Operation Optimization of Supply Chain Networks Under Disruptions

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Abstract

Supply and manufacturing networks throughout the chemical industry involve different processing steps at distinct locations. Their network complexity makes operation vulnerable to uncertainty and disruptions from unplanned events. Effective recovery is a major problem as clearly shown with the current disruptions of supply chains. An appropriate response needs to include order management: product allocation, delay shipments, price renegotiation, amongst others. We propose a multiperiod mixed-integer linear programming (MILP) model that integrates information from plant production and scheduling, shipping, and order management to generate an optimal response that minimizes the financial impact of supply chain disruptions. We present a motivating example based on a real life study case from the chemical industry. A comparison between the optimal solution and other intuitive approaches is presented where we demonstrate that the solutions provided by our model result in the highest profit. Also, three different disruption scenarios are presented to demonstrate the capabilities of the model of handling both discrete-event and continuous limitation disruptions. Finally, a sensitivity analysis on the cancellation penalty is performed to show the impact of this cost on the profit as well as the number of cancellations made.

Keywords

Supply chain, Mixed-integer optimization, Disruptions, Optimal response, Scheduling

Introduction

The supply chains of high value-added chemicals have more variability and uncertainty compared to commodity products since they need multiple steps for their production and distribution, and are tied to several types of demands. The customized product portfolio adds to the demand uncertainty as individual products may be aligned to single customers instead of being influenced by the combined demand of a larger market. This means the variability of the final demand depends on each individual customer leading to a higher uncertainty in prediction and assessment of operation plans. Similarly, many fine chemicals require several processing steps that are performed at distinct locations around the globe. This results in a significant increase of transportation requirements, and makes the supply chain more susceptible to logistics disruptions.

Given the above features, these supply chains are increasingly vulnerable to potential disruptions. Supply chain disruptions can lead to significant economic losses, including increased costs to meet customer orders, penalties associated with delayed orders, or even lost revenue due to failure in fulfilling existing orders. Furthermore, existing customer satisfaction can be significantly reduced when an order is delivered late or cancelled. Therefore, effective optimization of decisions along the supply chain is needed to reduce the economic and operational impacts of unplanned events. Rerouting current inventories, increasing production at non-disrupted locations, buying finished products from competitors or third parties, and managing existing order deadlines are some of the decisions that can be made to mitigate the impact of the disruption.

In many industrial cases, responses to disruptions are made in a decentralized and empirical way which is resource intensive, inefficient, and results in sub-optimal supply chain performance. Furthermore, localized decisions lead to cascading impacts on other parts of the supply chain, and can lead to unstable and suboptimal responses. In this work we develop a comprehensive framework for optimizing supply chain decisions under disruptions. More specifically, this work presents a mathematical programming model to opti-

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mize the immediate response to an unplanned disruption.

Literature Survey

Research in the design and operation of supply chains under disruptions is a burgeoning area. The literature on the subject has been growing, with contributions coming from various disciplines. The purpose of this section is to introduce some of works that have addressed this problem such as network analysis, discrete-event simulation, and optimization. Leveraging knowledge of the network topology can be helpful to design and operate supply chains under disruptions. If the network has a specific structure, assumptions can be made about its behavior, allowing for development of tailored algorithms or heuristics. Specific structure approaches consider the number of elements in the supply chain and the ways they are connected. For instance, Kim et al. (2005) studied a single supplier three-component supply chain network and developed a centralized model to satisfy a target service level. To achieve this, they used artificial intelligence techniques to design an evolving policy that changes throughout realizations.

In a similar way, other authors have developed several approaches that include both heuristics and exact algorithms for specific topologies of the problem that rely on graph and queuing theory (Snyder and Shen, 2019). These approaches include a reduction of the system to one with partial systems or using heuristics for base-stock levels. These authors exploit different types of topologies in the chain to determine the optimal replenishment and allocation policies.

