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Periodic Scheduling of Multiproduct Continuous Plants Using a Multiple Time Grid Continuous-time Formulation

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Abstract

This paper presents a new mixed integer nonlinear model (MINLP) for the periodic scheduling of multistage continuous plants with a single equipment unit per stage, where all units are subject to sequence dependent changeovers. The formulation is based on the resource task network (RTN) process representation, features combined processing and changeover tasks and does not require an iterative procedure over the total number of event points in the time grid. The new multiple time grid formulation is compared to a single time grid formulation through the solution of a few example problems taken from the literature with the results showing that the proposed formulation is significantly more efficient computationally.

Keywords: Optimization, Inventories; Wrap-around.

1. Introduction

Extensive reviews of optimization approaches for scheduling have recently appeared in the literature [1,2]. These focus mostly on batch processes, which have received considerable attention in the literature, whereas much less work involving the scheduling of continuous plants has been reported, despite their practical importance. Furthermore, one of the trends in the chemical industry is to move towards continuous flexible multiproduct plants that can respond more quickly to demand changes and that can handle a large product portfolio. The short-term mode of operation has also been preferred over a cyclic mode. Periodic scheduling is suitable wherever product demands are stable over extended periods of time and can be used to address non-cyclic problems whenever a long time horizon is involved, by periodically repeating a production pattern.

Time representation is the most important issue concerning the classification of scheduling models [1] and continuous-time formulations have been in the spotlight in the last 10 years. Those relying on time grids can be divided into single and multiple time grid formulations and Shaik et al. [3] identified the formulations of Janak et al. [4] and Castro et al. [5] as the best multiple and single time grid formulations for multipurpose batch plants, respectively. For multistage problems, Castro et al. [6] presented two very efficient short-term multiple time grid formulations for which specifying the total number of event points that lead to global optimal solutions is not an issue. This paper extends their work and presents a new periodic formulation that accounts for the material processed by the several tasks and also for inventory profiles. The definition of the general problem is taken from [7].

2. Problem definition

Given a set of orders $i \in I$ that must follow a sequence of processing stages $k \in K$ (with a single equipment unit per stage) to reach the condition of final products and their: i) maximum processing rates $\rho_{i,k}^{\max}$ (kg/h); ii) sequence dependent changeovers $cl_{i,i',k}$ (h); iii) minimum demand rates d_i (kg/h); iv) value c_i (\$/kg); v) intermediate storage $ci_{i,k}$ (\$/kg); vi) final storage cf_i (\$/kg/h) and vii) transition costs $ct_{i,i',k}$ (\$/h); the objective is to find a cyclic schedule that maximizes the profit. More specifically, we will determine the: a) cycle time; b) order sequencing; c) length of processing and cleaning tasks; d) processing rate of continuous tasks; e) amounts of products to be produced; f) levels of intermediate storage and final product inventories.

3. New multiple time grid formulation

The new continuous-time formulation is conceptually similar to formulation CT4I developed by Castro et al. [6]. It uses multiple time grids, one per unit, with a number of pre-specified time points |T|, where all tasks span a single time interval. Thus, with |T|=|I| we ensure optimality. Binary variables with 4 indexes, $N_{i,i',k,t}$ (with $i \neq i'$), are required to identify the execution of order i in stage k at time t, together with the required changeover task for order i' to immediately follow (same stage, next event point). Four other sets of binary variables $Y1_{i,k}$-$Y4_{i,k}$ identify the location of order i in stage k with respect to the processing of the same order in stage k+1 (see Figure 1). These are required to accurately determine the capacity of the intermediate storage vessel $V_{i,k}$.

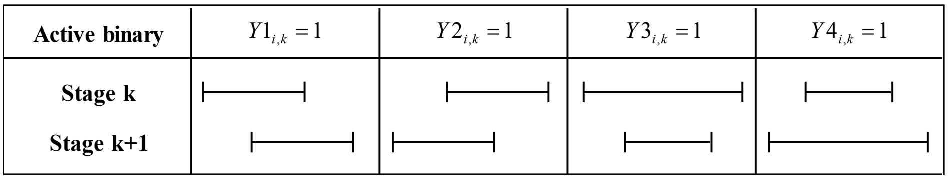

Figure 1. Four different possibilities of task interaction between consecutive stages

