Stochastic programming of energy system operations considering terminal energy storage levels

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Abstract

Energy storage units offer vital balancing power for energy systems with an increasing amount of variable renewable energy sources. The operation of such systems can be optimized by stochastic programming, which anticipates the uncertainty related to the variable renewable energy sources. However, these optimization problems can only be formulated for optimization horizons with a finite length (e.g., 24 h), due to the rapidly increasing problem size. Realistic valuation of the stored energy at the end of a horizon is important for long-term operation of the system. In this work, we investigate suitable valuation methods for a hybrid energy system, consisting of photovoltaic power generation and an energy storage unit, which sells electricity to the day-ahead market. The best of the tested approaches is to value the stored energy based on the predicted maximum electricity price during the next period. On the studied test cases, the approach increased the profit by 11.7 to 18.7% in comparison to a myopic approach, often used in the literature, where no value is given for the stored energy at the end of a horizon.

Keywords

energy storage, stochastic programming, day-ahead market

Introduction

The increasing proportion of variable renewable energy (VRE) sources, such as wind and solar, is a challenge for stable operation of an electricity grid. The availability of these energy sources are both variable in time and inherently uncertain. One solution to mitigate the variability of VRE is to install energy storage systems in the proximity of VRE plants. Recently, significant research efforts have been put on the optimal operation of such hybrid energy systems under uncertainty [Weitzel and Glock, 2018, Zakaria et al., 2020]. Many of the proposed models are based on stochastic programming, i.e., a framework to formulate optimization problems under uncertainty such that the expected outcome is optimized [Birge and Louveaux, 2011].

Stochastic programming is based on scenarios of realized uncertainty, each of which has dedicated optimization variables. Thus, stochastic programming problems can only be formulated for limited horizon lengths, as the optimization problems quickly become intractable due to the increasing number of variables. The finiteness of the optimization horizon causes a phenomenon, referred to as the *end-effect* [Grinold, 1983, Fisher et al., 2001]. In the described energy system, the end-effect causes the energy storage system to be drained empty at the end of the current optimization horizon when no value is given for the terminal energy level (see, e.g., Su et al. [2013] and Singh and Knueven [2021]). Such decisions are sub-optimal over long time horizons. Often used methods to handle the end-effect are i) to enforce the terminal inventory (i.e., in our case, the level of stored energy) to be the same as the starting inventory or ii) to use the rolling horizon method, such that the current optimization horizon spans beyond the next re-optimization time. Dong and Maravelias [2021] propose multi-material terminal inventory constraints, which are applicable to online production scheduling.

Recently, Shin and Lee [2019] and Shin et al. [2017] propose a multi-timescale operation strategy by integrating mathematical programming and reinforcement learning. Particularly, Shin et al. [2017] mitigate the end-effect of the energy storage system in wind power-based energy grid systems using the value function. Han and Lee [2021] propose a method to determine the optimal design and operation strategy of energy grid systems based on stochastic programming while assuming that the terminal level of stored energy (at the end of a day) is equal to the starting level (at the beginning). In the context of process control, Oh et al. [2022] propose a method, referred to as Q-MPC, which combines reinforce-

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ment learning and model predictive control.

In this work, we investigate the valuation of the end-of-horizon energy storage level based on predicted mean/maximum electricity prices during the next optimization period (i.e., the day following the day-ahead). The valuation is incorporated into a stochastic programming model of an energy system that consists of photovoltaic power generation and energy storage and sells energy to the day-ahead market. We generate the scenarios of photovoltaic electricity generation based on probabilistic predictions obtained by Solcast [Solcast, 2019]. In addition, we demonstrate the obtained improvement in the long-term profit of the system in comparison to a myopic approach that does not look beyond the current optimization horizon.

Methods

Figure 1 illustrates the studied energy system, consisting of a photovoltaic power station and an energy storage unit, that sells electricity to the day-ahead market. In this section, we first describe the used stochastic programming model and the generation of scenarios. We then describe the evaluation of the method on multi-period optimization and how the level of stored energy (at the end of a planning horizon) is valuated.

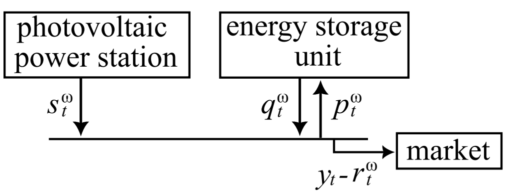


Figure 1: A hybrid energy system, consisting of a photovoltaic power station and an energy storage.

Stochastic programming model

We use here a modified version of the two-stage stochastic programming model by Singh and Knueven [2021], who use a chance constraint to model the probability of delivering the promised energy. In the original model, a set of binary variables (at the second-stage) indicate the small number of scenarios where the energy balance constraint is relaxed. In order to avoid the binary variables at the second stage, we use a soft constraint for delivering the promised energy. The first stage decision variables are promised power to be delivered, y_t at hour $t \in T$. The second stage decision variables, for a given scenario $\omega \in \Omega$, are the amount of stored energy x_t^ω, the power of charging the storage system p_t^ω, the power of discharging the storage system q_t^ω, and the reduction of power r_t^ω at hour $t \in T$ (see Fig. 1). The reduction of power r_t^ω at hour t is defined with respect to the promised power y_t. We denote the electricity price by R_t, and the cost of charging and discharging the storage system by C_c and C_d, respectively. The penalty coefficient of reducing power at hour t is C_p.