Optimization under the framework of mathematical programming is widely used to study supply chains under disruption. This approach is flexible for different structures and is amenable to algorithms that provide optimality guarantees. One optimization model that is currently considered for supply chain management is the guaranteed service model (GSM). This model assumes stationary normal demand, no back orders, and decentralized control with base stock policies (Simpson Jr, 1958; Graves and Willems, 2000; Bossert and Willems, 2007). The resulting model is a mixed-integer nonlinear programming (MINLP) formulation which considers decentralized decisions and nonlinear terms in the modelling equations. Recently, Achkar et al. (2021) showed that the MINLP model can be reformulated as a quadratically constrained program with significant improvements in computational efficiency.

Process control systems are addressed using optimization to develop optimal control policies. Perea-Lopez et al. (2003) integrated concepts of model-predictive control with the supply chain to find optimal inventory policies. This work uses a multiperiod model to capture system behaviour based on past and current information. The resulting model is a mixed-integer linear program (MILP) that is solved using closed-cycle control techniques.

Problem Statement

Given an arbitrary supply chain and manufacturing network topology, the goal is to determine production, shipping, inventory and order management schedules to optimally mit-igate the impact of a disruption, and bring the system back to a stable operation while maximizing its profit. Most of the literature concerning supply chain under disruptions tackles the design problem. That is, they seek to find the optimal node placement and inventory level under a steady behaviour. However, optimal recovery is an entirely operational problem that has not been fully addressed in the literature.

We extend the framework proposed by Perea-Lopez et al. (2003) to handle general network topologies meaning that there are no assumptions in the structure of the network and the connections between its components. The echelons for which decisions need to be optimized in a supply chain and manufacturing network are plants and warehouses. These echelons face both internal or external demand, meaning that orders can be placed from other internal echelons or directly from external customers. These general networks allow any arbitrary number of suppliers, plants, warehouses and customers, as well as arbitrary connections amongst them. Figure 1 shows the possible edges that can exists between the different components of the network and indicates which echelons hold inventory.
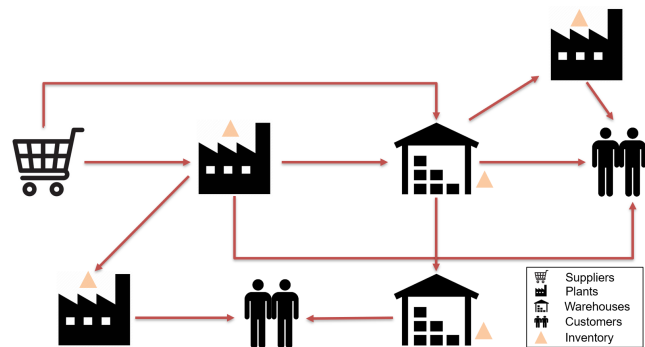

Figure 1: General Topology Sketch

Supplier-plant and supplier-warehouse connections are considered, and both plants and warehouses are allowed to store raw material in inventory. Similarly, these two types of nodes are permitted to store both intermediate and final product as well, given the existence plant-plant and plant-warehouse edges. Under this framework, plants are capable of both producing and storing materials (whether it is raw material, intermediate product or final product). Storing material in plants enables them to satisfy their internal and external demands by depleting inventory, manufacturing final product or a combination of both. This feature permits a much more flexible and broad response to satisfy the independent demand from customers as well as the dependent demand imposed by other downstream echelons.

We also, extend the work of Perea-Lopez et al. (2003) to support late delivery and order cancellation decisions. These are necessary given that, once a disruption occurs, it might not be feasible to deliver the original order schedule on time. Hence, the model accounts for late deliveries at a higher price to be able to satisfy customer orders. Given the case where an order can only be delivered past its original due date, it might be worth considering paying a high fixed price and cancelling the order instead of delivering the order significantly late. Cancelling an order may relieve the network

allowing allocation of resources to other orders more efficiently, resulting in a more cost effective response.

