

## **A joint control framework for supply chain planning**

José Miguel Laínez, Antonio Espuña and Luis Puigjaner

*Chemical Engineering Department, Univesitat Politècnica de Catalunya, Av. Diagonal 645, E08028, Barcelona, Spain. Email: luis.puigjaner@upc.edu*

### **Abstract**

The dynamics and uncertainty of the business and the market makes difficult to coordinate the activities of a supply chain (SC) and its echelons causing deviations from previous plans and schedules. Therefore, it is important to review systematically and to take into account the variability in the planning formulation in order to manage a plant or supply chain efficiently. These actions will permit to provide a more agile response to the market changes with more accurate decisions, improving by this means the overall performance of the business. In the literature, a control oriented decision framework has been proposed to carry out the review process. Specifically, model predictive control (MPC) is presented as a way to manage SC in the presence of uncertainty by incorporating the most recent information of the external market and internal business into the planning process. From another standpoint, an additional approach presented as a robust manner of decision making under uncertainty is solving the planning problem using stochastic optimisation. The main aim of this work is to analyse the consequences of implementing a MPC that comprises into the control algorithm a stochastic optimisation model. The potential of this approach is highlighted through a case study, in which the presented methodology is compared to the solely utilisation of MPC.

### **Keywords**

Supply chain planning, stochastic programming, and model predictive control.

## 1. Introduction

Supply Chain Management (SCM) is a challenging problem, since comprise the operations synchronization of a structure that includes several interconnected components (i.e., suppliers, manufacturing sites, distribution centres) in order to maximise financial returns while maintaining a competitive customer service. Nowadays to sustain a competitive customer service, the market is requiring environmentally friendly products, a good portfolio mixture, fast development of new products, high quality and reliability, after sales services, etc. Furthermore, SC managers also need to consider the dynamics of this fast-changing market environment, such as changes in demand, cancellations, returns; and the dynamic of SC internal operations, such as processing times, production capacity, availability of materials, among others. With the purpose of achieving their goal, SC managers need tools flexible enough to help them support the planning decision making process. These tools should be capable to account and to systematically review the uncertainty factors during the planning process. This work presents an approach to tackle this challenge.

## 2. Problem Statement and background

In literature, a well-known approach to overcome the problem of planning under uncertainty is MPC. MPC is a control strategy based on the explicit use of a process model to predict the process output over a long-range period of time<sup>1</sup>. The model attempts to predict the control variables for a set of time periods. Predicted control variables depend on a set of given parameters that are known in the control literature as control input. The MPC algorithm tries to optimise a performance criterion that is a function of the future control variables. By solving the optimising problem all elements of the control signal are defined. However, only a portion of the control signal is applied to the system. At next period of time, as new control input information is collected, the whole procedure is repeated as illustrated in figure 1.

On the other hand, Stochastic Programming (SP) is a framework for modelling optimisation problems that involve uncertainty. SP models assume that probability distributions governing the uncertain factors are known or can be estimated. The aforementioned fact and the computational effort required to obtain solutions are the main drawbacks of this approach. Its goal is to find a solution that is feasible for all the possible data scenarios and that maximises the expectation of a performance indicator. Discrete scenarios can be used to describe how uncertain parameters might play out in the future. The most widely applied SP models are two-stage programs. In this kind of models, the

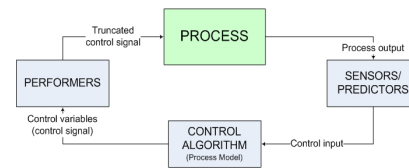


Figure 1: Simplified schematic of MPC

decision maker takes some action in the first stage, after which a random event occurs affecting the outcome of the first-stage decision. A recourse decision can then be made in the second stage that compensates for any bad effects that might have been experienced as a result of the first-stage decision.

The authors considered that a more robust tool to deal with planning/scheduling under uncertainty can be devised by merging the aforementioned two strategies and that is the aim of this work. With the recent advances in optimisation theory, software applications and hardware capacity, there is an appealing scenario to make this effort.

### 3. Paper approach

The present work proposes a general framework for SC planning/scheduling based on the inclusion of a two-stage stochastic MILP model within the control algorithm of a MPC. The control input considered under uncertainty is market demand. In order to predict demand and to have a support to generate the scenarios required by the SP model, a forecasting tool has been utilised.

#### 3.1. Joint framework strategy formulation

In figure 2, a general schematic of the joint control framework (JCF) proposed for SCM is shown. Next, each of the control strategy elements is briefly explained.

