

# **Integrating strategic, tactical and operational supply chain decision levels in a model predictive control framework**

José Miguel Laínez, Georgios M. Kopanos, Mariana Badell, Antonio Espuña,  
Luis Puigjaner

*Universitat Politècnica de Catalunya, Av. Diagonal 647, E-08028, Barcelona, Spain*

## **Abstract**

In this work an MILP model which achieves the integration of all three Supply Chain (SC) decision levels is developed. Then, the stochastic version of this integrated model is applied as the predictive model in a Model Predictive Control (MPC) framework in order to incorporate and tackle unforeseen events in the SC planning problem in chemical process industries. Afterwards, the validation of the proposed approach is justified and the resulting potential benefits are highlighted through a case study. The results obtained of this particular case study are analyzed and criticized towards future work.

**Keywords:** supply chain optimization, decision levels, MILP, model predictive control.

## **1. Introduction**

Although the Process Systems Engineering community (PSE) faces an increasing number of challenging problems, enterprise and SC remain subjects of major interest offering multiple opportunities. It is believed that further progress in this area will mean a unique opportunity demonstrating the PSE potential to enhance company's "value preservation". One of the key components in supply chain management (SCM) and enterprise wide optimization (EWO) is the decision making coordination and integration at all levels. Recent work offers models to separately tackle problems arising in the three standard SC hierarchical decision levels: strategic (long-term: network design), tactical (medium-term: aggregated planning) and operational (short-term: scheduling). These models, because of their nature and purpose, have very different timescales. It becomes evident the challenge of solving large size multi-scale optimization problems when considering the integration of decision levels. Since scheduling is also a basic building block in the more general area of EWO<sup>1</sup>, it is indispensable its incorporation into the already existed design-planning models.

Furthermore, it is noteworthy that SC planning is not a one time event, but a dynamic activity. Firms are in the need of a closed-loop planning approach in order to preserve competitiveness. This approach should be capable of revising planned activities, updating uncertain parameters (e.g., lead times, market demand and interest rates) and considering the effects of incidences; so that future plans are adapted to enhance SC performance under the current highly dynamic business environment. A MPC framework can be used as an appropriate approach to continuously improve the SC planning. The MPC framework attempts to optimize a performance criterion that is a function of the future control variables. By solving the optimization problem all

elements of the control signal are defined. However, only a portion of the control signal is applied to the SC system. Next, as new control input information and disturbance forecasts are collected, the whole procedure is repeated, which produces a feed-forward effect and enables the SC system to follow-up the dynamic business environment.

## 2. Problem statement

A novel stochastic multi-period design/planning/scheduling MILP model of a multi-echelon SC with financial considerations is used as a predictive model in this work. The model assumes that different technological equipment is available to be installed in potential sites and assists in their selection. Furthermore, the model allows the expansion of plant equipment capacities, not only in the first planning period. Regarding the financial area, the mathematical program endeavors to evaluate the shareholder value.

## 3. Control Strategy

In Fig.1, a general schematic of the proposed MPC framework for SCM is shown. It follows a brief description of the control strategy. When the SC process is disturbed, data required to describe the current SC state are captured and sent to the controller. Next, scenarios are calculated by the forecasting module. As it is indicated there, the control signal that is implemented in the SC processes only comprises the first stage variables resulting from the stochastic optimization problem. In fact, first stage variables are associated to next period decisions that are made prior to uncertainty realization.