There are two main types of disruptions that are addressed. The first one is the occurrence of a discrete event such as the unavailability of a specific route or the failure of equipment within a plant. The second type of event is associated with a continuous limitation. These disruptions can represent, for instance, a reduction in the transport capacity on a particular route, or partial delivery of a scheduled order from an upstream supplier.

Proposed Model

We propose a multiperiod mixed-integer linear programming (MILP) formulation to provide optimal supply chain decisions in response to disruptions. The model considers multiple elements in the supply chain, including suppliers, production facilities, warehouses, and customers; as well as the available transportation options and routes. Key decision variables include the shipment amount and route, material purchases (raw, intermediate, or final products), plant production schedule, and customer delivery dates and amounts. When a disruption occurs, the company seeks an effective response to reduce the impacts of that disruption considering financials and customer satisfaction. Examples of reactive decisions that can be considered to mitigate consequences include:

- Shipping adjustments: Adjusting the amount, route, or transportation mode for internal shipping of material can be an effective mechanism to mitigate the disruption. For example, if an important order for a key consumer is at a risk of being delivered late, incurring extra expenses to ship via air-freight could help to avoid a lost sale and reduce the overall economic impact of the disruption.

- Use of the existing stock: Given that there is inventory throughout the plants and warehouses in the network, we can consider reallocating this stock (i.e., reducing inventory levels) to help mitigate the disruption. Inventory can provide a buffer against disruption, and we may adopt different inventory policies during the disruption recovery period.

- Plant production and scheduling: Changing the production level, re-scheduling the manufacturing orders according to product priority, or buying intermediate materials from third parties can be considered to overcome disruptions at the expense of increasing the cost of operation.

While these options provide degrees of freedom for supply chain recovery, the impact of any of these decisions on the entire supply chain can be non-intuitive. The outcome of the model is a schedule that shows how to react to a given disruption, while considering financial and customer satisfaction objectives. The multiperiod MILP model is capable of optimizing inventories at appropriate facilities, can provide an optimal production schedule (e.g., including overproduction), and the possibility of purchasing additional raw material or products from third parties or suppliers. Moreover,

since the optimization is performed as soon as the unplanned event takes place, the model is able to alert supply chain managers if customer orders are not able to be delivered as scheduled to evaluate the financial impact of cancelling an order. The goal of the proposed optimization formulation is not only to effectively respond to the disruption, but also to return to baseline operations once the unplanned event has been addressed. This can be done by fixing the inventory variables in the last time period to the values they had before disruption. Note that the model integrates all the echelons in the supply and production network to develop schedules that account for the entire operation in a unified way. In that sense, decisions will be made in a manner that the overall optimality of the system is achieved as a whole rather than relying on local or empirical decisions which are suboptimal for the enterprise.

The model assumes known order quantities and frequencies for all customers. Similarly, time delays across the network are deterministic and known for all of the arcs. Routes that have same origin and destination but use different transportation modes are modeled using different arcs. All products are assumed to be manufactured in a continuous operation, hence, no batch scheduling dynamics are considered within the plants. Finally, the length and impacts of the disruptions are deterministic and known as they occur.

Model Formulation

In this section we present the multiperiod Mixed-Integer Linear Programming (MILP) model formulation for the optimal response given a disruption. This model considers a set of different materials M, as well as suppliers S, plants P, warehouses W and customers C that are connected through a set of arcs A. Similarly, the time periods are considered over a discretized time horizon T. The MILP formulation is as follows:

$$\max \sum_{t \in T} [\sum_{c \in C} \sum_{m \in M_c} (\lambda_{mct}^D D_{mct} - \lambda_{mct}^U U_{mct} - \lambda_{mct}^\delta y_{mct}^\delta)$$
$$- \sum_{a \in A} \sum_{m \in M_a} \lambda_{mat}^A F_{mat}^{out} - \sum_{s \in S} \sum_{m \in M_s} \lambda_{mst}^B B_{mst}$$
$$- \sum_{p \in P} (\sum_{r \in R_p} \lambda_{prt}^P P_{prt} - \sum_{m \in M_p} \lambda_{mpt}^{IP} I_{mpt}^P)$$
$$- \sum_{w \in W} \sum_{m \in M_w} \lambda_{mwt}^{IW} I_{mwt}^W] \quad (1)$$

s.t.

