Multi-period fair customer allocation in oligopolies

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Abstract

Contemporary process industries are constantly confronted with volatile market conditions that jeopardise their financial sustainability. Various approaches have been proposed in the literature to examine the impact of decentralisation on the optimal decision making within supply chain systems. Recently, a static game-theoretic approach for the fair customer allocation within oligopolies was proposed by Charitopoulos et al. (2020). Nonetheless, key issues related to the modelling and the impact of the related contractual agreements between firms and customers remain largely unexplored. In the present work, we examine the problem of fair customer allocation in oligopolies under different contractual agreements within a multiperiod setting. We consider an ensemble of contract types that vary in terms of pricing mechanisms and duration. The role of fairness is examined following the Nash bargaining scheme and the overall problem is formulated as a convex MINLP. For its efficient solution we employ two different solution techniques, i.e. (i) an outer approximation branch and refine global optimisation method and (ii) a piecewise linearisation strategy. The model impact and the benefits of fairness are evaluated through case studies from an industrial liquids market.

Keywords

Game theory, Supply chain optimisation, Customer allocation.

List of symbols

Sets

c	Customers
$cti_{(c,t,i)}$	Set of customer's tanks for product i
f	Oligopoly firms
i	Liquid products
k	Contracts
p	Time periods
t	Customer tanks

Parameters

α_f	Negotiation power of firm f
δ	Price increase parameter
$cost_{ictfp}$	Production cost of product i for customer c and tank t by firm f in time period p ($)
π_f^{sq}	Status quo profit of firm f prior to the fair allocation of the customers ($)
P_{ictfpk}	Price of product i for customer c and tank t served by firm f at time period p by contract k ($)
UPC_{ictfp}	Unit production cost of product i for customer c and tank t served by firm f at time period p ($)
USC_{ictf}	Unit service cost of demand of product i for customer c and tank t served by firm f ($/m^3$)

Binary Variables

W_{cfkp}	1, if a customer c is served by a company f with contract k in time period p
WS_{cfkp}	1, if a customer c is served by a company f with exactly one contract k in time period p

Continuous Variables

EC_{fp}	Electricity cost of firm f for time period p
IC_{fp}	Inventory cost of firm f for time period p
L_k	Duration of contract k
NC_{fp}	Forfeit cost of firm f for time period p
R_{fp}	Revenue of firm f for time period p
$RCfp$	Acquisition cost of firm f for time period p
SC_{fp}	Service cost of firm f for time period p

Introduction

Game Theory and Optimisation

Game theory has been extensively studied within the process systems engineering. Design of supply chains (Leng and Parlar (2005); Nagarajan and Sošić (2008)) and power system planning (Zhang et al. (2014)) have been facilitated from the use of a game theoretic approach. An early exam-

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ple of a SC design facilitated by Nash equilibrium analysis can be found in Sherali and Leleno (1988); they provide solution approached to reach the equilibrium solution of a two stage production model and evaluate the benefits of a coalition formation. Gjerdrum et al. (2002) presented two different solution approaches to solve the problem of fair transfer price and inventory optimisation. Zamarripa et al. (2012) have investigated the impact of game strategy, either cooperative or non-cooperative, as an extra degree of uncertainty in the original MILP supply chain (SC) problem with uncertain demand. Later on, Mahjoub and Hennet (2014) defined a minimal set of firms to achieve maximal expected profit and at the same time employed a profit sharing policy. It is noteworthy, that the game theoretic approach allows to take into consideration factors such as social or environmental impact of a design, which in a different framework would be difficult to quantify. Examples of game-theoretic approaches that have a sustainability impact include the modelling of a sharing economy framework in an organic food SC (Asian et al., 2019) and the research and development cooperation between supplier and manufacturer to mitigate spillover rates (Wu et al., 2021).

