TWO DIMENSION CYCLIC HOIST SCHEDULING

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Abstract

In multi-stage material-handling (MSMH) processes, such as electroplating and polymeric coating, cyclic hoist scheduling (CHS) is often employed for the maximum production efficiency. The previous studies on CHS, however, almost exclusively focus on optimal solution identifications for MSMH processes spatially designed as a straight line. Such a one-dimensional (1D) production line will cause hoists spending too much time on free and loaded moves to transferring jobs along the production line, which will inherently reduce the production efficiency. In this paper, a new two-dimensional (2D) CHS modeling method has been developed for general MSMH processes. The developed 2D CHS modeling methods can significantly increase the production efficiency compared with the previous 1D CHS methods, which has been demonstrated by case studies.

Keywords

Multistage material-handling process, Hoist scheduling, Two-dimensional production line.

Introduction

Multistage material-handling (MSMH) are widely used for manufacturing massive amounts of work pieces in industries. Surface finishing and printed circuit board fabrication are typical application examples. MSMH processes usually employ controlled hoist(s)/crane(s) to lift, transport, and drop jobs among different work stations along their production lines to accomplish their multi-stage processing. During the transportation, the hoist follows a given movement schedule based on the processing recipe of every job inline. The job-processing recipe restricts the job-processing sequence and the processing time window at different stages. Sometimes, an MSMH production line can be used to produce multiple types of jobs with different recipes as long as a proper hoist scheduling is provided. As a large amount of parts is processed in the production line, the system commonly runs in a cyclic mode. The hoist is programmed to perform a specified sequence of operations and finish a specified set of jobs repeatedly, which is called cyclic hoist scheduling (CHS). Each repetition of such a sequence of hoist operations is called a *cycle* and its duration is called *cycle time*. Note that in one cycle the produced product of one type of job on the unloading zone may not be the same one picked up from the loading zone; it could be the one from the previous cycle. The cycle time can be the indication of the process productivity. The less the cycle time is used to finish the same set of jobs, the more productivity the production line is. Thus, hoist scheduling must be well developed to satisfy the manufacturing request of each job, meanwhile to maximize productivity of the production line or minimize the cycle time.

The earliest report on computer-aided hoist scheduling was made by Phillips and Unger (1976). They used mixed integer programming (MIP) to schedule a process with one hoist in one-capacity processing units. Since then, a number of other new methods, especially mathematical programming based methods, had been introduced (Baptiste et al., 1992; Lei and Wang, 1994;

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Armstrong et al., 1994). Manier and Bloch (2003) summarized a classification scheme for hoist scheduling problems. These problems typically required a cyclic schedule involving one or more types of parts, and the most common objective is to minimize cycle time. To increase operability or productivity of a production line, research works on multi-hoist scheduling had also been studied as well (Lei and Wang, 1991; Manier et al., 2000; Leung and Levner, 2006; Aron et al., 2010). To increase the flexibility of the production line, dynamic hoist scheduling (DHS) was developed to handle a situation that new jobs come randomly (Goujon and Lacomme, 1996; Riera and Yorke-Smith, 2002).

No matter which kind of scheduling studied by above authors, the production lines were all designed in a 1D line. However, when the production line is too long or the hoist moving speed is slow compared to the processing time at different stages, the production efficiency will be low because lots of cycle time is wasted on hoist traveling. Adding additional hoists or work stations may solve the problem (Fu et al., 2013), but it requires additional cost. In this paper, the concept of 2D production line is proposed. The 2D design will not only reduce the cycle time in terms of saving hoist traveling time, but also save spatial working spaces. In this paper, a 2D CHS MILP model for the productivity maximization of general MSMH processes has been developed and demonstrated

Problem Statement

The studied MSMH processes contain a number of processing units in 2D dimension, which are used to process multiple types of jobs with different processing recipes. The objective of hoist scheduling is to maximize the production rates of each type of job in a cyclic way through optimizing the hoist movement schedule in the 2D dimension. The given information, information to be determined, and assumptions of the studied problem are summarized below.

Given information: (1) an MSMH production line in 2D dimension with one hoist; (2) job capacity of each processing unit; (3) number of different types of jobs to be processed; (4) the processing recipe of each type of job, including the processing sequence and processing time limits at each processing unit; and (5) the hoist travelling time between any spatial locations/processing units.