$$F_{mat}^{in} = F_{ma\{t+\tau_a\}}^{out} \quad \forall a \in A, m \in M_a, t \in T \quad (2)$$

$$B_{mst} = \sum_{a \in A_s^{out}} F_{mat}^{in} \quad \forall s \in S, m \in M_s, t \in T \quad (3)$$

$$D_{mct} = \sum_{a \in A_c^{in}} F_{mat}^{out} \quad \forall c \in C, m \in M_c, t \in T \quad (4)$$

$$I_{mpt}^P = I_{mp\{t-1\}}^P + \sum_{a \in A_p^{in}} F_{mat}^{out} + \sum_{r \in R_p} \phi_{rm} P_{prt}$$
$$\forall p \in P, m \in M_p^R, t \in T \quad (5)$$

$$I_{mpt}^P = I_{mp\{t-1\}}^P - \sum_{a \in A_p^{out}} F_{mat}^{in} + \sum_{r \in R_p} \phi_{rm} P_{prt}$$

$$\forall p \in P, m \in M_p^P, t \in T \quad (6)$$

$$I_{mwt}^W = I_{mw\{t-1\}}^W + \sum_{a \in A_w^{in}} F_{mat}^{out} - \sum_{a \in A_w^{out}} F_{mat}^{in}$$

$$\forall w \in W, m \in M_w, t \in T \quad (7)$$

$$U_{mct} = U_{mc\{t-1\}} - D_{mct} + \delta_{mct}(1 - y_{mct}^\delta)$$

$$\forall c \in C, m \in M_c, t \in T \quad (8)$$

$$I_{mc0}^P = I_{mc|T|}^P \quad \forall p \in P, m \in M_c \quad (9)$$

$$I_{mw0}^W = I_{mw|T|}^W \quad \forall w \in W, m \in M_w \quad (10)$$

$$D_{mct}, U_{mct} \geq 0 \quad y_{mct}^\delta \in \{0,1\} \quad \forall m \in M, c \in C, t \in T \quad (11)$$

$$F_{mat}^{in}, F_{mat}^{out}, B_{mst} \geq 0 \quad \forall m \in M, s \in S, a \in A, t \in T \quad (12)$$

$$I_{mpt}^P, I_{mwt}^W, P_{prt} \geq 0 \quad \forall m \in M, p \in P, w \in W, r \in R, t \in T \quad (13)$$

The objective of the model is to maximize profit from the sales (λ^D) as shown in Equation (1). This profit accounts for several costs such as buying raw material (λ^C), manufacturing cost (λ^P), shipping cost using different transportation types (road, marine or air-freight) (λ^A) and holding inventory cost at both plants and warehouses (λ^{IP} and λ^{IW} respectively). Furthermore, this model accounts for cost of delivering late (λ^U), and for a high fixed charge in case the order needs to be cancelled (λ^δ). Equation (2) models the time delay that takes place at shipping or manufacturing. Here, F_{mat}^{in} represents the quantity of material m that enters an arc a at a given time period t, while F_{mat}^{out} represents that same flow coming out of the arc after a delay τ ($F_{ma\{t+\tau\}}^{out}$). The input and output flows of the network are calculated in Equations (3) and (4), respectively. The quantity to buy of material m from supplier s at a time period t (B_{mst}) is calculated by accounting for all of the arcs that come out of supplier s (A_s^{out}). Similarly, to calculate the demand satisfied of material m to customer c at a time period t (D_{mct}) the model accounts for all arcs coming into that customer c (A_c^{in}), which might come from either plants or warehouses. Equations (5) and (6) correspond to the inventory balance of raw materials and products inside the plants. In these equations ϕ_{rm} corresponds to the bill of material m used in recipe r, which accounts for material transformation and its sign depends if m is being consumed or produced ($-$ or $+$ respectively). A simple inventory balance for the warehouses is calculated in Equation (7). The unmet demand of material m to customer c at a time period t (U_{mct}) is calculated in Equation (8) as an inventory balance to be able to account for accumulated unmet demand across time periods. Given that the unmet demand is non-negative, as stated in Equation (11), the model can choose to cancel a specific order ($y^\delta = 1$), which will relieve the system but at a high fixed price (λ^δ). To model the network recovery after the disruption, we include final time constraints for both plants and warehouses modeled in Equations (9) and (10). These constraints force the supply chain to return to the same inventories it had at the before the disruptive event since we do not want a complete depletion of the inventory as a solution. Finally, Equations (11)-(13) state the discrete/continuous nature of the variables.

