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Optimal Planning of Closed Loop Supply Chains: A Discrete versus a Continuous-time formulation

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Abstract

In this paper the planning of closed loop supply chains is studied where both. forward and return flows are integrated considering their simultaneous coordination. The product supply and returns are then considered explicitly at the planning level of the supply chain optimization.
Two different approaches are developed and implemented to model the supply chain optimal planning problem: a discrete and a continuous-time formulation. The former considers an uniform discretization of the planning horizon while the continuous-time counterpart considers the time space domain modeled through the definition of a set of time instances called *slots* of unknown duration. Each slot dimension is optimized simultaneously with the planning events. Both approaches account explicitly for the integration of topological, operational and market supply-demand constraints and requirements (Amaro and Barbosa-Póvoa, 2006).
The proposed formulations results both into Mixed Integer Linear Programming (MILP's) models that are solved using a standard Branch and Bound (B&B) procedure.
A detailed plan is obtained, at each formulation approach, that allows the improvement of the supply chain operability by exploiting general resource capacities (e.g. transforming, storage and transportation) and resource sharing policies based on equipment/tasks suitability's, economical performances and operational restrictions.

The applicability of the proposed formulations is illustrated by its implementation on the solution of an industry-oriented case study involving an international pharmaceutical supply chain.

Keywords supply chain management, products recovery, optimal planning, discrete and continuous-time formulations, optimization.

1. Introduction

Nowadays companies are facing new and important challenges at a world-wide scale due to the present economical globalisation context and due to the existence of new emerging market trends. The actuation over extended and geographically disperse markets results as mandatory to both, national and international organizations, in order to keep competitive. As a consequence their supply chain must be efficient at a world-wide scale supported by flexible operations (from purchase and supply till final products distribution) and managing decision procedures so as to respond promptly to the end consumer's requirements (Shah, 2005).

Also, new challenges were brought from the recognition of environment concerns and from the requirements of sustainable development. Thus, managing strategies based on the control of the forward material flows and on the feedback information flows are not enough to ensure global chain performances neither profit purposes. Important feedback flows of non-conform materials (reverse logistics – closed loop chains, French and LaForge, 2006) are strongly engaged with the provider's feasibility (producers and transportation capacity) and can not be economical nor operationally disregarded.

These new aspects lead to complex supply chain structures that need the usage of decision supporting tools to help the decision making associated process.

In this paper the optimal operation of these structure, the so called closed loop supply chains, is studied where both the forward and the reverse flows are optimized. The planning problem is addressed and as final result a detailed plan is obtained where the production, storage, distribution and recovery of products is defined.

Two alternative model formulations are analyzed. The first one was proposed by Amaro and Barbosa-Póvoa (2006) and uses a discrete time representation while the second one is developed along this paper and uses a continuous definition of time.

The developed models performances as well as the events allocation obtained within the two different time scale approaches are compared. This analysis is made through the solution of a real case-study proposed by Amaro and Barbosa-Póvoa (2006).

2. Problem Formulation Characterization

As it was previously reported, most of the SC planning approaches proposed for the close loop operation do not account for the explicit integration of aspects such as transport operations, forward and reverse simultaneous process structures operations and recovery practices. This paper proposes an approach that explicitly considers the details of these SC operability aspects.

In this paper two mathematical formulations (discrete and continuous) are proposed to model the closed loop SC planning problem.

The representation adopted describes the SC operability through a set of model entities that characterize the SC events and the associated resources. Two set of events are considered: tasks and flows. The former accounts for general processing operations as chemical and physical transformation of materials (e.g. reactions, packing, recovery, etc), while the later considers the materials mobility between SC partners and to the associated customer markets. Both events are described using an aggregated description of the involved operations, macro tasks and flows (Amaro and Barbosa-Póvoa, 2006).

Task events consider batch and semi-continuous tasks. The former are characterized by a fixed processing time (time required to perform a given operation within a specific suitable equipment), while the later are defined by an operational rate having minimal feasibility time requirements (e.g. minimal operating time that justifies the task assignment, 1 hr, 30 minutes, etc).

On the other hand, materials mobility is ensured by the transport operations represented by the flow events. Each transportation flow involves the assignment of a transport operation to an autonomous transportation facility available at a specific structure (e.g. fleet of vehicles, boat containers, etc) and a chain path with defined source and sink geographical locations.

In order to account for the multipurpose nature of the transportation facilities (simultaneous transport of compatible materials) the representation model formerly proposed by Amaro and Barbosa-Póvoa (2006) has been extended to account for the details associated with the assignment of transport operations to each autonomous facility within a transportation structure. This requires the implementation of the concept of flow families (e.g. set of compatible flows defined over a transportation structure) into the assignment conditions considered to each transportation facility.

These representation concepts are accounted for at both proposed formulations. The main modelling differences between the developed formulations are related with the representation of semi-continuous tasks. At the discrete model an integer set of variables is used to represent the allocation time devoted to each semi-continuous task. Although, at the continuous time formulation this integrality condition is relaxed and a set of continuous variables is used to report the residence time of those tasks.

3. Case Study

This paper follows a former study developed for an industrial pharmaceutical supply chain, SC presented by Amaro and Barbosa-Póvoa (2006). The industrial SC characterization was by the authors reported and described in detail. For further details please refer to the work cited.
The SC in study involves the production and distribution of different injection drugs, tablets and oral suspensions for different Portuguese and international markets. The production cluster comprises three medicine producers (I1 and I2, Portuguese partners, and I3 a Spanish partner). Four major intermediate medicines, IP1 to IP4, can be produced at I1 and I3 plants, while I2 produces only IP3 and IP4.
The closed loop supply chain operation is evaluated by comparing the supply chain economical performance for a given aggregated planning period where three independent operational scenarios are studied. A first scenario represents a disposal scenario where all the non conformed products are send to burning centres (while removed from the market places). The latter two scenarios, consider a recovery planning approach where the medicines are clustered based on the non conformity. Non-recoverable medicines must be send to burning centres while recyclable or remanufacture medicines can be recovered at I1 plant. Scenarios II and III consider respectively an unconstrained medicine recovery option and a recovery practice involving some minimal requirements for all recoverable medicines (III) .

