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A Model of Grinding-Classification Circuit Including Particles Size Distribution and Liberation of Material: Application to the Design and Retrofit of Flotation Circuit

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Abstract

In this study models were developed based on population balances which included the effects of redistribution of material of different composition classes and particle sizes with four configuration alternatives of grinding and classification circuits (grinding, grinding-classification, classification-grinding, classification-grinding-classification). The models for all these configurations were linear in character, and applicable in design or retrofit strategies in which there is an influence of particle size and composition in the recovery of materials such as in separation of plastics from metals and the recovery of ferromagnetic materials in the recycling industry, as well as in the separation of materials in treatments of urban solid wastes, refining of pharmaceutical products, and in the mineral industry. As an example of application, one of the models is incorporated into a strategy for the design of flotation circuits.

Keywords: particle separation, grinding, flotation, minerals, process design

1. Introduction

Particle size and chemical composition of particles are two important properties of materials which must be considered in the design and operation of various types of industrial processes in which reduction of particle size has an important role, from the food industry to pharmaceuticals, and from the mineral industry to chemicals and waste recycling [1,2,3,4,5].

The flow of particulate material entering a process is defined by both its granulometry and composition. These properties influence processes in which solid materials become separated out, such as in the waste recycling and mining industries. In mineral industries, grinding and classification steps in recovery circuits contribute to improvement of yields in terms of product grade values and the recovery of concentrate.

In general, grinding models are used in process design to quantify the effect of the redistribution of the mineral composition, without considering particle size [6,7]. The primary objective of the present study is the development of mass balances based on population balances which incorporate the effect of redistribution of the chemical composition of particles and the change in particle size. These models are adaptable for use in design or retrofit procedures.

2. Grinding model considering particle size and chemical composition

The comminution of particles includes the observation of two simultaneous phenomena, including (1) overall reduction of the material size and (2) release of material enclosed within the original particles. Both the granulometrical distribution and the composition can be described by a number of finite classes which include range of particle size and of compositions. That is, the grinding can be modeled as a process where populations of materials are redistributed under restrictions of material balance.

Let us consider, for example, a material which can be classified into classes k of composition, and j of particle size, which belong to the set K = (k/k is the mineralogical species) and J = (j/j is the size or range of particle size). Fig. 1 shows that the criterion of redistribution of material is that the particles that belong to larger size classes are able to shed material to the smaller size classes, and receive material from larger size classes. These larger size classes may also conserve part of their material within their class and lose material to other classes of the same size except having a different composition. The auxiliary sets used in modeling this condition can be defined as J1 = {(j1,j) / j1 - the range or size greater than j, j1>j, (j1,j) $\in$ J} and J2 = {(j,j2) / j2 - the size or range less than j, j2<j, (j,j2) $\in$ J}. Fig. 1 also establishes that there exists a distribution based on the composition, which leads to a classification by composition K1 = {(k1,k)/ k1 - original composition, and k is the final, k1,k, (k1,k) $\in$ K} and K2 = {(k,k2)/ k - the original composition, and k2 is the final, k,k2, (k,k2) $\in$ K}. That is, these sets establish a certain direction in the composition of the particle and serve as a basis for the development of a population balance fitting the problem.

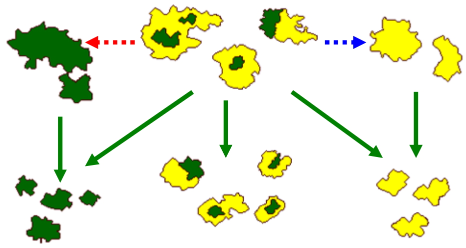

Figure 1. Scheme for the redistribution of composition and size classes in grinding.

To quantify the effect of grinding on the population balance, the parameter ϕ is introduced, which acts as a transference function between material which belongs to the original class and that of a given final class. Supposing a particle occurs in intermediate classes (k,j) $\in$ (K,J), the arrival of material from one class (k1,j1) $\in$ (K1,J1) can be written as $m_{k,j}^{(M)} = \phi_{k_1,j_1}^{k,j} m_{k_1,j_1}^{(M)}$ where $m^{(M)}$ is the mass of material in a respective class while the loss of material from class (k,j) to class (k2,j2) $\in$ (K2,J2) is $m_{k_2,j_2}^{(M)} = \phi_{k,j}^{k_2,j_2} m_{k,j}^{(M)}$. Thus, ϕ can be seen as a fitting parameter to guarantee both overall and by-composition mass balances, since the sum over all parameters ϕ from (k,j) to (k2,j2) $\forall$ (k2,j2) $\in$ (K2,J2) is 1.

