17th European Symposium on Computer Aided Process Engineering – ESCAPE17
V. Plesu and P.S. Agachi (Editors)

Weighted Residuals Method Applied to the Filament Winding Process

Rogério Luz Pagano,[a] Evaristo Chalbaud Biscaia Jr,[a] Verônica Calado[b]

[a]*Programa de Engenharia Química, COPPE, Universidade Federal do Rio de Janeiro, Rio de janeiro, Brasil, rpagano@peq.coppe.ufrj.br., evaristo@peq.coppe.ufrj.br.*
[b]*Escola de Química, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brasil, calado@eq.ufrj.br*

Abstract

Simultaneous process of filament winding and curing a reinforced polymer is widely used in composite industries. This process is normally used for manufacturing cylindrical structures of the composite materials, which have been used in many fields including aerospace and chemical industries, sports and military. The process consists in winding fibres passing through a resin bath on a mandrel at room temperature so that the resin is crosslinked via chemical reaction. In the chemical step, physical properties of the composite material obtained depend on temperature and cure behaviour; therefore, modelling and simulation of this process are important to investigate the quality of the final process product. The numerical solution of the mathematical model of the filament winding process is the main objective of this contribution. In this work, the modelling consists of the mass and thermal balances by using the autocatalytic model for describing the kinetic reaction rate. The model equation system was solved through the method of lines applying the weighted residuals method to discretize the radial direction and integrating the resulted differential-algebraic system using the DASSL code. The temperature and degree of cure behaviours were obtained and validated with experimental data. A good agreement between experimental data and simulated results was verified associated with low computational cost. Based on simulated results, the process variables can be optimised aiming to the quality of final product.

Keywords: Composite structures; Filament winding; Mathematical models; Weighted residuals method; Differential-algebraic systems; Curing reactions.

1. Introduction

Polymer composites are materials consisting of a polymer matrix and reinforcement. Polymer matrix, or resin, changes its physical and chemical properties during a cure reaction, when resin is converted into a three-dimensional cross-linked thermo set network. Cured resins have good thermal, electrical and mechanical properties, but they are brittle and have poor resistance to crack propagation. Due to this fact, the reinforcement is used for improve the resistance of the composite to break. The reinforcement in composite materials can be the continuous or discontinuous fibres; nevertheless, the first is generally used to filament winding process. The continuous fibres used are glass, carbon, synthetic organic, and silicon carbide fibres. The ceramic or metallic fibres can be used in high-temperature applications [1]. The cure process in composite polymers releases a significant heat of reaction during processing and has been studied in several papers [2-4].

To manufacture composite cylinders, the filament winding process has been presented as a competitive method. In this process, a continuous fibre tow impregnated by a resin is wound and deposited onto a rotating mandrel. The resin bath and winding head move parallel to the cylinder axis. The head speed and the mandrel angular speed determine the fibre angle.

There are three basic types of winding patterns used to produce filament winding products: polar, helical, and hoop [5]. Polar winding is used to lay down fibre close to 0º to the longitudinal axes, unlike hoop winding that uses an angle close to 90º. In helical winding, the fibres are wound with angles from 5º to 80º. The advantage to filament winding is its low cost, the elimination of autoclave, and high processing speed [1]. Some disadvantages of this technique are related to an appropriate outer contours, and to the mandrel removal from a closed-end winding that is difficult or even impossible [5].

The most significant process variables are mandrel speed, temperature and degree of cure. The knowledge of a temperature profile is very important for the degree of cure because of the exothermic nature of the reaction. The process modelling includes both thermal and mass balances. This system has been solved by using neural network [6], finite differences [7,8], finite element [9,10] and finite volumes [11,12]. The neural network [6] predicted the thermal behaviour of composite tubes during a cure step with network parameter fit. A neural network was trained with experimental data and was validated with results from literature. The good agreement was achieved by using this strategy, but this model is a closed package. Finite element method (FEM) [9] was successfully applied in order to solve a fibre consolidation/compaction model coupled with a thermochemical model of the resin. Finite volume method has

been used to simulate on-line curing of thermoset composites in filament winding [12], where the resin/resin is locally heated by radiative heater, when it wound onto the mandrel.

Other approach can be done with the well-known weighted residuals method, whose main characteristic is to eliminate weighted integral residuals. Orthogonal collocation, moments and Galerkin are methods used as weighted residuals methods [13]. Among these methods, orthogonal collocation has been widely applied to solve chemical engineering problems. In this method, N interior collocation points are the roots of orthogonal Jacobi polynomial of Nth degree. The number of collocation points needed is usually much smaller than in other methods.

The main purpose of this paper is to solve the equation system of a filament winding process through the method of lines, applying a method of weighted residuals (MWR) to discretize the radial direction and integrating the resulted differential-algebraic system through DASSL code.