Motivating Case Study

The supply chain network we consider manufactures ten different products, and consists of two raw material suppliers, two plants, two warehouses, and nine customers that are satisfied from both manufacturers and warehouses. We consider the operation over a four month period ($|T| = 120$ days) and a sketch of the motivating example is presented in Figure 2.
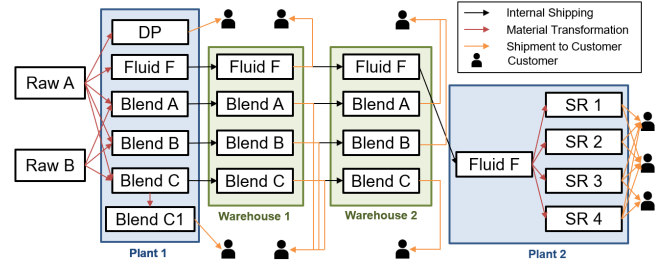

Figure 2: Supply Chain and Manufacturing Network Topology for the Motivating Example

The first plant (Plant 1) uses the two different raw materials (Raw A and Raw B) to make six products (DP, Fluid F, Blend A, Blend B, Blend C and Blend C1). The production processes of Plant 1 are assumed to be continuous for all the products that are being manufactured. In this plant, two products (labeled in the figure as DP and Blend C1) are sold to a customer directly, while the rest of the products are packed in steel drums and shipped to Warehouse 1 through road transportation. Note that Blend C is both an intermediate material in the production of Blend C1 as well as a final product that is delivered to Warehouse 1.

Once in Warehouse 1, these four products can be sold directly to local customers, or shipped to Warehouse 2 through marine transportation. There, three products (labeled in the figure as Blend A, Blend B, and Blend C) are sold directly to local customers. Fluid F is transshipped to Plant 2 to be further processed into four different final products (SR 1 - SR 4) and sold to local customers. All nine customers have independent demand frequencies and order quantities.

Due to its size and complexity, the network can be exposed to unplanned events that cause disruptions in its operation. This particular network can face, for example, three different types of disruptions that will negatively impact the overall operation.

- Plant equipment failure: This causes a reduction or complete loss of production at the plant facility, impacting downstream consumers and inventory levels for both upstream and downstream elements.

- Route unavailability: A given arc in the supply chain is not capable of transporting any material for a specific time window. In this case, the network becomes partially disconnected, hence, re-routing material or using other transport modes is required.

- Logistics resource limitations: The transport of material between supply chain elements can be impacted by availability of appropriate logistics resources (e.g.,

packing materials like steel drums or availability of drivers for truck transport).

These examples can cause a reduction in available material for downstream processes and delays in deliveries of customer orders. However, these can also impact upstream operations and cause inventory management challenges. For example, a plant failure causes an increase of available raw material that can impact upstream suppliers and producers. There are several objectives to consider when optimizing the supply chain operation. The cost of recovery is important, and one of the goals is to maximize the profit of the operation by delivering orders on time while reducing costs. Similarly, it is key to account for customer satisfaction to maintain the reputation of the company for future business. Therefore, delivering late (U_{mct}) or cancelling orders (y^{δ}_{mct}) are heavily penalized in the objective.