Fairness Schemes

Fair profit allocation is one of the main concerns in a cooperative game theory. An extensive review on fairness measures for decision-making and conflict resolution has been provided by Sampat and Zavala (2019). The selection of a fairness approach is not always straightforward, given that there may be conflicting objectives such as profit with sustainability pay-off. Lou et al. (2004) have evaluated such a conflicting problem in order to find the economically and environmentally optimal status of an industrial ecosystem. Fairness consideration has been recently accounted in resource allocation problems such as load balancing (Bertsimas et al. (2011)), electricity markets (Zavala et al. (2015)) and cost distribution (Liu and Papageorgiou, 2018). Even though it is common practice to evaluate fairness under the scope of a specific scheme, recent studies aim to compare the impact of different fairness schemes. Sampat and Zavala (2019) have examined different schemes for two interdisciplinary case studies, that of a power allocation problem and a geographical nutrient balancing. Later on, Cruz-Avilés et al. (2021) have evaluated different fairness schemes for the optimal allocation of water networks in eco-industrial parks.

Contractual Agreements

Park et al. (2006) have addressed the problem of contract modelling in a multi-period framework, including contract selection in supply chain models. Disjunctive programming was employed to address both long term and short term operations. The contracts proposed for supply/demand are: a) fixed price, b) discount after certain amount, c) bulk discount, and d) fixed duration. A classification of different supply contract in a multi-period programming problem for optimal contract selection was examined by Bansal et al. (2007). The study incorporated the dominant real-life contract features, such as purchase commitments and flexibility, commitment duration and bulk prices/ discounts. Qin et al. (2007) evaluate a non-cooperative Stackelberg game, where supplier acts as the leader and decides on a pricing policy, the buyer reacts a follower and determines the annual sales volume. For this application volume discounts are considered. Calfa and Grossmann (2015) have incorporated the optimal contract selection in the scheduling problem of a chemical process network, by choosing among the contracts proposed by Park et al. (2006). Recent publications examined different discount contracts for supplier/ manufacturer agreements in the process industry (Martín and Martínez (2018); Kirschstein and Meisel (2019)).

Problem Statement

For the detailed problem formulation we refer the reader to Charitopoulos et al. (2020), the equations used in the static model are augmented by a time series index and hence transformed to a multi-period setting. We will introduce here the new equations concerning the contractual agreements.

Customer assignment and contracts scheduling

At any time period (p) customers (c) must be served by one firm (f) and under one type of contract (k). This condition is modelled by introducing the binary variable W_{cfkp} and Eqs.(1)-(2).

$$\sum_f \sum_k W_{cfkp} = 1 \qquad \forall c, p \tag{1}$$

$$W_{cfkp} - W_{cfk,p-1} \leq WS_{cfkp} \qquad \forall f, c, k, p \tag{2}$$

Furthermore, at any time period (p) customers (c) may sign at most one contract (k) with one firm (f). To model this instance, the binary variable WS_{cfkp} is introduced which denotes whether at time period p customer c signed contract k with firm f along with Eq. (3).

$$\sum_f \sum_k WS_{cfkp} \leq 1 \qquad \forall c, p \tag{3}$$

To eliminate the possibility of a customer signing a contract prior to the end of their current one Eq.(4) is introduced.

$$\sum_f \sum_{p'=p-L_k+1}^{p} WS_{cfkp'} \leq 1 - \sum_f \sum_{k' \neq k} \sum_{p'=p-L_{k'}+1}^{p} WS_{cfk'p'}, \ \forall c, k, p \geq L_k \tag{4}$$

Effectively what Eq.(4) is implying is that if a contract of duration L_k starts at time period p then for the subsequent L_k periods no other contract (including a renewal of the incumbent) is allowed.

To ensure that once a customer signs a contract they must stay on for the entire duration of their contract Eq.(5) can be employed.