Results
Based on the above characteristics the supply chain planning is performed for a planning period of three months with the objective of evaluating the best recovery portfolio scenario. An aggregate time description is used at both proposed formulations. For the discrete model an uniform time grid, defined on a weekly basis, is used to represent the planning horizon. At the continuous model counterpart this weekly operational break (week ends) is also considered to represent the SC operability conditions (partners working schedules). Therefore, a fixed set of common time points is generated for both developed formulations. Although, for the continuous model a detailed time description (refined grid) is allowed. This accounts for the generation of further time evaluations enclosed within each weekly time points fact that is not allowed within the discrete time model.
To analyze both formulations compatibility a mutual consistency concerning the representation of the planning problem has been performed considering exclusively the set of common time grid points (k=14, weekly aggregation) and the disposal scenario. The achieved results translate a very small difference (0.22%) between the optimal values of the objective function raised within both formulations. This difference is due to the balance of residence times accounted for the semi-continuous tasks at each model formulation (integer versus

continuous durations) and to their subsequent economical evaluation. At this scenario the continuous time formulation is dimensionally larger than the discrete counterpart, except in what concerns the discrete variables. Although, a better computational performance is observed. Based on these results the models representation consistency is checked but no further differences can be pointed out between the proposed formulations.
To analyze the consistency of the maximal *k* value considered, a set of computational tests were performed for *k* values between 14 (weekly discretization) and 20. For each test the computational and economical performances were analyzed. For the planning disposal scenario, the optimal *k* value obtained is 15 since for the remaining tested values the optimization procedure generates the same number of different time slots (13), corresponding to an optimal *k* of 15 time points. Accordingly, the consideration of any *k* value greater than 15 increases the model dimension, dropping down the computational performances, without improving the optimal planning solution.
An equivalent procedure was developed for the remaining planning scenarios in order to identify the optimal number of time slots required to represent the problem. The values raised are *k*=16 (14 slots) and 17 (15 slots) respectively for scenarios II to III. The results are summed up in table 1.

Table 1. – Continuous and discrete models characterization for the operational scenarios studied.

Formulation Statistics	Scenario - I		Scenario - II		Scenario – III	
	Continuous	Discrete	Continuous	Discrete	Continuous	Discrete
Constraints	45172	27775	51391	29583	54780	29587
Variables	23688	16860	27004	18032	28735	18032
Discrete	8280	7742	9482	8240	10070	8240
CPUs*	823,7	688,6	8678,7	376,1	4987,8	606,95
Iterations	66580	90004	1051600	55945	410350	91674
Obj. Funct.	17345507	17209297	17715019	17475128	17622441	17448796
R.Gap ≤3%	2.88 %	0,15 %	2.99 %	0,23 %	2.99 %	0,09%
Nº slots/ Δt	13	12	14	12	15	12

*GAMS/CPLEX (v.10), Pentium III

As it can be observed, in terms of model dimension and computational statistics the continuous model performs globally worst than the discrete formulation (table 1). Although, the continuous formulation presents the better optimal planning solutions for the set of studied scenarios. The differences observed are not very significant (≤ 2 %) while compared with the optimal value of the objective functions achieved. The continuous formulation performs economically a bit better due to the increment of the time points generated to achieve the optimal solution. At this time points (not balanced by the discrete formulation) the materials existences are actualized and accordingly a different feasible set of tasks can result for the next, and further, time slots. Moreover, this positive profit differences are justified by a relatively larger increase of the economical incomes (delivers and existences) while compared with the associated cost balance. The incomes balance reports the existence of different materials availability to perform both delivers and existences. The former should be significant than the existences since the increment of existences also drops up the storage costs. This is one of the reasons why the time slots that apart the formulations are generated closer to the end of the planning horizon.

Concerning the operability and economical analysis for the planning scenarios studied the proposed formulations pointed out that a recovery of non conform materials is always the profitable operability policy. If a non constrained recovery option is accounted (Sc II) an higher planning profit can be raised. This allows us to conclude that recovery must be a selective industrial practice that should account for a full recovery of the profitable non conform products while satisfying the regulatory legislation to the remaining materials (constrained recovery). Although, in any case recovery is a profitable scenario than the disposal solution and accordingly, the costs incurred with the non conform products transportation and burning practices can be saved by the implementation of recovery practices.

4. Conclusions

This paper presents two different approaches that were developed and implemented to model the supply chain optimal planning problem: a discrete and a continuous-time formulation. Both formulations account for detailed supply chain characteristics such as production, storage, distribution and recovery of products. Different recovery portfolio scenarios are analysed.
The discrete formulation performs better than the continuous counterpart in what concerns the model statistics. It requires less iterations and less CPUs to reach the optimal solutions. Although, the continuous counterpart gives a better optimal solution for the planning conditions studied. The improvement on the net global profit observed for the continuous time model is due to the increment of the number and location of the evaluated points on the time domain fact that is not allowed at the discrete model since a fixed discretization time is used .
A real case study taken from a pharmaceutical industry was studied and the results obtained were promising.
The study made identified some interesting points, when comparing the two models, that need further attention. These essentially concern the application of the models over a high number of cases so as to generalize the results obtained.

References

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