Information to be determined: (1) the minimum cycle time which is equivalent as the maximum production rate for each type of job; (2) the detailed schedule of the hoist movement in the 2D dimension; and (3) every job processing time in each processing unit.

Assumptions: (1) there are no job capacity limits in the loading/unloading zones; (2) a processing unit may have multiple slots and every slot could at most hold one job at a time; and (3) the time for the hoist to change movement direction in 2D dimension is negligible.

General MILP CHS Model

In this section, an MILP model addressing 2D CHS is introduced, which includes the objective function, hoist travel time table, hoist movement constraints, processing time constraints, unit processing-capacity constraints, and variable bounds.

Objective function

As aforementioned, the optimal hoist scheduling is to maximize the productivity, which is equivalent to minimize the cycle time, as shown in Eq. (1).

$$Obj = \min T \tag{1}$$

where T is the cycle time of the hoist schedule.

Determination of Hoist travel time table

Assume the investigated production line is composed of i units ($SI_n = \{1, \cdots, i_n, \cdots, I\}$). The first unit is the loading zone and the last unit is the unloading zone. Different types of jobs ($SN = \{1, \cdots, n, \cdots, N\}$) will be processed in each unit sequentially and stay for a required period according to their processing recipes. Because units are evenly separated and the hoist movement times between two adjacent units are the same, we can build a hoist movement time table, which shows the required hoist travelling time, $Mv_{i,i'}$, between any two units i and i'.

Hoist movement constraints

Hoist movement constraints are used to define and restrict variables of the pickup time ($S_{i,n}$) and drop-off time ($E_{i,n}$) of job n in unit i. The hoist movement can be separated into two kinds: loaded moves which the hoist carries a job and free moves which the hoist carries nothing. Zhao etc. (2013) used Eq. (2) to define the sequence of two loaded movements by a binary variable $w_{i,n,i',n'}$. If $w_{i,n,i',n'}$ is 1, the loaded move of job n' from unit i' is ahead of another loaded move of job n from unit i; otherwise, if the loaded move of job n' from unit i' is behind another one of job n from unit i, $w_{i,n,i',n'}$ equals to 0. Meanwhile, another equation is needed to fulfill above constraints as shown in Eq. (3).

$$M(w_{i,n,i',n'} - 1) \le E_{i+1,n} - S_{i',n'} \le M w_{i,n,i',n'} \tag{2}$$

$$-M w_{i,n,i',n'} \le E_{i'+1,n'} - S_{i,n} \le M(1 - w_{i,n,i',n'}) \tag{3}$$

$$\forall i, i' \in SI_n; \forall n, n' \in SN; i \ne i', or, n \ne n'$$

Cyclic scheduling problem is a round-table problem. Thus, it does not matter to select a unit/job to the hoist scheduling. In this paper, $S_{1,1}$ as shown in Eq. (4) is designated as the starting time of the cyclic schedule, which means the loading zone (i=1) of job 1 is assigned as the initial starting unit of a cyclic operation.

$S_{1,1} = 0$ (4)

Equation (5) describes the relation between the starting and ending time for every loaded move. It suggests the ending time of a loaded move is equal to the starting time of the loaded move, plus the hoist travelling time between unit i and unit i' ($Mv_{i,i'}$). Similarly, the starting time of a loaded move should be larger than the ending time of the last loaded move plus the hoist transitional time as shown in Eq. (6), where the larger sign indicates the possible hoist waiting time after the hoist arrives at unit i .

$$E_{i++1,n} = S_{i,n} + Mv_{i,i++1} \ , \ \forall i, i++1 \in SI_n; \forall n \in SN \quad (5)$$

$$S_{i,n} \geq (E_{i',n'} + Mv_{i',i}) w_{i',n',i,n} \quad (6)$$

$$\forall i, i' \in SI_n; \forall n, n' \in SN; i \neq i', or, n \neq n'$$

Nonlinear Eq. (6) can be reformulated as Eqs. (7) through (9), where a positive variable $h_{i',n',i,n}$ is introduced to substitute $\left(E_{i',n'} + Mv_{i',i}\right) w_{i',n',i,n}$.