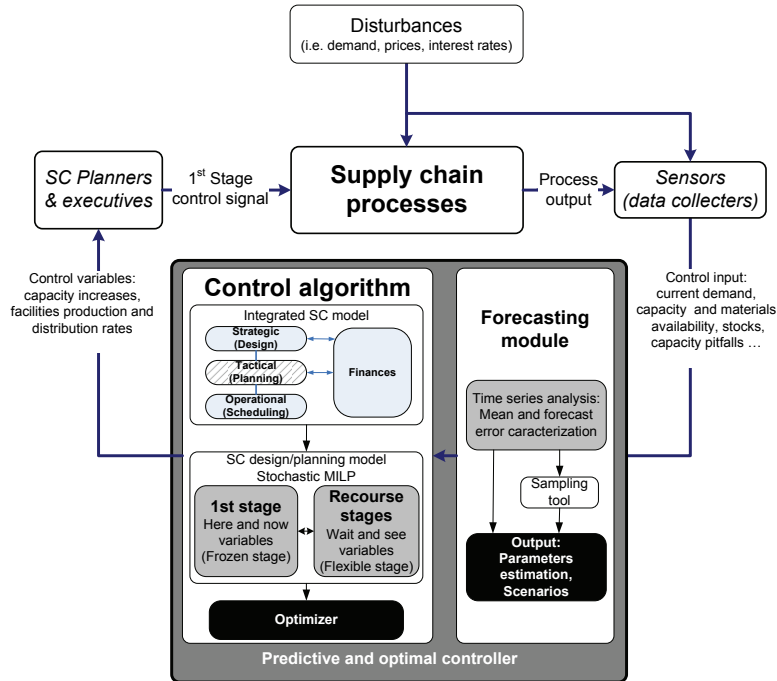


Figure 1. Control strategy

### 3.1. The control algorithm

The predictive model within the control algorithm consists in a multistage stochastic MILP. A scenario based approach is applied. Refer to Puigjaner and Laínez<sup>2</sup> for scenario tree description and stochastic formulation indices details ( $l, \hbar_l$ ).

#### 3.1.1. Process operations formulation

**Design-Planning formulation.** The stochastic design-planning approach presented in this work is inspired from the STN formulation<sup>3</sup>. The presented approach relies on the flexible echelons concept. The connectivity between echelons is not imposed; consequently, a facility may play the role of either a processing or a distribution site. Material flows among facilities are allowed. Moreover, the proposed design-planning formulation simplifies the representation of batch and/or continuous process into the same framework, which facilitates its integration with scheduling models.

The basic equations are next presented. Eq.(1) is the mass balance equation for each echelon  $f$  and material  $s$ . The design decisions are modeled through Eqns.(2)-(3). Eq.(4) forces the production to be within installed capacity and a minimum utilization factor. Eqns.(5)-(6) force materials flow from suppliers and to markets to be lower than an upper bound given by their capacity limitations.

$$S_{sft\hbar_l}^l = \sum_{f'} \sum_{i \in T_s} \sum_{j \in (J_i \cap J_{f'})} \alpha_{sij} P_{ijf'f\hbar_l}^l - \sum_{f'} \sum_{i \in \bar{T}_s} \sum_{j \in (J_i \cap J_f)} \bar{\alpha}_{sij} P_{ijff'\hbar_l}^l + S_{sft-1\hbar_{l^*}}^{l^*} \quad \forall s, f, \hbar_l, t \in T_l, l^* \in L_{t-1}^*, \hbar_{l^*} \in AH_{l^*\hbar_l} \quad (1)$$

$$F_{jft\hbar_{l-1}}^l = F_{jft-1\hbar_{l^*-1}}^{l^*} + FE_{jft\hbar_{l-1}}^l \quad \forall f, j \in J_f, l, \hbar_{l-1}, t \in T_l, l^* \in L_{t-1}^*, \hbar_{l^*-1} \in AH_{l^*-1, \hbar_{l-1}} \quad (2)$$

$$V_{jft\hbar_{l-1}}^l FE_{jft}^l \leq FSE_{jft\hbar_{l-1}}^l \leq V_{jft\hbar_{l-1}}^l FE_{jft}^U \quad \forall f, j \in J_f, l, \hbar_{l-1}, t \in T_l \quad (3)$$

$$\beta_{jf} F_{jft\hbar_{l-1}}^l \leq \sum_{f'} \sum_{i \in I_j} \theta_{ijff'} P_{ijff'\hbar_l}^l \leq F_{jft\hbar_{l-1}}^l \quad \forall f, j \in J_f, l, \hbar_l, t \in T_l, \hbar_{l-1} \in AH_{l-1, \hbar_l} \quad (4)$$

$$\sum_{f'} \sum_{i \in IT_s} \sum_{j \in J_i} P_{ijf'f\hbar_l}^l \leq Dem_{sft\hbar_l}^l \quad \forall s \in fp, f \in m, l, \hbar_l, t \in T_l \quad (5)$$

$$\sum_{f'} \sum_{i \in IT_s} \sum_{j \in J_i} P_{ijff'\hbar_l}^l \leq A_{sft} \quad \forall f \in e, s \in rm_f, l, \hbar_l, t \in T_l \quad (6)$$

**Scheduling formulation.** The scheduling formulation is an extension of STN representation<sup>2</sup> permitting scheduling in multiple facilities. Eqns.(7)-(8) are used for mass balances and assignment decisions, respectively.