3. Grinding-classification circuits

Fig. 2 shows the different grinding – classification circuits considered. In the case of grinding without classification shown in Fig. 2a, the balance of the population leads to a linear equation shown in (a) of Table 1, which describes the redistribution of the masses, assuming that all the particles remain for time τ^m within the mill, and that they depend on parameters $\phi_{k_1,j_1}^{k,j}$ and $\phi_{k,j}^{k_2,j_2}$, and fresh mass feed flows into the grinding circuit $W_{F,k,j}^{m}$ for all classes as input data, in a population balance at steady state. These conditions also hold for the other circuits. Mass flows can be obtained for the product of the circuit $W_{P,k,j}^{m}$ in recursive form, given that W_{P,k_1,j_1}^{m} has been previously calculated. For circuits (b), (c) and (d) in Fig. 2 we obtain models (b), (c) and (d), respectively in Table 1, with the input of the term $\varphi_{k,j}$ which is the typical value of the Tromp curve for class (k,j), which defines the fraction of material of a given size which passes to the coarse discharge of the hydrocyclones. The equations are also linear and recursive and retain a similar form, fulfilling with both overall and by-component mass balances.

In spite of the complex notation, the equations follow the same format of the first model, *i.e.* there is a constant multiplier with each fresh feed to the circuit and the grinding time together with other constants multiplying the sum which shows the import from the larger size classes.

4. Application example

The present study shows the use of the model given in Table 1(b) for the design of flotation circuits using a design strategy and equipment selection based on the work of Cisternas et al. [7,8] which allows for the selection of flotation equipment (mechanical cells, columnar cells) and the existence of grinding and classification circuits based on mixed integer linear programming (MILP) using a grinding model considering composition changes only, as previously stated.
The choice of flotation equipment makes it possible for a stream to be treated by banks of mechanical cells, or by a columnar cell, or by neither of these. On the other hand, the existence of grinding and classification circuits is considered in the cleaner and scavenger steps.

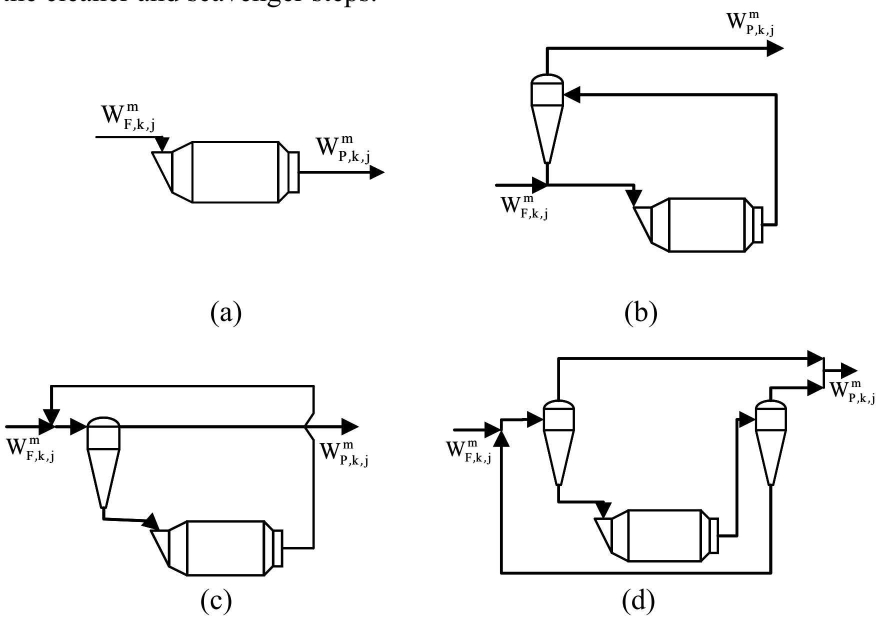

Figure 2. Options for grinding and classification circuits (a). Grinding without classification.(b). Grinding-classification (c) Classification-grinding (d) Classification-grinding-classification.

Since the grinding models are linear, these equations fit within the conditions required for establishing an MILP problem. The main supposition is that the hypothetical mineral is composed of three classes of composition (100% Calcopyrite, 50% Calcopyrite - 50% Quartz, and 100% quartz) and three particle size classes are maintained throughout the duration of the operation. The proposal for this design is to obtain a configuration which maximizes the difference between the sales benefits of the concentrate and the annualized costs associated with the process.
The strategy suggests use of the configuration presented in Fig. 3, where banks of mechanical cells are chosen for the primary scavenger and re-scavenger

steps, and columnar cells are chosen for the primary cleaner and re-cleaner steps. A grinding-classification step is included which treats the concentrates from the primary step and the scavenger, increasing the degree of liberation of the mineral although generating a large amount of fine material. The product of this circuit enters the treatment at the cleaner stage where it is remixed with the tailing of the re-cleaner prior to entering the primary cleaner, the tailing of which is returned to treatment by primary flotation.

