OFFSET-FREE NMPC WITH ROBUST CONSTRAINT SATISFACTION USING MODEL-ERROR MODELING*

Sakthi Thangavel†[1], Sankaranarayanan Subramanian[1] and Sebastian Engell[1]

[1]Process Dynamics and Operations Group, Technische Universität Dortmund, Germany

Abstract

Robustness and offset-free tracking remains a challenge in the field of nonlinear model predictive control (NMPC) in the presence of plant-model mismatch. This paper focuses on offset-free tracking with robust constraint satisfaction using NMPC. In the proposed approach, the discrepancy between the plant and a fundamental model is captured using a model-error model. The fundamental model is augmented with the model-error model to predict the future evolution of the states to obtain the optimal sequence of control inputs. Whenever new information from the plant is available, the model-error model is adapted. This helps the controller to react to time varying model-errors and disturbances and to achieve offset-free tracking with robust constraint satisfaction. The advantages of the proposed approach when compared to existing approaches for offset-free tracking are demonstrated with a continuous stirred tank (CSTR) example.

Keywords

Robust control, Nonlinear model predictive control, Offset-free tracking, ARX models, Constrained systems, Adaptive control

1 Introduction

Model predictive control (MPC) is widely used in the process industries because of its ability to handle multivariate systems with constraints. MPC usually uses a fundamental model to predict the future evolution of the plant. An open-loop optimization problem is solved at each sampling instant and the first control input in the sequence is applied to the plant. At the next sampling instant, the optimization problem is reinitialized using the measurement information available from the plant and it is solved again in a receding horizon fashion, thus providing feedback information (Rawlings and Mayne, 2009). The performance of MPC depends on the accuracy of the model used in the optimization.

In industry, it is often required to track a changing reference (Cairano, 2012). The advantages of model predictive control for a tracking problem were demonstrated in Stephens et al. (2013). Offset-free tracking with constraint satisfaction using an MPC in the presence of plant-model mismatch however remains a challenging task.

Several key contributions have already been made in this area. Pannocchia and Bemporad (2007) used a disturbance model along with the fundamental model to achieve an offset-free control. Maeder and Morari (2010) used target constraints in addition to the disturbance model to achieve offset-free tracking. These methods use an observer to estimate the states and the disturbances present in the augmented model. These approaches show that offset-free tracking can be achieved if the disturbance model can estimate the plant-model mismatch at steady state. These methods use a linear model in the controller design. An adaptive MPC strategy to obtain offset-free tracking using online model estimation is proposed in Bemporad et al. (2005). It uses an auto-regressive exogenous inputs (ARX) model as in Ljung (1999b) to identify the system at each control interval and uses this adapted ARX model in the MPC.

*The research leading to these results has received funding from the European Commission under grant agreement number 291458 (MOBOCON)

†Contact: *sakthi.thangavel@bci.tu-dortmund.de*

Morari and Maeder (2012) extended the approach in Maeder and Morari (2010) by using a nonlinear model and a constant disturbance model. A nonlinear estimation scheme was employed to estimate the states and the disturbances to achieve offset-free tracking. In the proposed approach, we build a linear model-error model to capture the unmodeled dynamics of the plant. Various approaches for model-error modeling are explained in Ljung (1999a). The proposed approach uses a model-error model along with the fundamental model of the plant in order to obtain offset-free tracking. The advantage of the proposed approach over the scheme in Bemporad et al. (2005) is that we are not approximating the nonlinear dynamics using a linear model, but only the dynamics of the plant which are not captured by the nonlinear model by a linear model. The proposed approach tries to capture the unmodeled dynamics of the plant in contrast to other approaches (Rawlings and Mayne, 2009) that try to estimate the plant-model mismatch using a constant disturbance model. A linear error-model approximates the transient response of the unmodeled dynamics of the plant better than a constant disturbance model. Hence the proposed approach can converge to the tracking-point faster than using a constant disturbance model and achieve robust constraint satisfaction.

We compare the tracking performance and robust constraint satisfaction of the proposed approach with bias update Seborg et al. (2011) and offset-free NMPC Morari and Maeder (2012) with the help of simulation study on a continuous stirred tank reactor (CSTR) example.