The positive continuous variables $\xi_{i,k,t}$ give the amount of material processed, while the excess resource variables $C_{i,k,t}$ refer to the availability of unit k at point t at a condition that enables the unit to process order i. Note that the sum over i is linked to the availability of the equipment resource. The time of event point t belonging to grid k relative to the start of the cycle is given by $T_{t,k}$, while H is used for the cycle time. Combined tasks starting at time point t unit k end their processing at $Tp_{t,k}$. The start and finishing time of order i in stage k is given by $Ts_{i,k}$ and $Tf_{i,k}$, respectively. To avoid numerical problems, we use ΔT_i for the processing time of order i in the last stage. Finally, $\rho_{i,k}$ holds the processing rate and D_i the delivery/demand rate of product i. The model constraints are given next. A wrap-around operator [8] has been used, so tasks starting at the last event point and ending after the cycle boundary can be viewed simply as wrapping around to the beginning of the cycle and continuing from there.

$$\Omega(t)=\begin{cases} t-|T|, t>|T| \\ t \quad ,1\le t\le |T| \\ t+|T|, t<1 \end{cases}; C_{i,k,t}=C_{i,k,\Omega(t-1)}+\sum_{i'\in I,i'\neq i}(\overline{N}_{i',i,k,\Omega(t-1)}-\overline{N}_{i,i',k,t})\ \forall i,k,t \tag{1-2}$$

$$\sum_{i\in I}C_{i,k,1}=1-\sum_{i\in I}\sum_{i'\in I,i'\neq i}\overline{N}_{i,i',k,1}\ \forall k\in K;\ \sum_{t\in T}(\xi_{i,k,t}-\xi_{i,k+1,t})=0\ \forall i\in I,k\neq |K| \tag{3-4}$$

$$\xi_{i,k,t}\le \rho_{i,k}^{\max}\overline{H}\sum_{i'\in I,i'\neq i}\overline{N}_{i,i',k,t}\ \forall i\in I,k\in K,t\in T;\ D_i\cdot H=\sum_{t\in T}\xi_{i,|K|,t}\ \forall i\in I \tag{5-6}$$

$$\sum_{t\in T}\sum_{i'\in I,i'\neq i}\overline{N}_{i,i',k,t}=1\ \forall i\in I,k\in K;\ \sum_{i'\in I,i'\neq 1}N_{1,i',1,1}=1;\ C_{i,k,t}=0\ \forall i\in I,k\in K,t\in T \tag{7-9}$$

$$T_{\Omega(t+1),k}+H\big|_{t=|T|}-T_{t,k}\ge\sum_{i\in I}(\xi_{i,k,t}/\rho_{i,k}^{\max}+\sum_{i'\in I,i'\neq i}\overline{N}_{i,i',k,t}cl_{i,i',k})\ \forall k,t;\ T_{1,1}=0 \tag{10-11}$$

$$T_{\Omega(t+1),k}+H\big|_{t=|T|}-Tp_{t,k}=\sum_{i\in I}\sum_{i'\in I,i'\neq i}\overline{N}_{i,i',k,t}cl_{i,i',k}\ \forall k\in K,t\in T;\ T_{|T|,k}\le H\ \forall k \tag{12-13}$$

$$T_{t,k}-\overline{H}(1-\sum_{\substack{i'\in I\\ i'\neq i}}\sum_{\substack{t'\in T\\ t'\ge t}}\overline{N}_{i,i',k,t'})\le Ts_{i,k}\le T_{t,k}+\overline{H}(1-\sum_{\substack{i'\in I\\ i'\neq i}}\sum_{\substack{t'\in T\\ t'\le t}}\overline{N}_{i,i',k,t'})\ \forall i,k,t \tag{14-15}$$

$$Tp_{t,k}-\overline{H}(1-\sum_{\substack{i'\in I\\ i'\neq i}}\sum_{\substack{t'\in T\\ t'\ge t}}\overline{N}_{i,i',k,t'})\le Tf_{i,k}\le Tp_{t,k}+\overline{H}(1-\sum_{\substack{i'\in I\\ i'\neq i}}\sum_{\substack{t'\in T\\ t'\le t}}\overline{N}_{i,i',k,t'})\ \forall i,k,t \tag{16-17}$$

$$\rho_{i,k}(Tf_{i,k}-Ts_{i,k})=\sum_{t\in T}\xi_{i,k,t}\ \forall i,k;\ \rho_{i,k}\le\sum_{i'\in I,i'\neq i}\sum_{t\in T}\overline{N}_{i,i',k,t}\rho_{i,k}^{\max}\ \forall i\in I,k\in K \tag{18-19}$$

$$\Delta T_i = Tf_{i,|K|} - Ts_{i,|K|}\ \forall i \in I; \sum_{i\in I}\Delta T_i + \sum_{i\in I}\sum_{i'\in I, i'\neq i}\sum_{t\in T}\overline{N}_{i,i',|K|,t}cl_{i,i',k} = H \tag{20-21}$$

$$Ts_{i,k+1} - \overline{H}(1 - Y2_{i,k} - Y4_{i,k}) \le Ts_{i,k} \le Ts_{i,k+1} + \overline{H}(1 - Y1_{i,k} - Y2_{i,k})\ \forall i, k \neq |K| \tag{22-23}$$