$$\sum_{i \in J_i} \sum_{t'_s=t_s}^{t'_s=t_s-pt_i+1} W_{ijft'_s} \leq 1 \quad \forall f, j \in (J_{batch} \cap J_f), t_s \quad (7)$$

$$\begin{aligned}
Ssched_{sft_s} - Ssched_{sft_{s-1}} &= \sum_{i \in T_s} \sum_{j \in J_i} \alpha_{sij} B_{ijft_s - pt_i} - \\
&\sum_{i \in T_s} \sum_{j \in J_i} \bar{\alpha}_{sij} B_{ijft_s} + RM_{sft_s} \quad \forall s, f, t_s
\end{aligned} \tag{8}$$

Integration of decision levels. The integration between the models for design-planning and scheduling is carried out through Eqns.(9)-(11). Eq.(9) states that production allocated in equipment  $j$  is identical in both models. In Eq.(10) the availability of raw material is computed from received materials according to the planning formulation. Raw material availability is then included in the scheduling mass balance of Eq.(8). Scheduling equations may be applied in more than one planning period. The appropriate equations for incorporating scheduling in first planning period ( $t = 1$ ) are next presented.

$$P_{ijffth_l}^l = \sum_{t_s} B_{ijft_s} \quad \forall f, j \in (J_{batch} \cap J_f), i \in I_j, t = 1, l \in L_t, h_l \tag{9}$$

$$RM_{sft_s} = \sum_{f' \neq f} \sum_{i \in T_s} \sum_{j \in J_i} \bar{\alpha}_{sij} P_{ijf'fth_l}^l \quad \forall s, f, t_s = 1, t = 1, l \in L_t, h_l \tag{10}$$

Eq.(11) is included to rectify capacity availability in the planning model. This correction is done based on the scheduling model task assignment ( $W_{ijft_s}$ ). Eq.(11) should be merely applied to those equipments which are production bottlenecks. Additionally, it is worth to mention that it must be checked that market demand is not actually the bottleneck process in the planning period, where scheduling is performed.

$$\sum_{i \in I_j} \theta_{ijff} P_{ijffth_l}^l \leq \sum_{i \in I_j} \sum_{t_s} W_{ijft_s} pt_i \quad \forall f, j \in (J_{bottle} \cap J_f), t > 1, l \in L_t, h_l \tag{11}$$

As it can be noticed, Eqs.(7)-(11) can be easily unplugged from the whole model in case the decision maker decides not to consider scheduling issues.

### 3.1.2. Financial formulation and objective function

The financial side of the problem is tackled through the inclusion of a set of constraints that characterize economical issues, such as payments to providers, short and long term borrowing, pledging decisions, buying/selling of securities, fixed assets acquisition. Furthermore, the expected corporate value ( $E[CV]$ ), which is calculated using the discounted-free-cash-flow method (DFCF) as described by Eqns.(12)-(13), is the objective function used in this work. The complete set of financial constraints as well as the equations that permit the integration between finances and process operations models can be found in the work of Puigjaner and Lainez<sup>2</sup>.

$$E[CV] = \sum_{h_L} P_{h_L}^L \left( DFCF_{h_L}^L - NetDebt_{Th_L}^L \right) \tag{12}$$

$$DFCF_{T\bar{h}_L}^L = \left( \sum_t \sum_{l \in L_t} \sum_{h_l \in AD_{t\bar{h}_L}} \frac{FCF_{t\bar{h}_l}^l}{(1 + WACC_{t\bar{h}_l}^l)^t} \right) + \frac{SV_{\bar{h}_L}^L}{(1 + WACC_{T\bar{h}_L}^L)^T} \quad \forall \bar{h}_L \quad (13)$$

#### 4. Illustrative example

The special characteristics of the proposed approach are highlighted by solving the design-planning of a SC comprising three potential facility locations that can act as distribution and/or processing sites. A set of potential equipment technologies are assumed to be available for the processing sites. Five products ( $P1-P5$ ) can be manufactured into seven different equipments types ( $TA$  to  $TG$ ); final products can be transferred to three markets ( $M1-M3$ ) even without passing through distribution centers. Batch products  $P4$  and  $P5$ , which are produced in batch equipments  $TD-TG$ , follow the STN example presented in Kondili et al<sup>2</sup>. A time horizon of five years is considered. It is composed of twelve planning periods with a length of one month each. In this example, market demand, prices of final products and interest rates are regarded as uncertain factors which unfold every year. A scenario tree which contains 32 leaf nodes (scenarios) is considered. It takes 27,970 CPU seconds to reach a solution for the design problem with a 5% integrality gap on an Intel Core Duo 2 computer using the MIP solver of CPLEX.