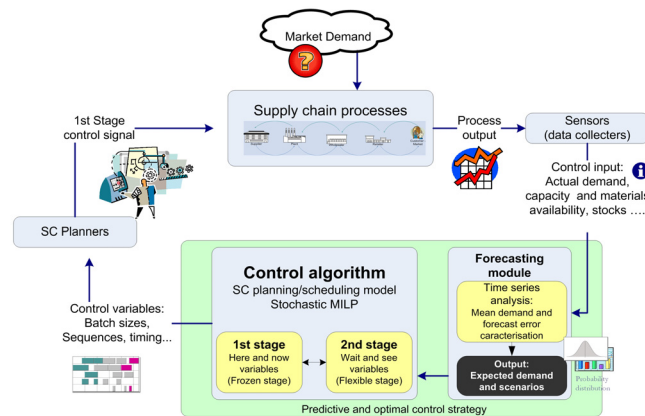


Figure 2: Schematic of joint MPC – stochastic optimisation proposed framework for SCM

##### 3.1.1. Forecasting module

The parameters that the proposed control strategy requires from a forecasting module are 1) estimates of demand mean ( $d(t)$ ) and 2) Forecast Error (FE) distributions.

The FE distributions depend on previous errors ( $e(t)$ ) and how many periods ahead the forecast is being done. If the correct forecasting model has been chosen, and if the statistical procedure used to estimate parameters in the model yields unbiased estimates, then the expected FE will be zero<sup>2</sup>. When the FE ( $\varepsilon$ ) is assumed to be normally distributed ( $N(0, \sigma_e)$ ),  $\sigma_e$  of the single-period ahead FE can be calculated by means of equation (1).

$$\sigma_e \approx 1.25 \frac{\sum |e(t)|}{N} \quad (1)$$

This approximation holds well even for non-normal errors<sup>2</sup>. For more details about FE estimation please refer to Montgomery and Johnson<sup>2</sup>. Finally, future market demand in period  $t$  ( $D(t)$ ) can be expressed as depicted in equation (2).

$$D(t) = d(t) + \varepsilon(t) \quad (2)$$

Therefore, a Monte Carlo sampling to generate  $\varepsilon(t)$  assuming that this parameter is governed by a  $N(0, \sigma_e)$  probability distribution and equation (2) can be utilised to create the demand scenarios needed in stochastic mathematical models.

### 3.1.2. Control algorithm

In the suggested strategy, the control algorithm corresponds to a stochastic SC planning/scheduling model. Currently, there is a big amount of deterministic and stochastic approaches available in the literature to model and optimise the SC planning. Here, the deterministic State-Task-Network (STN) based formulation presented by Guillén et al.<sup>3</sup> has been transformed into a stochastic formulation in order to be applied as the control algorithm. This formulation uses a time representation that allows addressing the integrated planning-scheduling of SC with embedded multipurpose batch chemical plants.

*The integrated planning-scheduling formulation.* The formulation is divided in two phases: detailed scheduling and production planning. In first phase, the detailed schedules of the sites of the SC as well as the transport decisions are optimised. At the production planning phase, neither the exact sequence of batches produced nor the initial and finishing times of the involved tasks are calculated within every period, but estimations by means of an aggregated STN representation are made. Three types of constraints are considered within both phases: the assignment, the mass balance and the capacity constraints.

The model proposed by Guillén et al.<sup>3</sup> divides the planning and scheduling horizon into periods of length  $H1$  where production is planned. The periods where detailed scheduling is carried out are divided into time intervals of lower length  $H2$  as shown in figure 3. Here, it has been defined detailed scheduling horizon equal to two planning periods to prevent problems in resources availability at subsequent detailed scheduling time intervals. This problem can

appear when using the aggregated model due to the approximations made by this formulation.