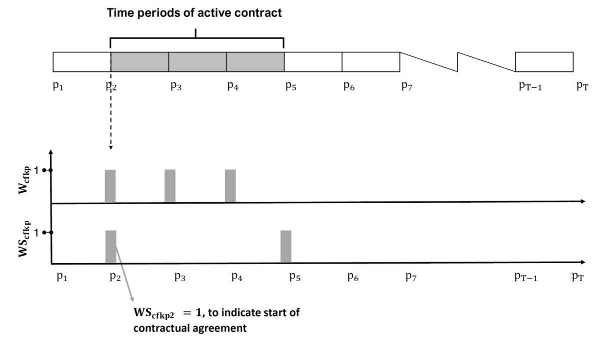

Figure 1: Relation between the binary variables WS_{cfkp} which denote the initiation of a contract and the binary variables W_{cfkp} which denote that a contract is active over the periods spanning its agreed duration L_k.

$$\sum_{p'=p}^{p+L_k-1} W_{cfkp'} \geq L_k WS_{cfkp} \quad \forall c,f,k,p \tag{5}$$

While Eq.(5) can guarantee that such condition will be stipulated, its relaxation can be poor and thus hinder the computational efficiency of the model. To this end, we also propose the Eq.(6) which is equivalent to Eq.(5) but has tighter relaxation.

$$\sum_{p+L_k-1}^{p} WS_{cfkp'} \leq W_{cfkp} \quad \forall c,f,k,p \geq L_k \tag{6}$$

It can be trivially shown that the relaxation of Eq.(6) is tighter than Eq.(5). Notice that in order to account for instances where a contract is signed but does not cover its entire duration within the incumbent planning horizon, Eq.(6)-(5) are only considered for $\forall p \leq |P| - L_k + 1$. A conceptual representation of the contract scheduling notions is given in Fig. 1.

Profit calculation

The profit of each firm f is calculated as the difference between the revenue and the total costs incurred by the customers' activity. The revenue is calculated as the selling price of product i multiplied by the resulting product demand from customers served by each firm. The profit $\pi_{f,p}$ for each firm f and a given time period p is given by Eq.(7).

$$\pi_f = \sum_p (R_{fp} - SC_{fp} - RC_{fp} - NC_{fp} - EC_{fp} - IC_{fp}) \; \forall f,p \tag{7}$$

Note that the formulation of Eq.(7) is equivalent to the one from Charitopoulos et al. (2020) augmented by a predefined time set.

Contract formulation

In the examined case studies three different contracts, Open, Formula and Firm, with varying duration have been introduced. Open contract (Eq. 8) secures a minimum profit for the manufacturer over the Unit Production Cost (UPC).

$$P_{ictfp,Open} = \begin{cases} (1+\delta_1) \cdot UPC_{ictfp}, \; 2 \leq p \leq P-1 \\ UPC_{ictfp}, \; p = \{1, P\} \end{cases} \tag{8}$$

The Formula contract (Eq.9) tailors the selling price based on the Unit Service Cost (USC) of each customer by a firm and the Employment Cost Index (ECI) which is fixed to 0.03 for this paper. Note that this contract results in the highest selling price for every time period

$$P_{ictfp,Formula} = (\delta_2 + \delta_3 USC_{ictf} + \delta_4 ECI) UPC_{ictfp}, \; \forall p \tag{9}$$

Finally the Firm contract (Eq.10) the manufacturer's profit is safeguarded for all time periods.

$$P_{ictfp,Firm} = (1+\delta_5) UPC_{ictfp}, \; \forall p \tag{10}$$

Fair game-theoretic solution

While perceiving the firms as players who want to simultaneously maximise their profit, two fairness schemes approaches are evaluated. The Naive scheme maximises the sum of the distinct profits as in Eq.(11).

$$\Phi_{NV} = \sum_f \pi_f \tag{11}$$

By formulation, the Naive approach does not account for the market share before the commencement of the game, to this extend the Nash fairness scheme is employed. The objective of the game is to maximise the geometrical mean of the profit increase over the status quo. Applying a separable linearisation approach as proposed by Gjerdrum et al. (2001) the final fair objective is formulated in Eq.12.

$$\widetilde{\Phi_{NS}} = \ln\left(\prod_f (\pi_f - \pi_f^{sq})^{\alpha_f}\right) = \sum_f \alpha_f \ln(\pi_f - \pi_f^{sq}) \tag{12}$$

Despite the above transformation, the corresponding problem is a MINLP problem with non-linear terms only in the objective function. Charitopoulos et al. (2020) have proposed two additional linearisation approaches based on branch and refine and SOS2 piecewise linearisation strategy which result in a MILP problem.