2 Problem statement

The dynamics of the plant to be controlled is given by

$$x_{k+1}^p = f_p(x_k^p, u_k), \tag{1a}$$

$$y_k^p = h_p(x_k^p, u_k) + w_k, \tag{1b}$$

where $x_k^p \in X \subseteq \mathbb{R}^{n_x}$ is the state, $y_k^p \in Y \subseteq \mathbb{R}^{n_y}$ is the output, $u_k \in U \subseteq \mathbb{R}^{n_u}$ is the input and $w_k \in W \subseteq \mathbb{R}^{n_y}$ is the measurement noise which is assumed to be white Gaussian noise. We assume that the true dynamics of the plant is not known but it can be approximately obtained using the fundamental model given below:

$$x_{k+1} = f(x_k, u_k), \tag{2a}$$

$$y_k = h(x_k, u_k). \tag{2b}$$

The number of states, output and control inputs of the plant is assumed to be known and it is given by n_x, n_y and n_u. The reference signals to be tracked are defined by r and are assumed to be asymptotically constant ($r_k \to r_\infty$) and are measured. The tracking error is given by

$$t_k^r = y_k^p - r_k. \tag{3}$$

We assume that the constraints are functions of measured states and control inputs. The goal of the controller is to achieve offset-free tracking with robust constraint satisfaction ($t_\infty^r = 0$).

3 Model-error modeling

Model-error models capture the discrepancy between the true plant and the predicted output (Reinelt et al., 1999). It is shown in Ljung (1999a) that the model-error can be modeled as a linear model such as Finite Impulse Response model (FIR), ARX model, Box Jenkins model and as a nonlinear model such as neural network finite impulse response model.

We choose an ARX model in order to model the model-error because of its simplicity and because it has less unknown model parameters to be estimated when compared to other approaches. The residual error between the plant measurements and the fundamental model is given by

$$e_k^p = y_k^p - y_k, \tag{4}$$

where $e_k^p \subseteq \mathbb{R}^{n_y}$. We approximate the nonlinear plant-model mismatch using a linear model-error model as

$$e_k^p = y_k^p - y_k = \frac{B(z)}{A(z)} u_k + \delta_k, \tag{5}$$

$$A(z) = I + A_1 z^{-1} + \ldots + A_{n_a} z^{-n_a}, \tag{6}$$

$$B(z) = B_1 z^{-1} + \ldots + B_{n_b} z^{-n_b}, \tag{7}$$

under the assumption that the process is sufficiently excited. $A_i, \forall i \in \{1, \cdots, n_a\}$ are $n_y \times n_y$ matrices, $B_j, \forall j \in \{1, \cdots, n_b\}$ are $n_y \times n_u$ matrices that have to be estimated using the measurement information obtained from the plant, δ_k represents the dynamics which are not captured by the linear model-error model (e.g. non-linearity, disturbances), z^{-i} is the backward shift (delay) operator. n_b denotes the number of zeros, n_a denotes the number of poles of the ARX model and these are tuning parameters which can be chosen offline.

The unknown parameters A_i and B_i can be estimated from the observed data by solving an optimization problem. The model parameters will be time varying if the plant-model mismatch is highly nonlinear. Hence we propose to use a weighted recursive least squares estimates with variable forgetting factor adapted from Golden and Ydstie (1989). The model-error model approximates the nonlinear plant-model mismatch locally about the current operating region. The formulation of such an optimization problem is given below

$$\min_{A_i, B_j} \sum_{k=q}^{N_m} [e_k^p - e_k]^T R_k [e_k^p - e_k], \tag{8a}$$

subject to

$$e_k = -\sum_{i=1}^{n_a} A_i e_{k-i}^p + \sum_{j=1}^{n_b} B_j u_{k-j}, \tag{8b}$$

$$h(e_k^p, e_k, u_k) \leq 0, \tag{8c}$$

where q is $\max(n_a, n_b)$, e_k is the plant-model mismatch predicted by the error model, N_m is the total number of measurement information available. R_k, the variable forgetting factor is a tuning parameter and it is a diagonal matrix whose values can be chosen from 0 to 1. If we choose the variable forgetting factor as an identity matrix ($R_k = I$), all the available information will be used to estimate the model-error model. Any additional constraint ($h(.)$) can be added to the estimation problem if required. The resulting model-error model is given as

$$e_k = \frac{B(z)}{A(z)} u_k. \tag{9}$$

The ARX model is transformed to a linear state space model to be used along with the fundamental model (Ljung, 1999b). The reduced model-error model in state space form is given as

$$x_{k+1}^e = A^e x_k^e + B^e u_k, \tag{10a}$$

$$e_k = C^e x_k^e + D^e u_k, \tag{10b}$$

where x^e denotes the state vector of the error dynamics.