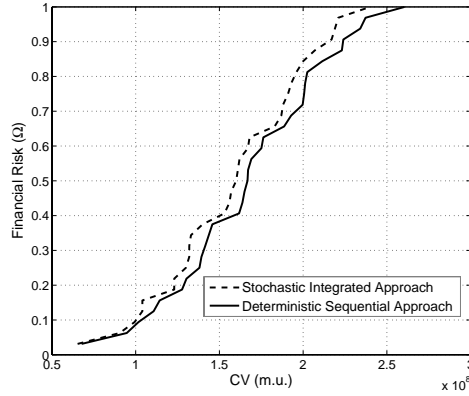


Figure 2. Financial risk curve of Corporate Value

The two stage shrinking horizon approximation presented in the work of Balasubramanian and Grossmann<sup>4</sup> was used to solve the multistage stochastic problem. In the first step of the control strategy, a design problem is solved. For the first month the scheduling model is taken into consideration. The problem has been also solved using a sequential manner (bi-level optimization) in order to compare it with the proposed approach. In this sequential manner the scheduling is not included when dealing with the design of the SC network. As shown in Fig. 2, the  $E[CV]$  and financial risk for the traditional approach seem to yield to better values. It should be noted that differences may arise when executing detailed scheduling in the sequential approach. Obviously, this fact occurs because of the aggregated capacity overestimation.

Recalling this fact, the MPC algorithm has been repeated during 10 planning month periods. Here, the uncertainty is assumed to unveil every month. The results are shown in Fig. 3. It can be noticed that the proposed integrated approach gives higher accumulated free cash flows (which are a key element in corporate value calculation) than the sequential one after the implementation of scheduling. An improvement of 12.33% is achieved.

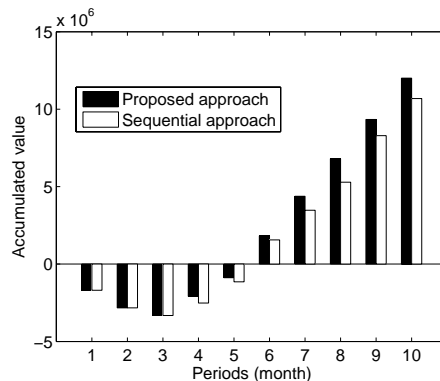


Figure 3. Accumulated value for both approaches

## 5. Final considerations and future work

A means of integrating the three standard SC decision levels is presented. Moreover, a novel SC design-planning model that permits a simple integration with scheduling models is proposed. The results show that significant improvements can be gained when all of these decision levels are incorporated into a single model. Moreover, the absence of scheduling can lead to apparently better corporate value which does not correspond to real scenario; resulting in myopic decision making. A drawback of the proposed approach is the computational burden that is required to solve the stochastic integrated monolithic model. Future work will be focused on applying decomposition strategies for tackling the aforementioned problem.

## Acknowledgments

Financial support received from the "Generalitat de Catalunya" (FI grants) and Ministerio de Educación y Ciencia (FPU grants). European Community (projects PRISM-MRTN-CT-2004-512233) is fully appreciated. Besides, financial support from Xartap (I0898) and ToleranT (DPI2006-05673) projects is gratefully acknowledged.

## References

1. I.E. Grossmann, 2005, Enterprise-wide optimization: A new frontier in process systems engineering. *AICHE J.*, 28, 260-275
2. L. Puigjaner, J.M. Lainez, 2007, Capturing dynamics in integrated supply chain management, *Comput. Chem. Eng.*, doi:10.1016/j.compchemeng.2007.10.003.
3. E. Kondili, C. Pantelides, R. Sargent, 1993, A general algorithm for short-term scheduling of batch operations-I: MILP formulation, *Comput. Chem. Eng.*, 17, 211-227.
4. J. Balasubramanian, I.E. Grossmann, 2004, Approximation to multistage stochastic optimization in multiperiod batch plant scheduling under demand uncertainty. *Ind. Eng. Chem. Res.*, 43, 3695-3713.