Case studies

To illustrate the proposed model, two case studies from an industrial liquid market will be examined. Initially, the formation of a duopoly is investigated and then, the formation of an oligopoly comprised of three firms. The examined time horizon is one year and is discretised into monthly time intervals, $p = 1, \cdots, 12$. The duration of the contracts is fixed for both case studies to 2 months for Open contract, 1 month for Formula and 4 months for Firm. The computational experiments were carried in an Intel® Core™i9-10900K CPU @ 3.70GHZ machine using GAMS v38.2.1.

Duopoly

In the duopoly case study the formation of a coalition comprised of two firms $f = 2$, $c = 97$ customers with $t = 315$ tanks and $i = 2$ trading products has been evaluated. Table 1 summarises the results of the computational experiments with Nash consideration approaches. Branch and Refine (BR) approach results in the greatest Nash objective followed by SOS2 approach. Given the fact that the CPU time is an order of magnitude greater in BR and that the global optimisation convergence is not always guaranteed, SOS2 approach is considered the most favourable linearisation approach. In addition, SOS2 approximations where initially tailored to find global optimum solutions of problems with a single nonlinear function in an otherwise linear programming problem (Beale and Forrest, 1976). Solving the MINLP problem with a local solver results in increased computational time and the worst objective.

Table 1: Result comparison for different Nash models.

	Solver	CPU time [s]	Nash objective
MINLP local	SBB/CONOPT4	2931	18.48
MILP SOS2	CPLEX	70	18.55
MILP BR	CPLEX	973	18.56

Having decided the linearisation approach it is worth evaluating different fairness schemes as proposed in the previous Section. The question of the proposed cooperative approach is to consider whether the formation of a coalition between the two firms would increase their profits and which fairness scheme will facilitate the market. Table 2 suggests that even though the Naïve scheme maximises the total coalition's profit, this is achieved by reversing the market share from 63/37 in the status quo to 46/54. Additionally, the Naïve scenario results in a 1.2% decrease of Firm's A profit and the same time increases the profit of Firm B by almost 100%. In this solution, Firm A would not agree to form a coalition. In contrast, the Nash scheme maintains the market share ratio and maximises both the total profit and each firm's profit. Despite the fact that Firm B has the smallest market share in status quo it meets the highest profit increase at 50% with the Nash scheme.

Table 2: Profit analysis for different fairness schemes and status quo in Duopoly.

	Status quo	Naïve	Nash
% Profit change A	-	-1.2	+25.0
% Profit change B	-	+99.5	+50.7
Market share A	0.63	0.46	0.58

An interesting aspect of the multi-period formulation is the customer's mobility. Increased mobility results in Forfeit/Acquisition costs in addition to delays in supply of the customer duo to un-instalment/instalment of each firm's equipment. Even though the Naïve scheme results in a static customer allocation, see Figure 2, no customers change firms during the examined time horizon, this does not apply in the Nash scheme where 4 customers change firms one time and 2 customers change 2 times. Despite some customer mobility, Figure 3 suggests that the dominant cost of Firm A is the Service cost, comprising the 50.8% of the cumulative cost from all time periods, followed by the Electricity cost (42.8%). Service cost accounts the unit service cost, the cost of swapping product and outsourcing production and is the dominating cost in the status quo market. Note that with the Naïve scheme the cost allocation maintains the structure of the status quo.

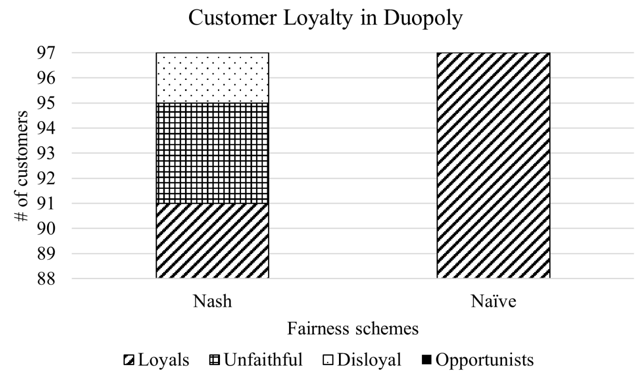

Figure 2: Customer loyalty for different fairness schemes. Total number of customers changed firm: 0 times (Loyal), 1 time (Unfaithful), 2 times (Disloyal), 3+ times (Opportunist).