4 Nonlinear Model Predictive Control with model-error model

The aim of the proposed NMPC with model-error model (NMPCMEM) is to obtain offset-free NMPC with robust constraint satisfaction, by reducing the plant-model mismatch. NMPCMEM achieves this by capturing the error dynamics using a model-error model.

The fundamental model is augmented with the model-error model in order to account for the plant-model mismatch. The augmented model is given as:

$$\underbrace{\begin{bmatrix} x_{k+1} \\ x_{k+1}^e \end{bmatrix}}_{x_{k+1}^{aug}} = \underbrace{\begin{bmatrix} f(x_k, u_k) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & C^e A^e \\ 0 & A^e \end{bmatrix} \begin{bmatrix} x_k \\ x_k^e \end{bmatrix} + \begin{bmatrix} C^e B^e + D^e \\ B^e \end{bmatrix} u_k,}_{f_{aug}(x_k^{aug}, u_k)} \tag{11a}$$

$$y_k^{aug} = h_{aug}(x_k^{aug}, u_k) = h(x_k, u_k) + e_k, \tag{11b}$$

Rawlings and Mayne (2009) and Morari and Maeder (2012) consider $A^e = B^e = C^e = D^e = 0$ and estimate x_k^e using an observer whereas the proposed approach estimates A^e, B^e, C^e, D^e using the plant-model mismatch data (e_k^p) and the control input (u_k).

The formulation of the NMPC problem using the augmented model is given as:

$$\min_{u_k} \sum_{k=0}^{N-1} J_k(r_k, x_k^{aug}, u_k) + J_N(r_N, x_N^{aug}, u_k), \tag{12a}$$

subject to

$$x_{k+1}^{aug} = f_{aug}(x_k^{aug}, u_k), \tag{12b}$$

$$g(x_k^{aug}, u_k) \leq 0, \tag{12c}$$

where N represents the length of the prediction horizon. $g : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \rightarrow \mathbb{R}^{n_g}$ denotes the constraints of the optimization problem where n_g denotes the total number of constraints. The stage cost is given by $J_i : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \rightarrow \mathbb{R}$ as

$$J_i(r_i, x_i^{aug}, u_i) = t_i^{r\,T} P t_i^r + \Delta u^T Q \Delta u, \tag{13a}$$

$$t_i^r = r_i - h_{aug}(x_i^{aug}, u_i), \tag{13b}$$

$$\Delta u = u_i - u_{i-1}, \tag{13c}$$

where P is a positive definite matrix with the dimension $n_y \times n_y$. Q is a positive definite matrix with the dimension $n_u \times n_u$.

The schematic diagram of the proposed approach is given in Figure 1. NMPCMEM solves the optimization problem (12) at every-time step and the first control input is applied to the plant in a receding horizon manner. We assume that full state measurement is available from the plant. The same control input is applied to the fundamental model and the discrepancy between the measurement data and fundamental model (e_k^p) is obtained. The ARX model is updated at every time-step by solving the optimization problem (8) using the observed data. The ARX model is converted to a state space model and the model-error model in the augmented model is updated. During the initial stages, enough information about the plant-model mismatch is not available, hence for $k \leq N_{model}$ we use the fundamental model in the NMPC without the model error model. N_{model} is chosen such that there is enough information available to estimate all the parameters present in the model-error model. We assume that the state trajectories are away from the constraints during this period. One of the key differences in the proposed approach when compared to the standard approach is that when $k > N_{model}$ the optimization problem is not reinitialized with the plant measurements (y_k^p), instead the model-error model is reinitialized at e_k^p (4). The plant measurement information enters the fundamental model via the model-error model. The necessary condition for offset-free tracking is $|e_\infty - e_\infty^p| = 0$.

