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A Discrete/Continuous-Time MILP Model For Medium-Term Planning of Single Stage Multiproduct Plants

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Abstract

The objective of this work is to develop an optimization model for the medium-term planning of single stage continuous multiproduct plants. Several types of SKUs (Stock Keeping Units) are produced. Customers place orders that represent multiples of SKUs and these orders must be delivered at the end of each week. When different SKU types are processed, sequence-dependent changeover times and costs are incurred. The problem is represented as a mixed-integer linear programming (MILP) model with a hybrid time representation. The objective is to maximize profit that involves sales revenues, production costs, product changeover costs, inventory costs and late delivery penalties. The proposed optimization-based model is validated in a real-world polymer processing plant.

Keywords: medium-term production planning, mixed-integer optimization, multiproduct plants, polymer production

1. Introduction

Medium-term planning matches production requirements so as to meet demands by generating plans that determine the flow of materials and the use of resources over a given planning horizon of several weeks to a few months.

Traditionally, continuous plants have been associated with single product manufacturing. Nowadays, there is an increasing need for more flexible continuous processing facilities being suitable for more than one product. The efficient planning and scheduling of such facilities (decisions related to when, where, and how much of each product should be manufactured) is a challenging problem, usually a non trivial one [1-7].
One key characteristic in the operation of continuous plants is the sequence-dependent changeovers/transitions incurred when changing from one product to another. A transition cost and a transition time arise whenever a new product starts to be processed. These factors often refer to the cost and time associated with out-of-specification products generated while the unit is adjusted to a new set of operating conditions. The planning and scheduling of continuous multiproduct plants usually involves tradeoffs such as quantities produced, storage levels, backlogs and transition costs. The above tradeoffs can be resolved at an optimal manner through a simultaneous optimization- based approach.
The objective of this work is to develop an optimization model for the medium-term planning of single stage continuous multiproduct plants based on a hyrbrid discrete/continuous time representation.

2. Problem Description

Several types of SKUs (Stock Keeping Units) are produced. Customers place orders that represent multiples of SKUs and these orders must be delivered at the end of each week. Customer demands are typically the result of a negotiation whereby order quantity, delivery date, and any variability on this is regulated by a contract. Penalties and manufacturer liability are usually agreed for late deliveries [5]. When different SKU types are processed, sequence-dependent changeover times and costs are incurred.
The strategic objective in this formulation is to maximize profit, which involves sales revenues, production costs, product changeover costs, inventory costs and late delivery penalties.

3. Mathematical Model

The main assumptions of the model are: (1) the plant is composed of a single stage with a single unit; (2) there are sequence dependent changeover times and costs; (3) demands and backlogs are only enforced at the end of each week. Due to the nature of the problem, the time domain is modeled in hybrid form: a discrete formulation represents the weeks of the planning horizon whereas each week is modeled with a continuous formulation, which is based on the one proposed by Casas-Liza and Pinto [5]. Hence, intervals of equal length represent each week, and each week comprises several time slots of variable length. The model relies on the following notation:

Indices and sets

c	customers
i, j	products
k	time slots
K_w	set of time slots in week w
w	weeks

Parameters

$CB_{c,i}$	backlog cost of i to customer c
$CI_{i,w}$	inventory cost of i in week w
$CT_{i,j}$	transition cost from i to j
$D_{c,i,w}$	demand of i from customer c in week w
$PS_{c,i}$	price of i to customer c
r_i	processing rate of i
$V_i^{max/min}$	max / min storage of i
θ^L/θ^U	lower/upper processing time
$\tau_{i,j}$	changeover time from i to j

Binary Variables

$E_{i,w}$	1 if product i is produced in week w
$y_{i,k,w}$	1 if product i is processed in time slot k during week w
$Z_{i,j,k,w}$	1 if product i (slot k-1) precedes j (slot k) in week w

Continuous Variables

$P_{i,w}$	production of i in week w
$S_{c,i,w}$	sales of i to customer c in week w
$T_{k,w}$	time point k in week w
$V_{i,w}$	volume of product i in week w
$\Delta_{c,i,w}$	backlog of i for c in week w
$\theta_{i,k,w}$	processing time of product i in slot k during week w

Next, the mathematical model is presented.

- Assignment constraints

$$\sum_i y_{i,k,w} = 1 \qquad \forall k \in K_w, w \in W \qquad (1)$$

The unit processes exactly one product at every time interval. Although products are assigned at every interval, production may not be required.