$$0 \leq \left(E_{i',n'} + Mv_{i',i}\right) - h_{i',n',i,n} \leq M\left(1 - w_{i',n',i,n}\right) \quad (7)$$

$$h_{i',n',i,n} \leq M w_{i',n',i,n} \quad (8)$$

$$S_{i,n} \geq h_{i',n',i,n} \quad (9)$$

$$\forall i, i' \in SI_n; \forall n, n' \in SN; i \neq i', or, n \neq n'$$

At the end of a hoist cycle, the empty hoist should come back to the loading zone, so that the next cycle could start. Equation (10) suggests that the total cycle time should be greater than any ending time of a loaded move plus its free move time to the loading zone.

$$T \geq E_{i,n} + Mv_{i,1}, \quad \forall i \in SI_n; \forall n \in SN \quad (10)$$

Equations (11) and (12) indicate the logic relation between $S_{i,n}$ and $E_{i,n}$ based on two scenarios: (i) if $S_{i,n} > E_{i,n}$, it suggests the hoist first drops the job and later lifts a job in the same operation cycle; then the binary variable $x_{i,n}$ is 1 and $y_{i,n}$ is 0; (ii) if $S_{i,n} < E_{i,n}$, it suggests the hoist first releases a job in the previous operation cycle, and later picks it up in the current operation cycle; then the binary variable $x_{i,n}$ is 0 and $y_{i,n}$ is 1.

$$-Mx_{i,n} < E_{i,n} - S_{i,n} < M\left(1 - x_{i,n}\right) \quad (11)$$

$$-My_{i,n} < S_{i,n} - E_{i,n} < M\left(1 - y_{i,n}\right) \quad (12)$$

$$\forall i \in SI_n; \forall n \in SN$$

Processing time constraints

The processing time for job n in unit i is represented as $p_{i,n}$. It is the time interval during which a job stays in the unit. Equation (13) gives a general formula for $p_{i,n}$. As discussed for Eqs. (11) and (12), if $S_{i,n} > E_{i,n}$, or the hoist first drops the job and later lifts a job in the same operation cycle; $y_{i,n}$ equals to 0, then $p_{i,n}$ just equals to $S_{i,n} - E_{i,n}$. When $S_{i,n} < E_{i,n}$, which means the hoist first releases a job in the previous operation cycle, and later picks it up in the current cycle; then $y_{i,n}$ is 1, and $p_{i,n}$ should be equal to $S_{i,n} + T - E_{i,n}$.

$$p_{i,n} = S_{i,n} - E_{i,n} + T y_{i,n} \ , \ \forall i \in SI_n; \forall n \in SN \quad (13)$$

The nonlinear Eq. (13) can be replaced by linear Eqs. (14)-(16).

$$p_{i,n} = S_{i,n} - E_{i,n} + g_{i,n} \quad (14)$$

$$0 \leq T - g_{i,n} \leq M\left(1 - y_{i,n}\right) \quad (15)$$

$$0 \leq g_{i,n} \leq M \ y_{i,n} \quad (16)$$

$$\forall i \in SI_n \ ; \ \forall n \in SN$$

Equation (17) bounds the processing time with the lower and upper limits, $RT_{i,n}^{lo}$ and $RT_{i,n}^{up}$, according to the job processing recipe.

$$RT_{i,n}^{lo} \leq p_{i,n} \leq RT_{i,n}^{up} \ , \ \forall i \in SI_n; \forall n \in SN \quad (17)$$

Unit processing-capacity constraints

For a unit with a single job capacity; the job stayed in the unit must be moved out before another job can be moved in. Liu etc. (2002) proposed a method for multi-function tank to ensure there is no conflict in using the same tank for different processing stages. Their method can be used here for single job capacity unit to ensure there is no conflict in using the same unit for different jobs. Two more binary variables, $u_{i,n,n'}$ and $v_{i,n,n'}$ are introduced: $u_{i,n,n'}$ equals to 1 if $E_{i,n'}$ is larger than $S_{i,n}$; otherwise, $u_{i,n,n'}$ equals to 0. Similarly, $v_{i,n,n'}$ equals to 1 if $E_{i,n}$ is larger than $S_{i,n'}$; otherwise, $v_{i,n,n'}$ equals to 0. The constraints are shown in Eqs. (18) and (19).