$$Tf_{i,k+1} - 2\overline{H}(1 - Y2_{i,k} - Y3_{i,k}) \le Tf_{i,k} \le Tf_{i,k+1} + 2\overline{H}(1 - Y1_{i,k} - Y4_{i,k})\ \forall i, k \neq |K| \tag{24-25}$$

$$Tf_{i,k} \ge Ts_{i,k+1} - \overline{H}(1 - Y1_{i,k} - Y2_{i,k} - Y3_{i,k} - Y4_{i,k})\ \forall i \in I, k \in K, k \neq |K| \tag{26}$$

$$Tf_{i,k+1} \ge Ts_{i,k} - \overline{H}(1 - Y1_{i,k} - Y2_{i,k} - Y3_{i,k} - Y4_{i,k})\ \forall i \in I, k \in K, k \neq |K| \tag{27}$$

$$V_{i,k} \ge (Ts_{i,k+1} - Ts_{i,k})\rho_{i,k} - M_i(1 - Y1_{i,k})\ \forall i, k \neq |K|;\ M_i = \min_{k\in K}\rho_{i,k}^{\max}\cdot\overline{H}\ \forall i \in I \tag{28-29}$$

$$V_{i,k} \ge (Tf_{i,k+1} - Tf_{i,k})\rho_{i,k+1} - M_i(1 - Y1_{i,k})\ \forall i \in I, k \in K, k \neq |K| \tag{30}$$

$$V_{i,k} \ge (Ts_{i,k} - Ts_{i,k+1})\rho_{i,k+1} - M_i(1 - Y2_{i,k})\ \forall i \in I, k \in K, k \neq |K| \tag{31}$$

$$V_{i,k} \ge (Tf_{i,k} - Tf_{i,k+1})\rho_{i,k} - M_i(1 - Y2_{i,k})\ \forall i \in I, k \in K, k \neq |K| \tag{32}$$

$$V_{i,k} \ge (Tf_{i,k+1} - Ts_{i,k+1})(\rho_{i,k+1} - \rho_{i,k}) - M_i(1 - Y3_{i,k})\ \forall i \in I, k \in K, k \neq |K| \tag{33}$$

$$V_{i,k} \ge (Tf_{i,k} - Ts_{i,k})(\rho_{i,k} - \rho_{i,k+1}) - M_i(1 - Y4_{i,k})\ \forall i \in I, k \in K, k \neq |K| \tag{34}$$

$$V_{i,k} \ge \sum_{t\in T}\xi_{i,k,t} - M_i(Y1_{i,k} + Y2_{i,k} + Y3_{i,k} + Y4_{i,k})\ \forall i \in I, k \in K, k \neq |K| \tag{35}$$

$$\underline{H} \le H \le \overline{H}; T_{t,k} \le \overline{H}; Tp_{t,k} \le 2\overline{H}\ \forall t \in T, k \in K \tag{36}$$

$$Ts_{i,k} \le \overline{H}; Tf_{i,k} \le 2\overline{H}; d_i\underline{H}/\rho_{i,|K|}^{\max} \le \Delta T_i \le \overline{H}\ \forall i \in I;\ \rho_{i,k}.l = \rho_{i,k}^{\max}\ \forall i \in I, k \in K \tag{37}$$

$$\max\left[\left(\begin{array}{l}\sum_{i\in I}\sum_{t\in T}c_i\xi_{i,|K|,t} - \sum_{k\in K}\sum_{i\in I}\sum_{i'\in I,i'\neq i}\sum_{t\in T}ct_{k,i,i'}cl_{i,i',k}\overline{N}_{i,i',k,t} - \sum_{k\in K,k\neq|K|}\sum_{i\in I}ci_{i,k}V_{i,k} - \\ \sum_{i\in I}cf_i/2\cdot[(\rho_{i,|K|} - D_i)\cdot(\Delta T_i)^2 + D_i\cdot(H - \Delta T_i)^2]\end{array}\right)/H\right] \tag{38}$$