- Timing constraints

$$T_{0,w} = 0 \qquad T_{|K_w|,w} = 168 \qquad \forall w \in W \qquad (2)$$

$$0 \le \theta_{i,k,w} \le \theta^U \cdot y_{i,k,w} \qquad \forall i, k \in K_w, w \in W \qquad (3)$$

$$\sum_{k \in K_w} \theta_{i,k,w} \ge \theta^L \cdot E_{i,w} \qquad \forall i, w \in W \qquad (4)$$

$$T_{k,w} - T_{k-1,w} = \sum_i \left(\theta_{i,k,w} + \sum_j \tau_{j,i} \cdot Z_{j,i,k,w} \right) \qquad \forall k \in K_w, w \in W \qquad (5)$$

The time points and time intervals are calculated by the duration of the processing of a product. The time points are determined from the time horizon defined between the initial point ($T_{0,w}$ = 0) and final point ($T_{K_w,w}$ = 168) as well as from the ordering of the intermediate points. The duration of a time interval k ($T_{k,w} - T_{k-1,w}$) is determined from the duration of the processing and transition.

- Transition constraints

$$\sum_j Z_{i,j,k,w} = y_{i,k-1,w} \qquad \forall i, k \in K_w - \{1\}, w \in W \qquad (6)$$

$$\sum_i Z_{i,j,k,w} = y_{j,k,w} \qquad \forall i, k \in K_w - \{1\}, w \in W \qquad (7)$$

$$\sum_j Z_{i,j,1,w+1} = y_{i,K_w,w} \qquad \forall i, w \in W \qquad (8)$$

$$\sum_i Z_{i,j,1,w+1} = y_{j,1,w+1} \qquad \forall i, w \in W \qquad (9)$$

Transition constraints (6) and (7) are enforced within the weeks, while constraints (8) and (9) represent transitions between consecutive weeks. Similar constraints have been proposed by Pinto and Grossmann [1].

- Process and storage capacity constraints

$$P_{i,w} = r_i \cdot \sum_{k \in K_w} \theta_{i,k,w} \qquad \forall i, w \qquad (10)$$

$$V_i^{\min} \le V_{i,w} \le V_i^{\max} \qquad \forall i, w \in W \qquad (11)$$

The amount of product i being produced is given by its constant product rate and processing time. The amounts of material to be stored are bounded by minimum and maximum capacities.

- Demand constraints

$$V_{i,w} = V_{i,w-1} + P_{i,w} - \sum_c S_{c,i,w} \qquad \forall i, w \qquad (12)$$

$$\Delta_{c,i,w} = \Delta_{c,i,w-1} + D_{c,i,w} - S_{c,i,w} \qquad \forall c, i, w \qquad (13)$$

Constraint (12) represents material balances, whereas constraint (13) addresses product backlogs per customer.

- Objective function

The optimization criterion adopted is the maximization of the operating profit by the sales of final products minus changeover, inventory and backlog costs.

$$Pro = \sum_i \sum_w \left[\sum_c \left(PS_{i,c} S_{c,i,w} - CB_{i,c} \Delta_{c,i,w} \right) - \sum_j \sum_{K \in K_w} \left(CT_{i,j} Z_{i,j,k,w} + CI_{i,w} V_{i,w} \right) \right] \qquad (14)$$

Integer cuts are proposed to eliminate degenerate schedules by allocating each selected product for each week to one slot while allowing the last product manufactured per week to occupy more than one consecutive time slot. Other cuts involve changeovers required during each week. Moreover, upper bounds in the total manufacturing time available per week are imposed.

4. Computational Results

The proposed optimization-based model is validated in a real-world polymer processing plant that produces ten SKU types (A to J) by investigating different contract scenarios. Weekly demands for each individual SKU are established for ten customers over a period of four weeks. All products are processed at a maximum rate of 110 ton/week. Inventory cost and backlogging coefficients are 10% and 20% of product prices, respectively; transition costs correspond to 10% of transition times, which are shown in Table 1. This table also shows the prices of products for all customers, except for one of them (50% higher).