$$M\left(u_{i,n,n'} - 1\right) < E_{i,n'} - S_{i,n} \leq M \ u_{i,n,n'} \quad (18)$$

$$M\left(v_{i,n,n'} - 1\right) < E_{i,n} - S_{i,n'} \leq M \ v_{i,n,n'} \quad (19)$$

$$\forall i, i' \in SI; \forall n, n' \in SN; n \neq n'$$

Since two jobs cannot be processed in the same unit which are single capacity, there are only four possible scenarios as discussed by Liu etc. (2002): (i) $E_{i,n} < S_{i,n} < E_{i,n'} < S_{i,n'}$, (ii) $E_{i,n'} < S_{i,n'} < E_{i,n} < S_{i,n}$, (iii) $S_{i,n'} < E_{i,n} < S_{i,n} < E_{i,n'}$, and (iv) $S_{i,n} < E_{i,n'} < S_{i,n'} < E_{i,n}$. They also proposed two equations to guarantee these four conditions, which are close to Eqs. (20) and (21).

$$x_{i,n} + y_{i,n} = 1, \quad \forall i \in SI; \forall n \in SN \quad (20)$$

$$x_{i,n} + x_{i,n'} + u_{i,n,n'} + v_{i,n,n'} = 3 \quad (21)$$

$$\forall i, i' \in SI; \forall n, n' \in SN; n \neq n'$$

Variable bounds

All the continuous variables T, $S_{i,n}, E_{i,n}, p_{i,n}, g_{i,n}, h_{i,n,i',n'}$ have a lower bound of zero and an upper bound of M. As described, $x_{i,n}, y_{i,n}, w_{i,n,i',n'}, u_{i,n,n'}$ and $v_{i,n,n'}$, are defined as binary variables.

In summary, the developed general MILP CHS model include Eqs. (1)-(5), (7)-(12), and (14)-(21). All the case studies generated from the model are programmed and solved in GAMS v23.3 with the MILP solver of CPLEX.

Case Study

1D CHS scheduling

The developed CHS model can also applied to 1D CHS. The traditional 1D production line is first studied for the purpose of comparison with the 2-dimenison production line. The studied MSMH process line is first designed as a 1D line as in Figure 1. All units, except the loading and unloading zone, only have single-job processing capacity. The production line is able to process three types of jobs, A, B, and C, whose recipes are also shown in Figure 1. In every operation cycle, only one product of each job is processed, i.e., one A, one B, and one C. The processing time constrains for each job in different units are listed in Table 1.

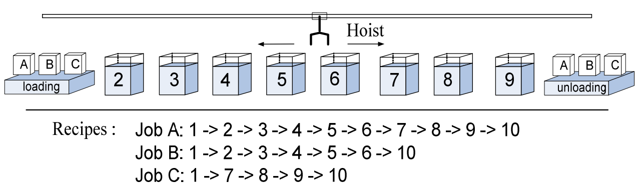

Figure 1. 1D Production line of an MSMH process

Table 1. Processing Time Limits for Case Studies

	Job n	$RT_{i,n}^{lo}$ (sec)			$RT_{i,n}^{up}$ (sec)		
		A	B	C	A	B	C
Unit i	1	-	-	-	-	-	-
	2	35	55	-	200	200	-
	3	55	55	-	200	200	-
	4	20	45	-	200	200	-
	5	40	35	-	200	200	-
	6	55	20	-	200	200	-
	7	20	-	40	200	-	200
	8	45	-	55	200	-	200
	9	35	-	45	200	-	200
	10	-	-	-	-	-	-

Table 2 shows the calculated hoist travel time between any two units. The optimized schedule of 1D production line is shown in Figure 2. The minimum processing time to produce each product of A, B, and C in one cycle is 151 seconds. The solid lines represent the hoist's loaded moves between two units, carrying product A, B or C which is labeled above the line. The dotted lines represent the hoist's free moves which carries nothing. The up arrow means the hoist lift a product from a unit and the down arrow means the hoist drop a product to a unit. Also note that there are two short zigzag lines for unit 8 between 22 s and 26 s and for unit 2 between 142 s and 145s. These two zigzag lines are the waiting periods when the hoist stays above unit 8 for 4 seconds before it picks up a C from unit 8 and waits for 3 seconds before picking up a B from unit 2. There is no other waiting period in the schedule.