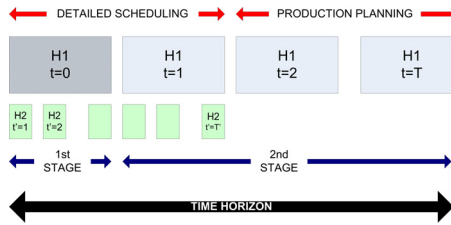


Figure 3: Model time representation

*The stochastic program.* The decisions and constraints of the system are classified into two sets. The first-stage variables, also known as ‘here and now’ variables, are determined prior to the resolution of the underlying demand uncertainty<sup>4</sup>. In the model proposed, the first stage variables are included in the detailed scheduling formulation as

shown in figure 3. The first stage variables are task assignment to units, batch sizes, materials and utilities purchases. As a contingent on these ‘here-and-now’ decisions and the realizations of the uncertain demand, the second-stage variables are determined to optimise in the face of uncertainty<sup>4</sup>. In this work, the second stage variables are included in both, the detailed scheduling and production planning formulation. Sales and inventories are second stage variables in the total detailed scheduling horizon ( $t=0$  y  $t=1$ ). Task assignment to units, batch sizes, materials and utilities purchases are second stage variables only in the last part of detailed scheduling horizon ( $t=1$ ). All variables included in the production planning formulation belong to the second stage.

*Control signal.* As shown in figure 2, just ‘here-and-now’ decisions resulting from the stochastic optimisation problem are applied to SC system. The whole strategy is repeated every planning period ( $H1$ ), as demand information is updated and treated by the forecasting module.

### 3.2. Case study

The capabilities of the proposed approach are highlighted by solving a problem of a SC comprising 3 DC, 6 markets and 2 plants that can manufacture 2 products. The structure of the SC has been taken from the case study presented by Guillén et al.<sup>3</sup> Please refer to this work so as to examine the STN representation of the SC. The detailed scheduling horizon has a length equal to 16 hours (2 days). The control strategy horizon is equal to 5 days and attempts to optimise the NPV. The control strategy is repeated every day. The implementation in GAMS of the integrated control algorithm model leads to a MILP model with 673,023 equations, 145,579 continuous variables, and 26,785 discrete variables. It takes an average of 343 CPU seconds to reach a solution with 5% integrality gap on an AMD Athlon 3000 computer using CPLEX.

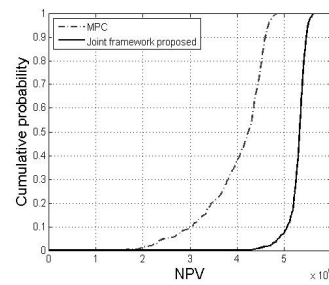


Figure 4: NPV cumulative probability distribution for first planning period

### 3.2.1. Results

Numerical results show that the proposed JCF is a more robust strategy. As illustrated by fig. 4, NPV variability due to demand uncertainty is reduced by applying the JCF. For instance, probability of achieving a NPV lower than 500,000 m.u. is shown to be merely 10% in the JCF case and 100% when carrying out MPC approach. Fig. 5 shows NPV behaviour of both control algorithms during fifteen planning periods. By using the MPC, the SC system yields a NPV equal to 4,491,539.88 m.u., whereas by using the JCF 8213029.40 m.u. are accomplished. The case study demonstrates, perhaps not surprisingly, that significant benefits are obtained when merging the advantages of MPC and SP on an integrated approach.

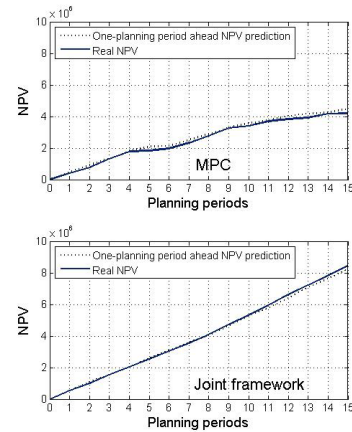


Figure 5: NPV behaviour comparison

## 4. Conclusions and future work

The main contribution of this work is that presents a novel approach for controlling and reviewing the SC plans to take into account the most recent information from market and operations environment. The novel joint framework consists of a MPC that comprises a SP into the control algorithm. Future work will be focus on devising how risk management features and decentralised policies can be taken in consideration within the control strategy proposed.

## Acknowledgements

Financial support received from "Generalitat de Catalunya" (FI programs) and European Community (project PRISM-MRTN-CT-2004-512233) is fully appreciated.

## References

1. Camacho, E.; Bordons, C. Model Predictive Control in the Process Industry; Springer: 1995.
2. Montgomery, D. C.; Johnson, L. Forecasting and Time Series Analysis; McGraw-Hill: 1976.
3. Guillén, G.; Badell, M.; Espuña, A.; Puigjaner, L. Comput. Chem. Eng., 30 (2006) 421.
4. Gupta, A.; Maranas, C. D. Comput. Chem. Eng., 27 (2003) 1219.