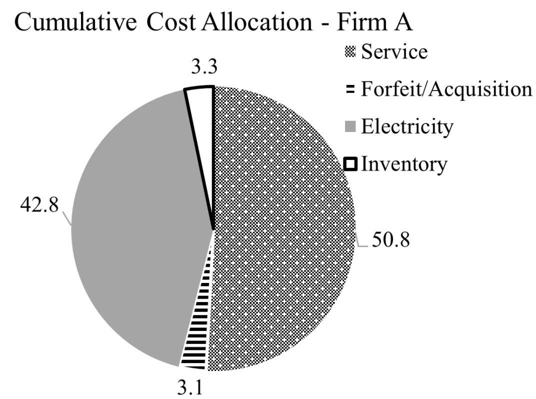

Figure 3: Cumulative cost allocation for Nash scheme over 12 time periods.

When it comes to the contract selection for both schemes and all time periods the only contract selected is Formula. The corresponding Gantt chart of a Disloyal customer based on the Nash approach is illustrated in Figure 4. Given that the utility function of the examined game is to maximise the firms' profit it is expected that the acquired solution will correspond to the contract with the highest product pricing.

Oligopoly

An additional case study has been examined to evaluate the formation of an oligopoly comprised of 3 firms. In this case an additional product is introduced in the market as well, so the total number of products $i = 3$, the total number of customers is $c = 81$ and the tanks $t = 119$. Table 3 showcases the results for the MINLP and MILP models respectively,

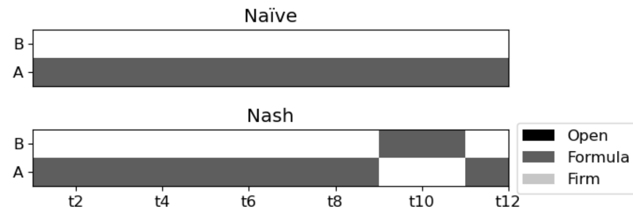

Figure 4: Gantt chart for Disloyal customer by Nash approach, #71. Comparison with the customer's mobility in Naïve approach.

only the SOS2 linearisation approach is selected for this case study. The MILP model with SOS2 variables has the fastest convergence and to a better final solution compared to the local MINLP approach.

Table 3: Result comparison for different Nash models.

	Solver	CPU time [s]	Nash objective
MINLP local	SBB/CONOPT4	1495	14.17
MILP SOS2	CPLEX	46	14.28

In the oligopoly case study, there is a clear leverage of Firm C over the other two firms which have a similar market share in the status quo as suggested by Table 4. The Naïve approach results in the greatest increase, almost 100%, for Firm B which has the lowest market share (0.24), while for Firm C there is a total profit decrease of 9.6%. In contrast, the Nash fairness scheme maintains the overall balance of the status quo in terms of market share and at the same time guarantees a profit increase for all of the involved firms. It can be observed that the % of profit increase is analogous to the market share of each firm.

Table 4: Profit analysis for different fairness schemes and status quo in Oligopoly.

	Status quo	Naïve	Nash
% Profit change A	-	+23.7	+19.6
% Profit change B	-	+98.2	+7.1
% Profit change C	-	-9.6	+28.0
Market share B	0.24	0.38	0.21
Market share C	0.50	0.36	0.53

Figure 5 suggests that in the Nash fairness scheme there are 1 Opportunist, 1 Disloyal and 5 Unfaithful customers while in the Naïve scheme there are 2 Unfaithful. The increased customer mobility has an impact on the total cost as observed in Figure 6, where the Forfeit/Acquisition cost comprises the 4.2% of the total cumulative cost for Firm C over 12 months. Overall, the multi-period game theoretic approach that allows customers to change firms and contracts in the examined time horizon, results in an improved customer allocation solution. The % of Service cost is decreased from 59% to 33% while the Electricity cost is increased from 30 to 47%. The game theoretic approach results in a marginal increase of 4% in the Inventory cost which increases the operational flexibility.