Table 2. $Mv_{i',i}$ (sec.) for 1D Production Line

i \ i'	1	2	3	4	5	6	7	8	9	10
1	0	2	4	6	8	10	12	14	16	18
2	2	0	2	4	6	8	10	12	14	16
3	4	2	0	2	4	6	8	10	12	14
4	6	4	2	0	2	4	6	8	10	12
5	8	6	4	2	0	2	4	6	8	10
6	10	8	6	4	2	0	2	4	6	8
7	12	10	8	6	4	2	0	2	4	6
8	14	12	10	8	6	4	2	0	2	4
9	16	14	12	10	8	6	4	2	0	2
10	18	16	14	12	10	8	6	4	2	0

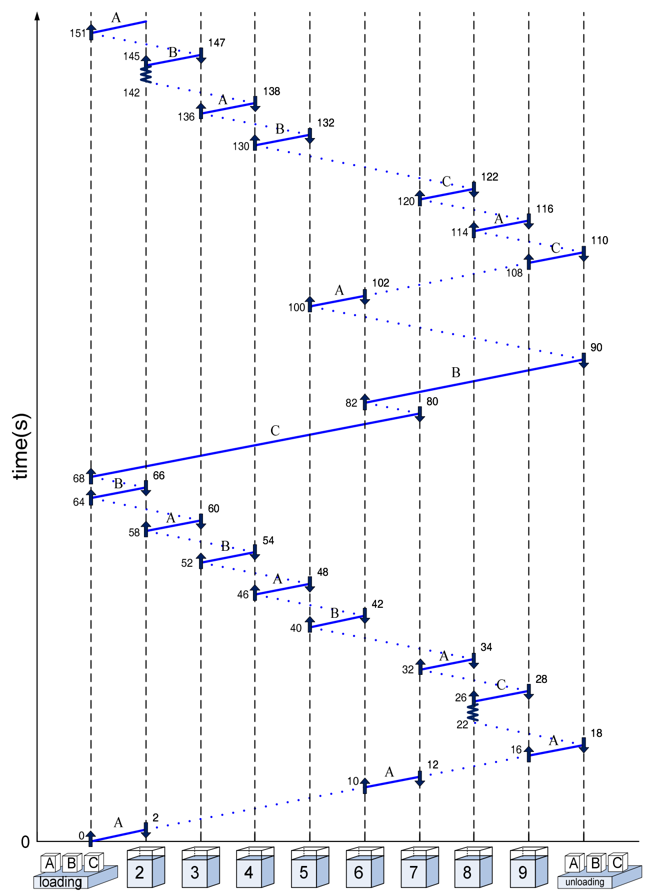

Figure 2. CHS solution for 1D Production Line

2D CHS scheduling

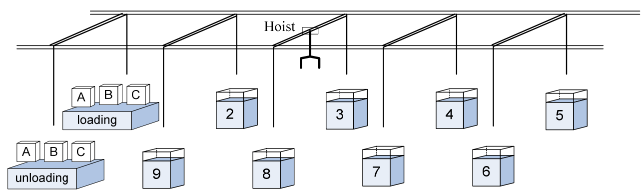

Figure 3. 2D Production line of the same MSMH process

Figure 3 shows the 2D production line of the same MSMH process, which is arranged in two rows. The hoist movement time information is shown in Table 3. The optimized schedule is shown in Figure 4. The optimal solution shows that the minimum cyclic time is 131 seconds, which is 20 seconds less than the 1D CHS case. The physical meanings of solid/dot lines and up/down arrows in Figure 4 are the same as those in Figure 2. There is a waiting period of 11 seconds above unit 6 before the hoist can lift up product A and carry it to unit 7. Note that when the hoist moves to a unit in the different x and y positions of the previous unit, the hoist will move along the x direction first, and then move along the y direction to reach the scheduled unit. For example, the hoist should move from unit 7 at 31 sec. to unit 5 at 35 sec. without carrying any job as shown in Figure 4. Thus, it will reach unit 4 at 33 second first in x direction; and then move to unit 5 in y direction as indicated in the figure.