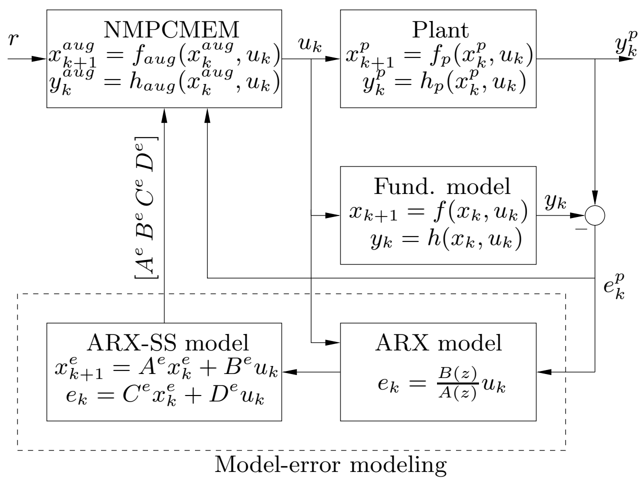

Figure 1. Schematic diagram of proposed nonlinear model predictive control with model-error model

If the error dynamics is highly nonlinear, it is difficult to approximate it using a linear model-error model. The forgetting factor can be tuned so that we can approximate the nonlinear plant-model mismatch in the operating region using a linear model-error model with a reasonable accuracy. Though the proposed scheme may not work for highly nonlinear error dynamics, the accuracy of the proposed scheme is higher than the other approaches that use a constant disturbance model because we consider the first-order information of the nonlinear error dynamics to update the fundamental model. This results in a better prediction accuracy and an improved closed loop performance.

5 Case study

A nonlinear CSTR bench mark problem from (Klatt and Engell, 1998) is adapted to illustrate the approach proposed in this paper via simulation results. Three chemical reactions take place inside the reactor:

$$A \rightarrow B \rightarrow C$$
$$2A \rightarrow D$$

where B is the desired product and C and D are unwanted by-products. The mathematical model of the CSTR is obtained from the molar balance of component A (of concentration c_A) and component B (of concentration c_B), the energy balance for reactor (of temperature T_R) and the jacket (of temperature T_J). The dynamics of the CSTR is given by the following differential equations

$$\dot{c}_A = F(c_{Ain} - c_A) - k_1 c_A - k_3 c_A^2, \tag{14a}$$

$$\dot{c}_B = -F c_B + k_1 c_A - k_2 c_B, \tag{14b}$$

$$\dot{T}_R = F(T_{in} - T_R) + \frac{k_w A}{\rho c_p V_R}(T_K - T_R) - \frac{k_1 c_A \Delta H_{AB} + k_2 c_B \Delta H_{BC} + k_3 c_A^2 \Delta H_{AD}}{\rho c_p}, \tag{14c}$$

$$\dot{T}_K = \frac{\dot{Q}_K + k_w A(T_R - T_K)}{m_K c_{pK}}, \tag{14d}$$

where k_i gives the reaction rate and follows the Arrhenius law

$$k_i = k_{0,\,i} e^{\frac{-E_{A,i}}{R(T_R + 273.15)}}. \tag{15}$$

Table 1. Activation energies of the true plant and the model

Parameter	Value		Unit
	True	Model	
$\frac{E_{A,1}}{R}$	9953.47	9758.3	K
$\frac{E_{A,2}}{R}$	9953.47	9758.3	K
$\frac{E_{A,3}}{R}$	7742.52	7704.0	K

The feed ($F = \frac{\dot{V}}{V_R}$), the inflow normalized by the volume of the reactor and the cooling capacity ($\dot{Q}_K$) are the control inputs with bounds $F \in [5, 100]\,\mathrm{h}^{-1}$ and $\dot{Q}_K \in [-13500, 0]\,\mathrm{kJ\,h}^{-1}$. We assume that the activation energy of the reaction taking place inside the plant f_p is not exactly known and only 99% of the feed (F) is fed into the reactor due to actuator error. The true values of the parameters that are known along with the initial condition of the states and their bounds can be obtained from Lucia and Engell (2015). The true values of the activation energy of the reaction along with the values that are used in the model equation is given in Table 1. The control task is to track the concentration of component B (c_B) respecting the constraint on the reactor temperature ($T_R \leq 150\,^\circ\mathrm{C}$).

6 Results

This section shows the simulation results obtained for the case study described above using standard NMPC (with no plant-model mismatch), NMPC with bias correction (with plant-model mismatch), nonlinear offset-free NMPC (Morari and Maeder, 2012) and the proposed scheme (NMPCMEM).

In all the cases the length of the prediction horizon is chosen as 10 time-steps. The sampling time of the plant is 0.005 h. The NMPC with bias correction is implemented as in Seborg et al. (2011) and integrates the tracking error as mentioned in Lucia and Engell (2015). The augmented model used for nonlinear offset-free NMPC considers that the disturbance model enters the system through the input feed rate F as mentioned in (Morari and Maeder, 2012). We use the Extended Kalman Filter as the observer with a state covariance matrix as diag([1, 1, 10, 10, 10]) and a measurement co-variance matrix as diag([0.01, 0.01, 0.1, 0.1]). The N_{model} of NMPCMEM is chosen as 10. The order of the denominator and numerator of the model-error model is chosen as 1 ($n_a = n_b = 1$). We consider that $R_k = I$ i.e. we use all the available information to build the model-error model.