In the oligopoly coalition Open contract has been selected

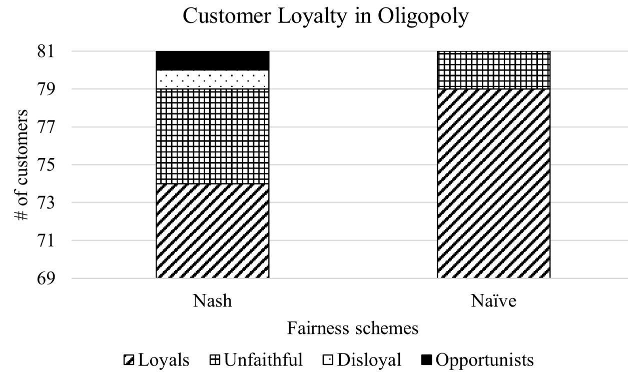

Figure 5: Customer loyalty for different fairness schemes. Total number of customers changed firm: 0 times (Loyal), 1 time (Unfaithful), 2 times (Disloyal), 3+ times (Opportunist).

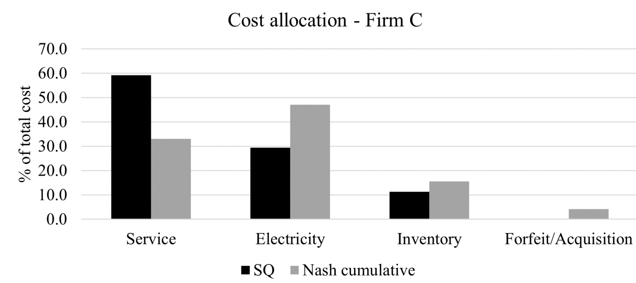

Figure 6: Comparison of cost allocation for status quo and cumulative over 12 time periods of Nash fairness scheme.

for a few customers both in the Naïve and Nash schemes. Figure 7 represents the Gantt chart of an Unfaithful customer in Nash approach. Even though the Nash approach selects the customer to change firms after $t8$, the Naïve approach initially allocates an Open contract with Firm B which is later transferred to a Formula contract with the same firm.

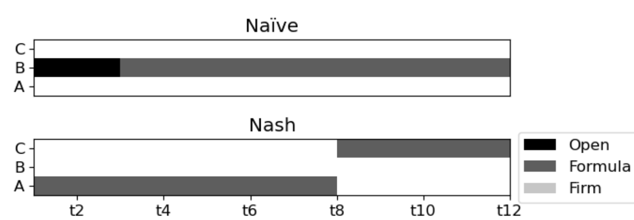

Figure 7: Gantt chart for an Unfaithful customer by Nash approach, #5. Comparison with the customer's mobility in Naïve approach.

Conclusion

For the examined case studies, the formation of coalitions was proven beneficial for the firms involved, since it resulted in increased profits compared to the status quo. The piecewise SOS2 linearisation has provided superior solution over the BR and the local MINLP approach for the examined instances. Among the two examined fairness schemes, the Nash scheme maintains the relative market structure of the status quo and at the same time allows for profit increase for all firms involved in the game. Comparing the games with different number of players, the formation of an oligopoly with more than 2 players using the Nash fairness scheme results in favouring the dominant player in the status quo. In the case of the duopoly the dominant player met 25.0% profit increase, while in the oligopoly case 28.5%. It is noteworthy, that for an increased number of firms an extra evaluation needs to be performed, either via the Shapley or Schmeidler's value, to determine the stability of the grand coalition in contrast to smaller coalition formations.

Future work aims to take into account customers as an extra player in the game whose utility function is to maximise their savings.

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