2. Modelling Approach

The mathematical model of filament winding process consists simply of the energy and mass conservation equations. In this model, only thermal gradients through the thickness have been considered. Due to exothermic nature, the energy release during the cure is included in the energy balance. The mass balance and composite and mandrel energy balances are given by Eqs. (1)-(3), respectively.

$$\frac{\partial \alpha(t,r)}{\partial t} = Da\,\Psi(\alpha,\Theta) \quad (1) \qquad \frac{\partial \Theta(t,r)}{\partial t} = \kappa \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \Theta(t,r)}{\partial r} \right) + \phi_R\, DaB\Psi(\alpha,\Theta) \quad (2)$$

$$\frac{\partial \Theta_m(t,r)}{\partial t} = \kappa_m \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r \frac{\partial \Theta_m(t,r)}{\partial r} \right) \right] \quad (3) \qquad \text{for } t > 0 \text{ and } r_0 > r > 1.$$

The degree of cure is α, the dimensionless temperature is Θ (defined below), the dimensionless radial directions is r, the dimensionless time is t, the fibre volume fraction is ϕ_R, and the Damköhler number is Da. κ, κ_m and B are defined by the following equations:

$$\kappa = \frac{k_r \tau}{\rho c_P R_{EXT}^2} \qquad \kappa_m = \frac{k_m \tau}{\rho_m c_{Pm} R_{INT}^2} \qquad B = \frac{\rho(-\Delta H_R)\sigma_1}{\rho c_P T_0} \qquad \Theta = \left(\frac{T - T_0}{T_0} \right)\sigma_1$$

Physical constants are defined in Table 1. The cure kinetic rate, $\Psi(\alpha,\Theta)$, was described by an auto catalytic model [14]:

$$\Psi(\alpha,\Theta)=\left[e^{\frac{\Theta\sigma_1}{\sigma_1+\Theta}}+\alpha^{m}\xi e^{\frac{\Theta\sigma_2}{\sigma_1+\Theta}}\right](1-\alpha)^{n}$$

The initial and boundary conditions for the previous process are as following:

$$\alpha(0,r)=\alpha_0,\ \Theta(0,r)=0,\ \ \Theta_m(0,r)=0,\ \left.\frac{\partial\Theta}{\partial r}\right|_{r=1}=-Nu\left(\Theta(t,1)-\Theta_\infty\right),$$

$$\left.\frac{\partial\Theta}{\partial r}\right|_{r=r_{INT}}=\gamma\left.\frac{\partial\Theta_m}{\partial r}\right|_{r=1},\quad \Theta_m(t,1)=\Theta(t,r_{INT}),\quad \left.\frac{\partial\Theta_m}{\partial r}\right|_{r=r_0}=-Nu_m\left(\Theta(t,r_0)-\Theta_{INT}\right)$$

Where $\gamma=\frac{k_m}{k_R}\frac{R_{EXT}}{R_{INT}}$ $\qquad Nu=\frac{h_\infty R_{EXT}}{k_R}$ $\qquad Nu_m=\frac{h_\infty R_{INT}}{k_m}$

Table 1. Physical constants used in filament winding model. The subscript "r" refers to resin and "m" refers to the mandrel. Parameters without subscript refer to composite.

k, thermal conductivity	kJ m^{-1} K^{-1}	h_∞, convective coefficient	kJ m^{-2} K^{-1}
ρ, specific mass	kg m^{-3}	ΔH_R, reaction enthalpy	kJ $kmol^{-1}$
c_P, specific heat	kJ kg^{-1} K^{-1}	T_0, reference temperature	K
R_{EXT}, composite external radius	m	R_{INT}, composite internal radius	m
R_0, mandrel internal radius	m	r, dimensionless radius	-
σ_i, dimensionless activation energy	-	ξ, ratio frequency factor	-

The internal mandrel temperature, Θ_{INT}, was obtained by solving the differential equation below. This equation expresses the mass balance in the inner mandrel coupled with the filament winding model through the boundary condition at $r = r_0$.

$\frac{\partial\Theta_{INT}}{\partial t}=\eta\left(\Theta_w-\Theta_{INT}\right),\ \Theta_{INT}=0$, where η is a parameter to be fit.

3. Results and Discussions

The method of lines was implemented in a FORTRAN code. In all examples, six collocation points were considered. First, the results were compared to those from a thin cylinder model. The physical properties and conditions of simulation were established in [15].

Due to very small cylinder width, the composite temperature profile can be considered as completely flat and cannot be used to compare the performance of the proposed method with former ones. Nevertheless, Fig 1 shows that similar

simulated degree of cure time profile was obtained using the proposed MWR method with a much smaller computational effort in comparison with the finite difference method used in [15].