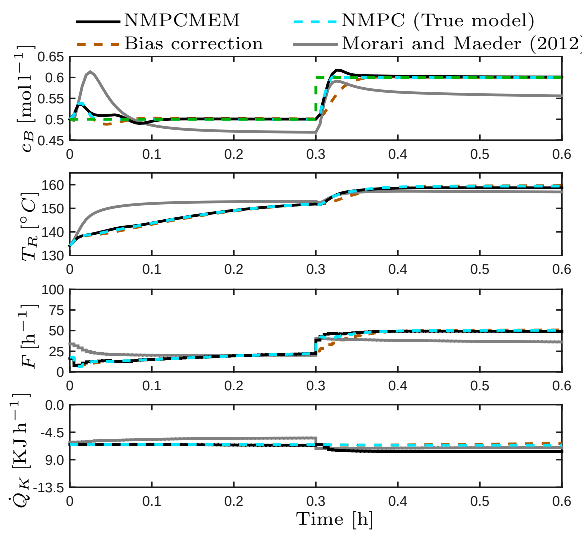

Figure 2. Concentration of component B, reactor temperature and control inputs obtained using different NMPC strategies without additional constraint on the reactor temperature.

The simulation results of the concentration of the component B (c_B), temperature of the reactor (T_R) and control inputs (feed rate (F) and cooling capacity ($\dot{Q}_K$) without the constraint on the temperature of the reactor are shown in Figure 2. The green-dotted line represents the set-point to the concentration of component B. It can be seen that all the controllers except the nonlinear offset-free NMPC proposed in Morari and Maeder (2012) achieve offset free tracking. The Morari and Maeder (2012) controller fails because the plant-model mismatch does not converge to the disturbance model present in the augmented model within finite time. NMPC with bias correction reaches the first set-point ($r_k = 0.5$) faster when compared to the proposed NMPC because during the initial stages when enough information about the plant-model mismatch is not available, NMPCMEM waits until $k \leq 10\,(N_{model})$ to build the model-error model. It can also be seen that NMPCMEM reach the second set-point ($r_k = 0.6$) faster than NMPC with bias correction .

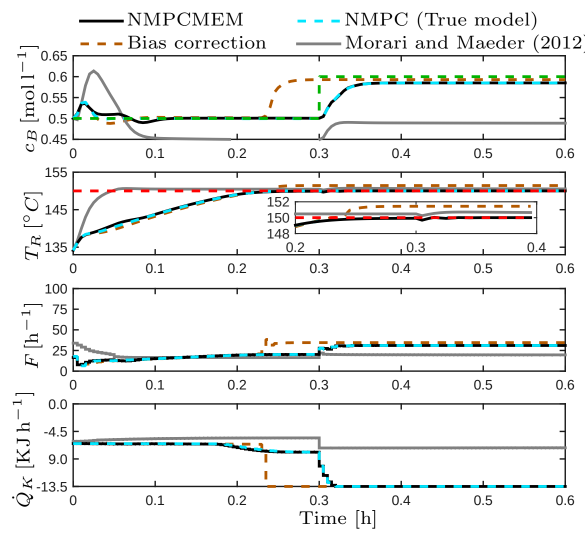

Figure 3. Concentration of component B, reactor temperature and control inputs obtained using different NMPC strategies with a constraint on the reactor temperature

Figure 3 shows the simulation results obtained using different controllers with an additional constraint on the reactor temperature. The red-dotted line shows the constraint on the reactor temperature. It is implemented as a soft constraint. It can be seen that both NMPC with bias correction and nonlinear offset-free NMPC violate the constraints. The NMPC with bias correction drifts away from the first set-point due to constraint violation. The fundamental model predicts that the temperature of the reactor would go down and satisfy the constraint for the chosen optimal control input but in reality this does not happen due to the plant-model mismatch. Hence there is a violation of the constraint. The proposed scheme reduces this plant-model mismatch with the help of the model-error model and does not violate the constraint even though the set-point is not tracked. The NMPC with bias correction also suffers from the windup effect due to the integration of the tracking error in the presence of an active constraint.