Table 1 – Changeover times (min) and product prices

	A	B	C	D	E	F	G	H	I	J	$PS_{i,c}$ ($)
A	-	45	45	45	60	80	30	25	70	55	10
B	55	-	55	40	60	80	80	30	30	55	12
C	60	100	-	100	75	60	80	80	75	75	13
D	60	100	30	-	45	45	45	60	80	100	12
E	60	60	55	30	-	35	30	35	60	90	15
F	75	75	60	100	75	-	100	75	100	60	10
G	80	100	30	60	100	85	-	60	100	65	8
H	60	60	60	60	60	60	60	-	60	60	14
I	80	80	30	30	60	70	55	85	-	100	7
J	100	100	60	80	80	30	45	100	100	-	15

Table 2 shows the demands for each SKU during each week as well as their total amounts. The same table shows the weekly aggregated backlogs. Note that the total backlog is 86.3 tons in the first week, because the maximum capacity of the plant is 110 tons, whereas overall demand is 195 tons. The backlog is slightly reduced in the following weeks due to spare capacity of the plant.

Table 2 – Product demands and backlogs

SKU	Weekly demands (ton)				Weekly backlogs (ton)			
	1	2	3	4	1	2	3	4
A	31	0	0	0	20.3	20.3	0.0	0.0
B	12	15	0	15	12.0	11.5	0.0	0.0
C	19	4	4	19	0.0	4.0	0.0	0.0
D	36	0	6	0	0.0	0.0	6.0	0.0
E	43	0	43	0	0.0	0.0	0.0	0.0
F	24	0	0	24	24.0	0.0	0.0	0.0
G	12	0	27	0	12.0	2.6	29.6	24.3
H	3	27	3	9	3.0	0.0	0.0	0.0
I	15	15	15	15	15.0	30.0	30.0	40.0
J	0	27	0	27	0.0	0.0	0.0	0.0
Total	195	88	98	109	86.3	68.5	65.6	64.3

Figure 1 shows the production schedule for the 4-week horizon.

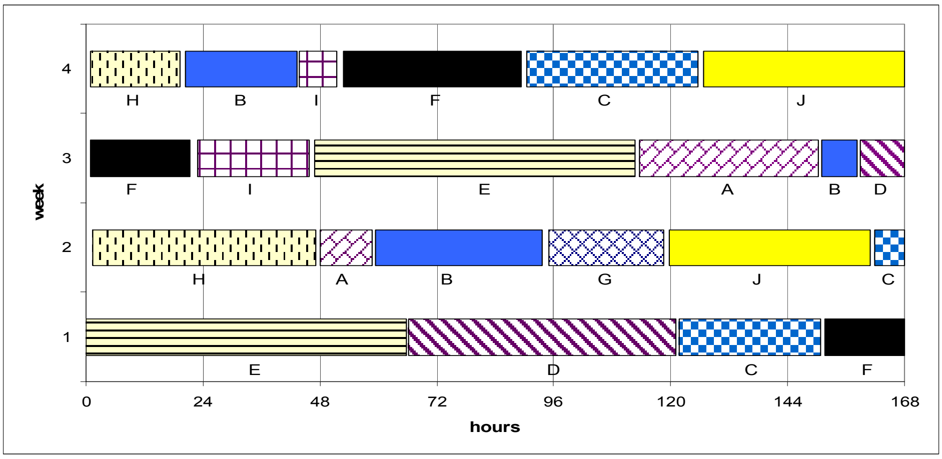

Figure 1. Gantt Chart for a four-week schedule

5. Conclusions

This paper presented an optimization model for the planning of single stage continuous multiproduct plants. The model is based on a discrete/continuous representation of the time domain and effectively represents changeover and backlog constraints. Results were obtained for a four-week horizon.

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References

1. J. Pinto and I.E. Grossmann, Comput. Chem. Eng., 18 (1994) 797.
2. M.G. Ierapetritou and C.A. Floudas, Ind. Eng. Chem. Res., 37 (1998) 4360.
3. A. Alle and J.M. Pinto, Ind. Eng. Chem. Res., 41 (2002) 2689.
4. C.A. Mendez and J. Cerda, , Comput. Chem. Eng., 26 (2002) 687.
5. N.F. Giannelos and M.C. Georgiadis, Ind. Eng. Chem. Res., 41 (2002) 2431.
6. P.M. Castro, A.P. Barbosa-Povoa, H.A. Matos and A.Q. Novais, Ind. Eng. Chem. Res., 43 (2004) 105.
7. M.E. Dogan and I.E. Grossmann, Ind. Eng. Chem. Res., 45 (2006) 299.
8. K. Lakhdar, Y. Zhou, J. Sarvey, N.J. Tichener-Hooker and L.G. Papageorgiou, Biotechnol. Progr., 21 (2005) 1478.
9. J. Casas-Liza and J.M. Pinto, Comput. Chem. Eng., 29 (2005) 1329.