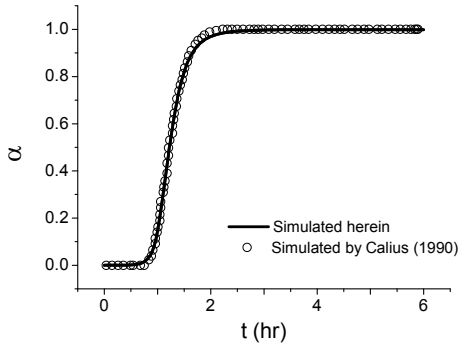

Figure 1. Degree of cure calculated by Calius, 1990 [15] at r = 0.5.

In order to realize other validations, experimental data were retrieved from [7] and the system reported elsewhere as F-4 was chosen for this purpose. This system was made of Fiberite prepeg tows wound onto aluminium mandrel. The composite wall thickness was 1.125 in. Temperatures at the inner mandrel and outer composite surfaces cylinders were measured using thermocouples. In this present contribution, more realistic boundary conditions were considered, by calculating heat fluxes in both sides of the equipment, in terms of temperature differences. The temporal variations of bulk air temperature, $\Theta\infty$ (dotted line), measured and simulated internal composite temperature are showed in Fig.2.

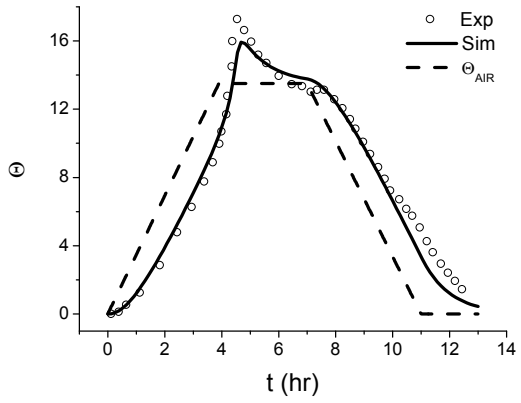

Figure 2. Experimental and simulated data during cure at r = 0.00 for a cylinder F-4 in [7].

The parameters η, Nu and Num were calculated to fit the measured temperature profiles. It must be mentioned that simulated results properly predicted the verified temperature overshoot.

4. Conclusions and Future Work

The method of weighted residuals was successfully used to simulate the filament winding process with a very low computational cost. Good agreement between the experimental data and simulated results was verified considering more realistic boundary conditions in comparison with meaningless boundary conditions adopted in former contributions [7] and [15]. New experimental data, temperature and degree of cure in a real pilot plant, are being collected and will be presented in a future work; new boundary conditions will be explored as well.

Acknowledgements

Authors would like to thank CAPES, CTPETRO/FINEP (# 0104081500) and CNPq for financial support.

References

1. J. Mackerle, Computational Materials Science, 31 (2004) 187-219.
2. D. Rosu, C. N. Cascaval, F. Mustata, and C. Ciobanu, Thermochimica Acta, 383 (2002) 119-127.
3. J. Lopez, I. Lopez-Bueno, P. Nogueira, C. Ramirez, M. J. Abad, L. Barral, and J. Cano, Polymer, 42 (2001) 1669-1677.
4. R. Seifi and M. Hojjati, Journal of Composite Materials, 39 (2005) 1027-1039.
5. F. C. Shen, Materials Chemistry and Physics, 42 (1995) 96-100.
6. V. M. A. Calado, S. Constant, and L. M. F. Lona, Polímeros: Ciência e Tecnologia, 14 (2004) 295-300.
7. E. P. Calius, S. Y. Lee, and G. S. Springer, Journal of Composite Materials, 24 (1990) 1299-1343.
8. S. Y. Lee and G. S. Springer, Journal of Composite Materials, 24 (1990) 1270-1298.
9. L. Y. Zhao, S. C. Mantell, D. Cohen, and R. McPeak, Composite Structures, 52 (2001) 499-510.
10. H. C. Park, N. S. Goo, K. J. Min, and K. J. Yoon, Composite Structures, 62 (2003) 51-57.
11. X. F. Wang and D. Y. S. Lou, International Journal of Energy Research, 27 (2003) 377-388.
12. X. F. Wang, D. Y. S. Lou, and N. L. Zhang, International Journal of Heat and Mass Transfer, 47 (2004) 4807-4820.
13. R. G. Rice and D. D. Do. Applied Mathematics and Modeling for Chemical Engineers. - 720. 1995. John Wiley & Sons.
14. M. R. Kamal and S. Sourour, Polymer Engineering and Science, 13 (1973) 59-64.
15. E. P. Calius and G. S. Springer, International Journal of Solids and Structures, 26 (1990) 271-297.