Table 1. Linear models for each circuit shown in Figure 2

keys	Model
(a)	$W^m_{P,k,j} = \left(1-\tau^m \sum_{K_2}\sum_{J_2} \phi^{k_2,j_2}_{k,j}\right) W^m_{F,k,j} + \tau^m \sum_{K_1}\sum_{J_1} \phi^{k,j}_{k_1,j_1} W^m_{P,k_1,j_1}$
(b)	$W^m_{P,k,j} = \alpha_1(k,j) W^m_{F,k,j} + \alpha_2(k,j) \sum_{K_1}\sum_{J_1} \phi^{k,j}_{k_1,j_1} \left(W^m_{F,k_1,j_1} + \frac{\varphi_{k_1,j_1}}{1-\varphi_{k_1,j_1}} W^m_{P,k_1,j_1} \right)$ $\alpha_1(k,j) = \frac{1-\tau^m \sum_{K_2}\sum_{J_2} \phi^{k_2,j_2}_{k,j}}{1+\frac{\varphi_{k,j}}{1-\varphi_{k,j}} \tau^m \sum_{K_2}\sum_{J_2} \phi^{k_2,j_2}_{k,j}}$ $\alpha_2(k,j) = \frac{\tau^m}{1+\frac{\varphi_{k,j}}{1-\varphi_{k,j}} \tau^m \sum_{K_2}\sum_{J_2} \phi^{k_2,j_2}_{k,j}}$
(c)	$W^m_{P,k,j} = \beta_1(k,j) W^m_{F,k,j} + \beta_2(k,j) \sum_{K_1}\sum_{J_1} \phi^{k,j}_{k_1,j_1} \frac{\varphi_{k_1,j_1}}{1-\varphi_{k_1,j_1}} W^m_{P,k_1,j_1}$ $\beta_1(k,j) = \frac{\left(1-\varphi_{k,j}\right)}{1-\varphi_{k,j}\left(1-\tau^m \sum_{K_2}\sum_{J_2} \phi^{k_2,j_2}_{k,j}\right)}$ $\beta_2(k,j) = \frac{\left(1-\varphi_{k,j}\right)\tau^m}{1-\varphi_{k,j}\left(1-\tau^m \sum_{K_2}\sum_{J_2} \phi^{k_2,j_2}_{k,j}\right)}$
(d)	$W^m_{P,k,j} = \gamma_1(k,j) W^m_{F,k,j} +$ $\gamma_2(k,j) \sum_{K_1}\sum_{J_1} \left(\frac{\phi^{k,j}_{k_1,j_1} \varphi^{(1)}_{k_1,j_1} \varphi^{(2)}_{k_1,j_1}}{1-\varphi^{(1)}_{k_1,j_1} \varphi^{(2)}_{k_1,j_1}} W^m_{P,k_1,j_1} + \frac{\phi^{k,j}_{k_1,j_1} \varphi^{(1)}_{k_1,j_1} \left(1-\varphi^{(2)}_{k_1,j_1}\left(1-\varphi^{(1)}_{k_1,j_1}\right)\right)}{1-\varphi^{(1)}_{k_1,j_1} \varphi^{(2)}_{k_1,j_1}} W^m_{F,k_1,j_1} \right)$ $\gamma_1(k,j) = 1-\varphi^{(1)}_{k,j} + \frac{\varphi^{(1)}_{k,j}\left(1-\varphi^{(1)}_{k,j}\varphi^{(2)}_{k,j}\right)\left(1-\tau^m \sum_{K_2}\sum_{J_2}\left(\phi^{k_2,j_2}_{k,j}\right)\right)}{1-\varphi^{(1)}_{k,j}\varphi^{(2)}_{k,j}\left(1+\tau^m \sum_{K_2}\sum_{J_2}\left(\phi^{k_2,j_2}_{k,j}\right)\right)}$ $\gamma_2(k,j) = \frac{\tau^m\left(1-\varphi^{(1)}_{k,j}\varphi^{(2)}_{k,j}\right)}{1-\varphi^{(1)}_{k,j}\varphi^{(2)}_{k,j}\left(1+\tau^m \sum_{K_2}\sum_{J_2}\left(\phi^{k_2,j_2}_{k,j}\right)\right)}$

5. Conclusions

Different grinding-classification circuits can be modeled using linear equations which can be used in the design and retrofit of processes where particle size and composition are key factors for technical and economic yield, with the presence

of grinding-classification circuits being important. These models provide a population balance in which a redistribution of material into different particle size classes and composition is considered, as well as the influence of the classification equipment and their location.
One of the models was used in the design of flotation circuits based on the proposal of Cisternas et al. [7,8] based on MILP, which included a grinding-classification circuit as the optimal solution. The influence of the number of classes is challenging when considering the design of circuits required for use in comminution stages.

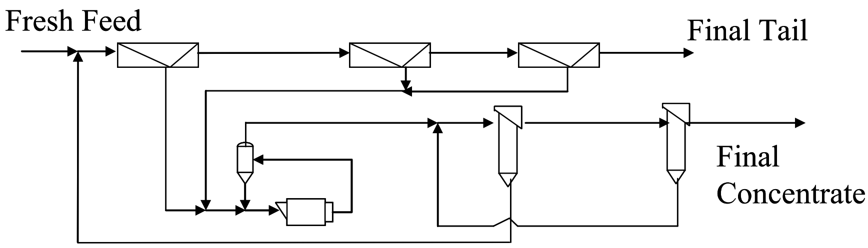

Figure 3. Resultant configuration from the application of the method for the grinding model.

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