Table 3. $Mv_{i',i}$ *(sec.) for the 2D Production Line*

i' \ i	1	2	3	4	5	6	7	8	9	10
1	0	2	4	6	8	10	8	6	4	2
2	2	0	2	4	6	8	6	4	2	4
3	4	2	0	2	4	6	4	2	4	6
4	6	4	2	0	2	4	2	4	6	8
5	8	6	4	2	0	2	4	6	8	10
6	10	8	6	4	2	0	2	4	6	8
7	8	6	4	2	4	2	0	2	4	6
8	6	4	2	4	6	4	2	0	2	4
9	4	2	4	6	8	6	4	2	0	2
10	2	4	6	8	10	8	6	4	2	0

Impact of hoist traveling speed on CHS results

Conceivably, the hoist traveling speed will affect the optimal CHS solutions. The impact has been studied and summarized in Table 4. Columns 2 and 3 in Table 4 show the cyclic time change with different hoist traveling speed, which characterized by the traveling time between two adjacent units. The first row shows the cyclic time when the hoist traveling time is 2 seconds as discussed above. The 2D production line can save 20 seconds or 13% in cyclic time compared to the 1D production line. When the hoist traveling time is increased to 3 seconds, the cyclic time for the 1D and 2D designs will increase to 204 and 170 seconds, respectively. The cyclic time can be reduced by 34 seconds or 17%. When the hoist traveling time increases further to 4 seconds, the cyclic time of 1D design is increased greatly by 60 seconds and the cyclic time of 2D designs only have 36 seconds increase, which is 22% reduction in cyclic time. Thus, Table 4 concludes that the slower the hoist travels, the more cyclic time 2D designs could save. This is understandable because when the hoist travel time increases, the 2D process design will save more time on hoist free/loaded moves than 1D design due to the spatially more compact advantage.

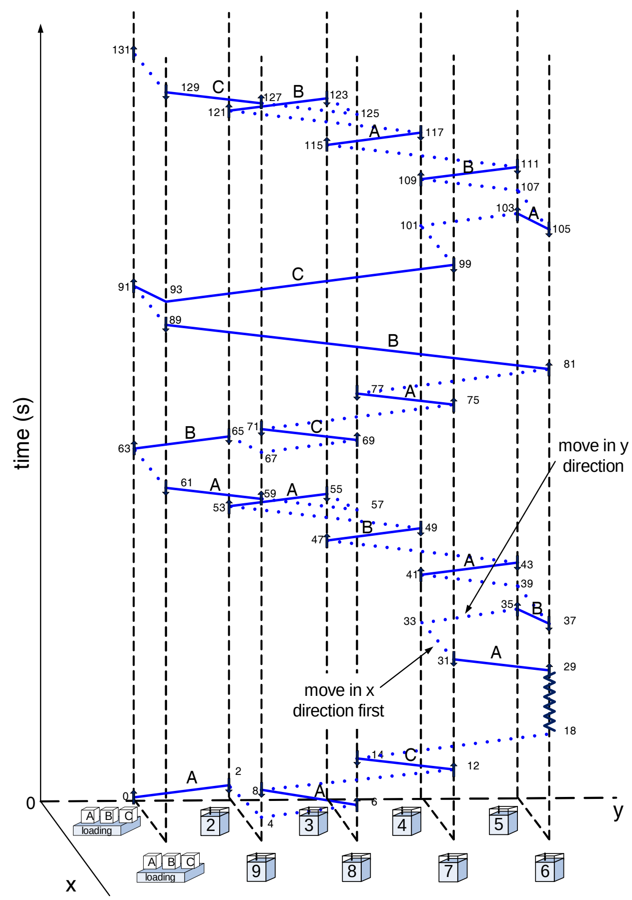

Figure 4. CHS solution for 2D Production Line.

Table 4. Effect of Hoist Traveling Time on Cyclic Scheduling Time

Hoist Traveling Time (Sec)	Cyclic Time (Sec)		Saved Cyclic Time (Sec)	Saved Cyclic Time (%)
	1D Case	2D Case		
2	151	131	20	13
3	204	170	34	17
4	264	206	58	22

Conclusion

In this study, we proposed the concept and MILP model of 2D CHS for general MSMH processes. Case studies show that the cyclic processing time can be greatly reduced by changing the 1D production line to 2D production line. The slower the hoist travels, the more cyclic time could be saved, which means the productivity can be more increased.

Acknowledgments

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