Results and Discussion
The proposed multiperiod MILP model (1)-(13) was formulated for the case study for a total of 120 time periods, each of 1 day length, leading to 2,280 0-1 variables, 15,360 continuous variables, 12,736 constraints and 36,171 nonzero elements for the base case using Pyomo (Bynum et al., 2021). The optimal solution was obtained in a Intel(R) Core(TM) I7-1165G7 at 2.80GHz with 4 cores using Gurobi v9.5.1 (Gurobi Optimization, LLC, 2022) as a solver. The optimality gap used was 0.1% and the solution time reported by Gurobi was 0.11 seconds for the base case. All the values presented in the case study were obtained using fabricated data, therefore, these numbers do not represent the real life values an industrial operation.

Comparison with intuitive approaches
The optimal response from the model is compared with other intuitive responses that may considered by local decision-makers. Consider the disruption where, out of the usual 1000 special pallets, there are only 300 of them available to pack material in the ships that go from Warehouse 1 to Warehouse 2. A possible intuitive approach could be sending all the units required using air-freight to overcome the shortage in the marine port. Another intuitive approach could also be, instead of using air-freight, to deplete the existing inventory in Warehouse 2 to satisfy the demand while the rest is sent using the available pallets. Table 1 compares the profit of the optimal solution given by the model with the profits resulting from these two intuitive approaches.

Table 1: Optimal Response and Intuitive Approach Comparison

Approach	Profit [10^3\$]	Decrease [%]
Optimal Response	3,601	-
Air-freight Only	3,156	12.38
Inventory Depleting	3,148	12.58

The optimal solution integrates different types of the decisions in a way that is not necessarily intuitive. Nevertheless, this response proves to be better than other approaches given

that it yields the highest profit. Particularly for this example, the output of the proposed optimization model results in a profit around \$445,000 larger than the approach where air-freight is used to overcome the disruption. This reduction in profit comes from sending all the required units via air-freight, which is the most expensive transportation mode. Similarly the optimal response has an objective approximately \$453,000 larger than the approach where inventory is depleted. Depleting inventory is one of the least expensive responses to a disruption. However, this approach yields a lower profit as well. This occurs because the available product in inventory is not sufficient to cover the total demand. Here, the model depletes all the existing inventory to satisfy part of demand but is forced to deliver late the other part which was not covered by the allocated inventory.

Scenario evaluation
Here, three different disruptions are presented to illustrate the capabilities of the model when handling different disruptions. These scenarios are compared against a base case where there is no disruption and the model is solved to optimality. Scenario 1 consists of the failure of two of the three main reactors that operate in parallel at Plant 1. This disruption causes the maximum production of the plant to be reduced to a third of the original capacity for the complete time horizon. Scenario 2 implies the total unavailability of the land routes that go from Plant 1 to Warehouse 1 for the first three months. Finally, Scenario 3 is the combination of Scenarios 1 and 2. Table 2 presents a profit comparison of the different scenarios. Furthermore, it also shows the total number of units of product that were delivered late, as well as the number of orders cancelled throughout the four month operation.

Table 2: Scenario Comparison

Scenario	Profit [10^3\$]	Late Units	Cancellations
Base	3,706	233	0
1	2,867	19	26
2	3,490	365	1
3	2,746	38	26

As one could expect, the base case has the largest profit given that there is no disruption occurring. However, even in the base case, given the high demand of the customers, the optimal schedule decides to deliver late a total of 233 units. Given the high penalty associated with cancelling an order, the model chooses to not make any cancellations for the base case. In Scenario 1 the optimizer decides to deliver 19 units late (considerably less than the base case) at the expense of cancelling 26 orders to overcome the disruption. Completely closing the truck routes proves to have a negative impact in the profit of the company. This is due to the fact that the other alternative (air-freight) is more expensive and has a smaller capacity than the trucks. Hence, the model chooses to deliver late 132 more units than in the base case, but it also decides to cancel one order. Finally, it can be observed that Scenario 3 has the greatest negative impact and lowest optimal profit. In this scenario, the optimal response requires the

same number of cancellations as Scenario 1, but the number of late deliveries increases compared to this scenario yielding a profit around $960,000 lower than the base case.