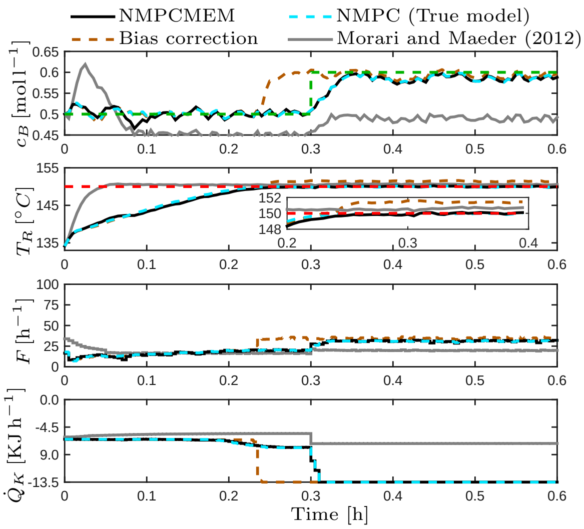

Figure 4. Concentration of component B, reactor temperature and control inputs obtained using different NMPC strategies with measurement error and a constraint on the reactor temperature

We also investigate the case where the measurement information is corrupted by additive white Gaussian noise and the bound on the temperature measurement is given by ± 0.1 and the bound on the concentration measurement is given by ± 0.01. The simulation results obtained using different NMPC strategies with measurement error and a constraint on the reactor temperature is shown in Figure 4. It can be seen from the figure that NMPC with bias correction and nonlinear offset-free NMPC violates the constraint where as the proposed NMPC satisfies the constraints also in the presence of the measurement noise.

7 Conclusion

This paper shows the possibility of achieving offset-free NMPC with robust constraint satisfaction using a model-error model by means of simulation studies. A linear ARX model is used to approximate the plant-model mismatch. The fundamental model is augmented to the model-error model and used to obtain optimal control movements to achieve offset-free tracking with robust constraint satisfaction. The accuracy of the proposed NMPC depends on the accuracy of the model-error model. Future work will focus on tuning of the forgetting factor and investigation on other model-error models for large plant-model mismatch.

References

Bemporad, A., Morari, M., and Ricker, N. L. (2005). Model predictive control toolbox. *The Mathworks Partner Series.*

Cairano, S. D. (2012). An industry perspective on mpc in large volumes applications: Potential benefits and open challenges. *IFAC Proceedings Volumes*, 45(17):52 – 59.

Golden, M. P. and Ydstie, B. E. (1989). Adaptive extremum control using approximate process models. *AIChE journal*, 35(7):1157–1169.

Klatt, K.-U. and Engell, S. (1998). Gain-scheduling trajectory control of a continuous stirred tank reactor. *Computers & Chemical Engineering*, 22(4):491 – 502.

Ljung, L. (1999a). Model validation and model error modeling.

Ljung, L., editor (1999b). *System Identification (2Nd Ed.): Theory for the User.* Prentice Hall PTR, Upper Saddle River, NJ, USA.

Lucia, S. and Engell, S. (2015). Potential and limitations of multi-stage nonlinear model predictive control. *IFAC-PapersOnLine*, 48(8):1015 – 1020.

Maeder, U. and Morari, M. (2010). Offset-free reference tracking with model predictive control. *Automatica*, 46(9):1469 – 1476.

Morari, M. and Maeder, U. (2012). Nonlinear offset-free model predictive control. *Automatica*, 48(9):2059 – 2067.

Pannocchia, G. and Bemporad, A. (2007). Combined design of disturbance model and observer for offset-free model predictive control. *IEEE Transactions on Automatic Control*, 52(6):1048–1053.

Rawlings, J. B. and Mayne, D. Q. (2009). *Model Predictive Control : Theory and Design.* Madison, Wis. Nob Hill Pub. cop.

Reinelt, W., Garulli, A., Ljung, L., Braslavsky, J. H., and Vicino, A. (1999). Model error concepts in identification for control.

Seborg, D. E., Edgar, T. F., Mellichamp, D. A., and Doyle, F. J. (2011). *Process dynamics and control*, chapter 20. John Wiley & Sons, Inc,, 3 edition.

Stephens, M. A., Manzie, C., and Good, M. C. (2013). Model predictive control for reference tracking on an industrial machine tool servo drive. *IEEE Transactions on Industrial Informatics*, 9(2):808–816.