Eq 2 is the excess resource balance, where combined tasks from order i to i' starting at t consume equipment condition i at t and produce condition i' at $\Omega(t+1)$. Eq 3 ensures that the start-up procedure does not use more equipment units than those available at the plant. Eq 4 states that the amount of order i handled in stage k equals that processed in k+1.Material production for order i can only occur if one of its combined tasks is active (eq 5). Eq 6 defines the delivery rate D_i. Every order is processed only once in each stage (eq 7). Eq 8 and 11 anchor the schedule [8]. We can enforce all equipment units to be occupied at all times (eq 9). Eq 10 is the most important timing constraint, where it is assumed that equipment units can be operated below their maximum processing rates. Eq 13 forces the time of the last event point in all grids to be lower than the time horizon. Eqs 12, 14-17 are big-M constraints relating the different timing variables. Eq 18 defines the processing rate and is a nonlinear constraint, while eq 19 ensures that it does not exceed its maximum value. Eq 20 defines the processing times in the last stage, which need to be properly bounded to avoid numerical problems with the solvers (eq 21). Eqs 22-27 are used to identify the interaction between processing tasks of the same order in consecutive stages (see Figure 1). Based on such variables, the capacity of the intermediate storage vessels can be determined (eqs 28-35). Eqs 36 and 37 place appropriate bounds on the timing variables where $\overline{H}$ and $\underline{H}$ are respectively the upper and lower bounds on the cycle time. In the optimal solution, the processing rates will regularly equal their maximum values, so these are used as starting points. Finally, the new MINLP formulation is completed with the profit maximization objective function (\$/h), eq 38. The first term in the numerator gives the revenue due to product sales, while the others give the total cost in terms of changeover, intermediate and final inventories, respectively.

4. Computational results

The MINLP model was implemented and solved in GAMS build 22.2 using DICOPT, with CONOPT as the NLP and CPLEX as the MILP solvers. The computer consisted on a Pentium-4 3.4 GHz processor with 2 GB of RAM. The performance of the formulation is illustrated through the solution of 7 example problems and compared to the MINLP single time grid formulation (STG) given in [5]. Most of the data was taken from [7] and is based in examples from continuous multiproduct plants for manufacturing lubricants. Table 1 lists the results obtained, where the examples gradually increase in size due to an increase in either the number of products or stages. Additional increments resulted in intractable problems, while STG could not solve Ex4 and Ex7.

The new formulation leads to the best solution for all but Ex1 and is faster by at least one order of magnitude. When compared to STG, it requires fewer event points to find the optimal solution (fewer binary variables), without requiring an iterative search procedure (recall that $|T|=|I|$). When compared to the one in [7], it is not limited to solutions that feature the same sequence in all stages, and the processing rates may be lower than the maximum values. Because of this, for Ex1 and Ex3, which match examples in [7], the new formulation obtained profits of 352.446 \$/h and 6514.49 \$/h vs. 297 \$/h and 6513 \$/h.

Figure 2 shows the optimal schedule for Ex7, the hardest 3-stage problem, for which the optimal product sequence is the same in all stages (ADEBC). Notice that stage 2 is the limiting stage since all tasks are performed at the maximum rate (100% is the default). Also, note that all orders are more or less being produced in the three stages simultaneously in order to reduce inventory profiles. All products are at their minimum demand rates except C.

5. Conclusions

This paper has presented a new continuous-time formulation for the optimal periodic scheduling of multistage continuous plants with a single unit per stage and sequence dependent changeovers. It uses multiple time grids where the number of event points that achieves optimality can be easily determined and is able to calculate the capacity that is required for the intermediate storage vessels. This is a substantial development since the vessels inputs and outputs result from equipment units belonging to different time grids that, because of that, are difficult to relate. When compared to a some extent similar single time grid formulation, the new formulation could find better solutions to all but one example problems in significantly less computational time.

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Table 1. Overview of computational statistics

			New multiple time grid formulation			Single time grid formulation		
Problem	|I|	|K|	Profit ($/h)	Binaries	CPU (s)	Profit ($/h)	Binaries	CPU (s)
Ex1	3	2	352.446	48	1.5	**355.087**	198	33.6
Ex2	4	2	**7099.19**	112	3.2	7074.61	288	239
Ex3	5	2	**6514.49**	220	287	6397.78	500	7899
Ex4	7	2	**5572.57**	616	71598			
Ex5	3	3	**5923.90**	78	4.9	5789.04	216	111
Ex6	4	3	**5312.76**	176	58.3	5166.12	384	1863
Ex7	5	3	**4986.59**	340	33284			

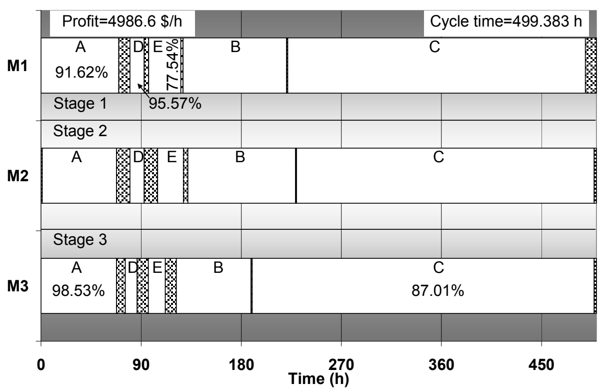

Figure 2. Optimal schedule for Ex7