gap metric method into mini-max optimisation algorithms (Wang et al., 2003). In spite of the advances witnessed in the field, a major limitation of the reported methods can be identified as: "It is very difficult to achieve desired robustness properties if the process consists of open-loop unstable regimes". It can be shown in the case studies carried out in this paper that, although acceptable control performance was obtained using reported methods for a class of chaotic processes (Morningred et al., 1990; Bartholomaus, 2002; Wang et al., 2003), the robustness criteria have not been reached. This can be demonstrated easily through the observation of chaotic dynamics in the stabilized systems with slightly disturbed controller gains. If the robust issues cannot be effectively addressed, the reported methods have little practical significance for unstable processes. Furthermore, the number of local models is still large for highly non-linear processes.

In this paper, a significantly modified multiple model approach is developed to achieve robust control with global stability. In the proposed approach, the original open loop unstable plants are first stabilized using a state feedback strategy followed by the local linearization within a regime classified by a gap metric measure. The smooth transition between regimes, as well as offset-free control can be assured through the incorporation of an adjustable pre-filter in the multiple model control framework. If the feedback states are not measurable, a Kalman filter is implemented for the state estimation. Two case studies, namely robust control of a continuous stirred tank reactor and a *Zymomonas mobilis* reactor, are carried out to demonstrate the advantages of the proposed approach over conventional ones. It can be shown that the chaotic dynamics are under robust control with a dramatically reduced number of local models.

2. Multiple Model, State Feedback Strategy

The proposed multiple model, state feedback control strategy is schematically shown in Figure 1, in which the left part is the mini-max optimiser, and the right part is the non-linear system.

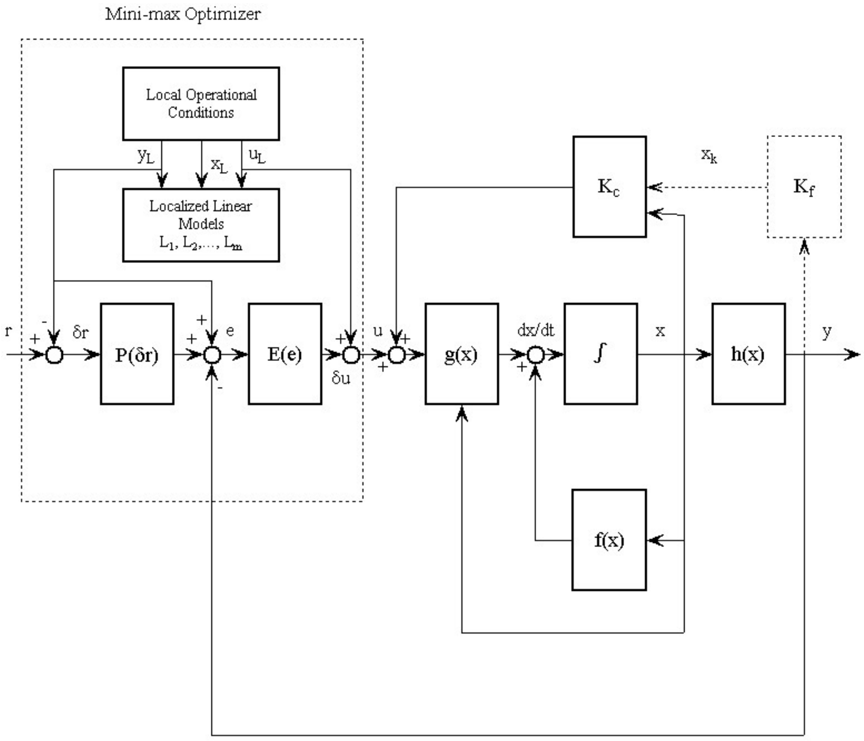

Figure 1. Schematic Diagram of Multiple Model Approach with State Feedback

In Figure 1, E is the global linear controller, K_c and K_f are the state feedback and the Kalman filter matrix, respectively, and P is the pre-filter with adjustable gains. The non-linear model is described by:

$$\dot{x} = f(x) + g(x)u \; ; \; y = h(x) \tag{1}$$

where x, y, and u are state, output and control variables, respectively. The non-linear model can be linearised into a set of local linear models L_i in m regimes identified by index i (i = 1, …, m) with state space matrices $\{A_i, B_i, C_i\}$ and transfer function matrix $G_i(s)$. It should be noted that the relationship between matrix A_i with and without state feedback is: $A_i = (A_i^0 + B_i K_c)$, where A_i^0 is obtained before state feedback. The key issue of the control study is the optimal determination of parameters in the global controller E and pre-filter P using mini-max optimisation algorithms. The gap metric method is used as a measure for regime classifications, which was clearly described in Samyudia et al. (1996).