Sensitivity analysis
Th case study assumes a constant cost for order cancellations for all scenarios as $\lambda^{\delta}_{mct} = \$10.000 \quad \forall m \in M, c \in C, t \in T$. In practice, calculating this penalty is far from trivial. Cancelling an order not only implies a lost sale but it also affects the good will of the company with its customers, which might affect the future orders from the customer or even result in the loss of the customer. Evaluating methods for calculating this penalty is beyond the scope of this work. Nevertheless, since this parameter has a significant influence in the overall decisions made by the model, a sensitivity analysis is presented in Figure 3 to illustrate how the optimal solution might change given variations in this cost.
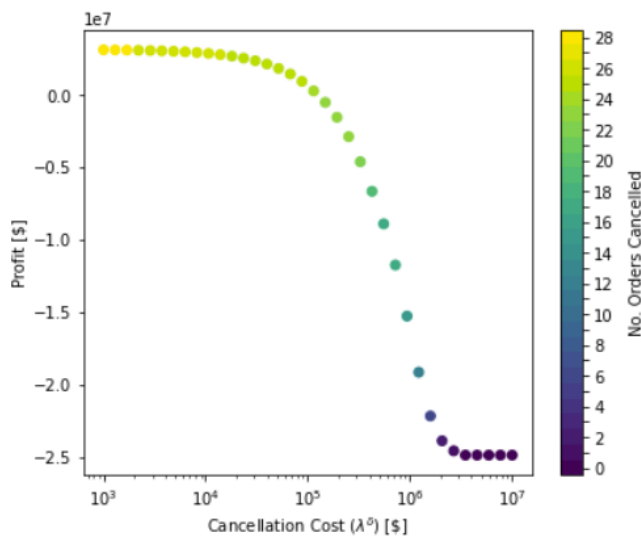

Figure 3: Cancellation Cost Sensitivity Analysis
This analysis was performed under Scenario 1 with a fixed value of λ^{δ} for all materials, customers and time periods. It can be observed that the profit drops as the the penalty for order cancellation increases. For small penalty values, the model chooses to cancel orders that are not being able to deliver on time instead of delivering them late. However, as the cancellation cost increases, it becomes more profitable to deliver the orders late instead of paying larger penalties for cancellation. This can be inferred by observing that the number of order cancellations also decreases with λ^{δ}.

Conclusions

This work proposes a multiperiod MILP model to address the optimal operational response of a multiproduct supply chain and manufacturing network once a disruption occurs. This model is able to handle general network topologies allowing plants and warehouses to have both internal and external demand. First, the output of the optimization model was compared with intuitive approaches and to show that our model yields a higher profit than other solution approaches. We also demonstrated the capabilities of the model by proposing scenarios with both continuous and discrete disruptions over different time windows.

The proposed model be can further extended to incorporate detailed production scheduling within the plants. Accounting for internal operations within the plant allows the optimization to re-schedule the production to satisfy orders according to the client priority and overall profit maximization. In a similar way, the currently deterministic model can be further extended to account for uncertainty in disruption occurrence. Note that accounting for all these decisions and features simultaneously, while considering supply chain complexity and the available degrees of freedom results in a large-scale MILP optimization problem that may require advanced solution approaches for efficient computational performance. Given that we showed that the model is highly sensitive to the value of the cancellation cost penalty, future work includes developing strategies that help to estimate that value. These estimations need incorporate not only the loss sales that cancellations imply, but also capture the negative impact on the good will of the company.

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