3. Optimal Design of Controller and Pre-filter

The optimal controller parameters are determined using mini-max optimisation algorithms. The controller E(Q) consists of n parameters $q_j \in Q \subseteq R^n$, The local objective function in H-infinity norm format can be formulated as:

$$J_i = \left\| \begin{matrix} W_u E(Q)\left(I + G_i E(Q)\right)^{-1} \\ W_e\left(I + G_i E(Q)\right)^{-1} \end{matrix} \right\|_\infty \tag{2}$$

where i = 0, 1, …, m with i = 0 as the starting operational point, J is the objective function, W is the weighting function matrix, the subscripts u and e identify control and error relevant functions, respectively. In principle, the mini-max optimisation can be formulated as **min max** $\{J_0,\ldots,J_m\}$ with respect to Q. Since it is very difficult to determine the starting point for optimisation (Bartholomaus, 2002), a sequential mini-max optimisation algorithm is proposed in this paper, which is represented as follows:

$$\textbf{min}\ \{J_0\} \xrightarrow{Q_0} \textbf{min max}\{J_0, J_1\} \cdots \xrightarrow{Q_{m-1}} \textbf{min max}\{J_0, \cdots, J_m\} \rightarrow Q \tag{3}$$

The overall parameter vector Q is applicable to all regimes rather than a single regime.

The following equation is used to determine the pre-filter gain for SISO systems:

$$\lim_{s\to 0}\left[P_i(s)(1 - G_i(s)E(s))^{-1} G_i(s)E(s)\right] = 1 \tag{4}$$

4. Case Studies

A continuous stirred tank reactor model originally proposed by Morningred et al. (1990) and further analysed by Bartholomaus (2002) is selected as the first case study. The model is represented as:

$$\begin{aligned} \frac{dC_a}{dt} &= \frac{q}{V}\left(C_{a0} - C_a\right) - k_0 C_a\, \mathbf{exp}(-E' / RT) \\ \frac{dT}{dt} &= \frac{q}{V}\left(T_0 - T\right) + k_1 C_a\, \mathbf{exp}(-E' / RT) + k_2 q_c\left[1 - \mathbf{exp}\left(-k_3 / q_c\right)\right]\left(T_{c0} - T\right) \end{aligned} \tag{5}$$

The nominal values of model parameters are available from Morningred et al. (1990). We treat $k_2q_c[1-\exp(-k_3/q_c)]$ together as the control variable u. The control objective is to drive the concentration C_a from the initial operating point $C_a = 0.06$ to the final operating point $C_a = 0.16$ along a specified staged trajectory by adjusting the coolant flow rate q_c. The process is open loop unstable with multiple steady states. The conventional (Bartholomaus, 2002; Wang et al., 2003) and improved performances are shown in Figure 2. The system becomes open loop unstable as $C_a > 0.140$. It can be shown from Figure 2a that the conventional control leads to notable deviations from desired trajectory in the unstable regime. The most unacceptable fact is that when the conventional controller gain increases 10%, chaotic dynamics appear as shown in Figure 2b. This implies that the conventional controllers are of little practical significance due to the robustness concern. The newly developed control scheme allows a broad range of controller gain variations. The control variable profile and its deviations from steady state are depicted in Figures 2c and 2d, and this is easy to achieve. The number of local models is reduced from 10 (Wang et al., 2003) to 5 using the proposed approach. Previous work by Bartholomaus (2002) suggests many more than 10 were used.

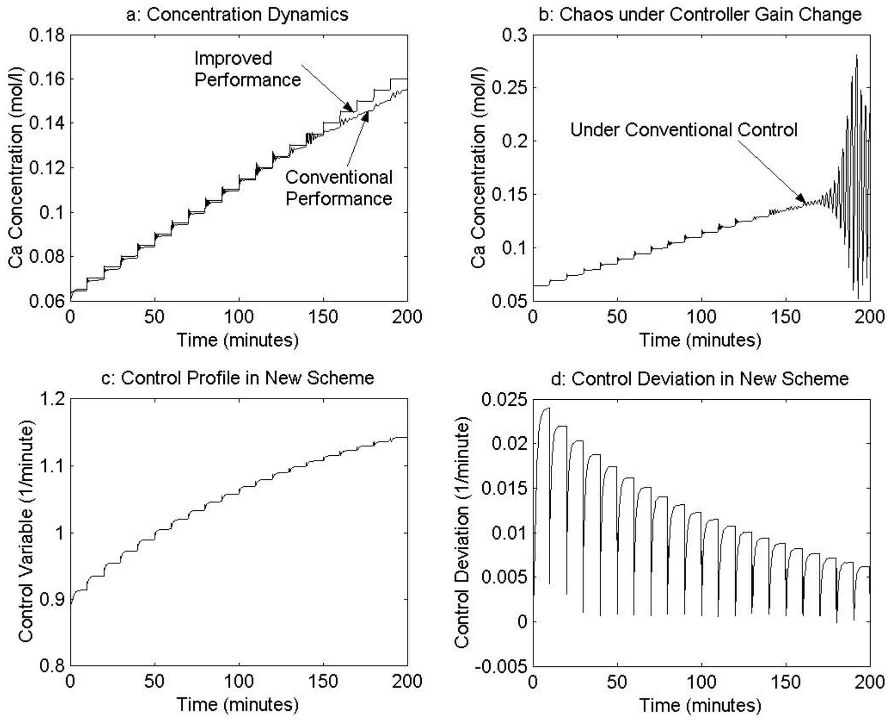

Figure 2. Control of a chaotic CSTR

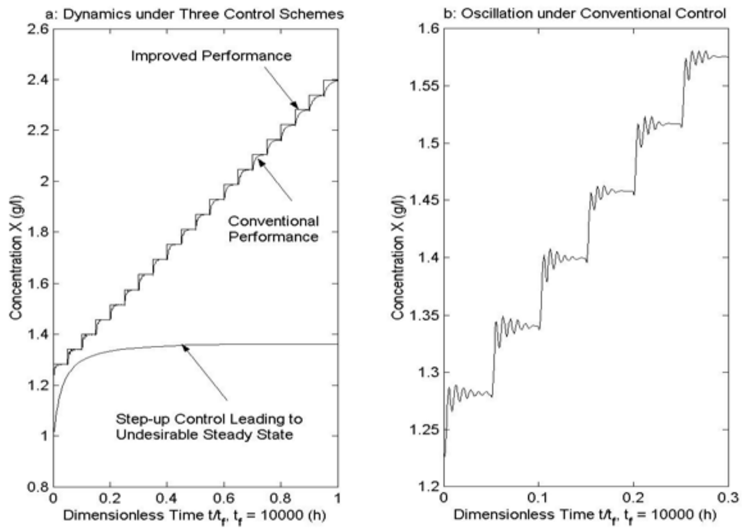

Figure 3. Control of ZMC with bifurcation behaviour

The second case study is a *Zymomonas mobilis* reactor. Its model was fully described by McLellan et al (1999) and further analysed by Zhang and Henson (2001). The model consists of 5 state variables. We choose biomass X as the output and dilution rate D as the manipulative variable. The conventional and modified performances can be seen in Figure 3. Figure 3a shows the performance with three different control schemes with an indication of multiple steady states. Figure 3b shows the oscillatory behaviour using conventional control schemes without state feedback. Similar to the first case study, oscillations become severe with a slightly disturbed controller gain. Both performance and robustness have been improved significantly using the proposed control scheme. Three local models are sufficient for effective control of this process.

For both processes, the controller format is: $(q_1s^2+q_2s+q_{3)/(}q_4s^2+q_5s+1)$, and the pre-filter equation is: $p_i/(s+1)$, where q_1-q_5 are determined through mini-max optimisation, and the regime dependent parameter p_i is computable using Equation (4).

5. Conclusions

Through the theoretical development and simulation studies on control of two non-linear processes with chaotic dynamics, the following conclusions can be drawn:

1. Although a class of non-linear processes with chaotic dynamics can be stabilised using conventional control schemes, this work has shown that robustness is the main issue preventing the industrial application of the reported methods.
2. State feedback for pole placement is an effective strategy amenable within the framework of the multiple model approach, leading to significantly improved performance and robustness with a dramatically reduced number of local models.
3. The mini-max optimisation techniques enable the design of a global controller without relying on membership and validity functions. An integration of mini-max optimisation, pre-filter design, state estimation using Kalman filter and state feedback leads to the development of robust, offset free control systems for non-linear, unstable processes.

Reference

Balas, G.J., J. C. Doyle, K. Glover, A. Packard and R. Smith, 1995, μ-Analysis and Synthesis Toolbox For Use with MATLAB, The Math Works, Natick.

Bartholomaus, R., 2002, Contr. Eng. Practice, 10, 941.

McLellan, P.J., A.J. Daugulis J. and J. Li, 1999, Biotechnol. Prog., 15, 667.

Morningred, J.D., B. E. Paden, D. E. Seborg and D. A. Mellichamp, 1990, Proc. ACC, 1614.

Murray-Smith, R and T. A. Johansen Eds., 1997, Multiple Model Approaches to Modelling and Control, Taylor and Frances, London.

Samyudia, Y., P. L. Lee, I. T. Cameron and M. Green, 1996, Comput. Chem. Eng. 20, S919.

Shorten, R., R. Murray-Smith, R. Bjorgan and H. Gollee, 1999, Int. J. Control, 72, 620.S.

Wang, F.Y., P. Bahri, P.L. Lee and I.T. Cameron, 2003, Proc. PSE 2003, 1065.

Zhang, Y. and M.A. Henson, 2001, Biotechnol. Prog. 17, 647.

Acknowledgements

The authors would like to acknowledge the financial support from the Australian Research Council (ARC) through a Large Grant Scheme for project A10030015.