The objective is to maximize the expected sum of i) the profit during the optimization horizon and ii) the value of stored energy at the end of the horizon, $Vx_{|T|+1}^\omega$, where V is the value of stored energy per kWh. The model is defined as follows:

$$\max \sum_{t\in T} R_t \tau y_t - \mathbb{E}[C_\mathrm{c}\tau p_t^\omega + C_\mathrm{d}\tau q_t^\omega + C_\mathrm{p}\tau r_t^\omega] + \mathbb{E}[V x_{|T|+1}^\omega]$$

$$(y_t - r_t^\omega) + (p_t^\omega - q_t^\omega) \le s_t^\omega, \qquad t \in T, \omega \in \Omega \tag{1}$$

$$r_t^\omega \le y_t, \qquad t \in T, \omega \in \Omega \tag{2}$$

$$x_{t+1}^\omega = x_t^\omega + \eta_\mathrm{c}\tau p_t^\omega - \frac{1}{\eta_\mathrm{d}}\tau q_t^\omega, \qquad t \in T, \omega \in \Omega \tag{3}$$

$$p_t^\omega \le \overline{P}, \qquad t \in T, \omega \in \Omega \tag{4}$$

$$q_t^\omega \le \overline{Q}, \qquad t \in T, \omega \in \Omega \tag{5}$$

$$\underline{X} \le x_t^\omega \le \overline{X}, \qquad t \in \{1,\dots,|T|+1\}, \omega \in \Omega \tag{6}$$

$$x_1^\omega = x_0 \qquad \omega \in \Omega \tag{7}$$

$$y_t, p_t^\omega, q_t^\omega, r_t^\omega \ge 0, \qquad t \in T, \omega \in \Omega$$

Constraints 1 enforce that (at each hour $t \in T$ under scenario $\omega \in \Omega$) the sum of the actual delivered power $y_t - r_t^\omega$ and the charged or discharged power from the battery, $p_t^\omega - q_t^\omega$, do not exceed the available solar power s_t^ω. Constraints 2 enforce that the power reduction at hour t cannot exceed the promised delivery. Constraints 3 track the amount of stored energy, where $\eta_\mathrm{c}, \eta_\mathrm{d} \in [0,1)$ are the efficiencies of charging and discharging, respectively, and $\tau = 1$ h is the time interval. Constraints 4 and 5 specify the upper bounds, $\overline{P}$ and $\overline{Q}$, for charging and discharging the battery, respectively. Accordingly, Constraints 6 specify the lower and upper bounds, $\underline{X}$ and $\overline{X}$, for the stored energy. Here, it is worth noticing index t being over set $\{1,\dots,|T|+1\}$, instead of T. Variable $x_{|T|+1}^\omega$ tracks the terminal stored energy after the last hour of the period (appears also in the objective function). Constraints 7 define the amount of stored energy at the start of the optimization horizon.

Terminal state valuation

The focus of this work is on valuation of stored energy at the end of an optimization horizon. In the above-described stochastic programming model, the valuation is controlled via the parameter V, which is the multiplier of the terminal storage level $x_{|T|+1}^\omega$ in different scenarios $\omega \in \Omega$.

We investigate four methods for the terminal storage level valuation, the determination of parameter V. In the first, no value is given for the stored energy at the end of an optimization horizon ($V = 0$). It is worth noticing that, as storing energy has the cost of $C_\mathrm{c} \ge 0$, this method only stores the minimum amount of available solar energy that is sufficient to the deliver promised power $y_t, t \in T'$ during the remaining time points $T' \subset T$ of the period. Our second method is designed to collect all available solar energy by setting V to be slightly greater than C_c.

The third and fourth method for terminal stored energy valuation are based on the predicted electricity price during the period following the current optimization period. We generate the predictions by computing the average mean and max price for different weekdays from historical data of day-ahead electricity prices. The parameter V is then set to the predicted mean (i.e., the third method) or maximum (i.e., the fourth method) day-ahead electricity price. The four methods are summarized in Table 1.

Table 1: Descriptions of the methods for the terminal storage level valuation

method	description
no value	$V = 0$
small value	V is a small value slightly greater than the cost of storing energy, C_c
prediction strategy 1	V is equal to the predicted *mean* electricity price during the next period
prediction strategy 2	V is equal to the predicted *maximum* electricity price during the next period

Scenario creation

The described energy system is highly affected by the uncertainty in the hourly solar irradiance. Solcast is a state-of-the-art prediction model for solar irradiance prediction [Bright, 2019]. Its predictions are based on satellite images from five geostationary weather satellites, as well as models for aerosol and vapor concentrations. In addition to the mean of the prediction, Solcast also yields estimates of the prediction uncertainty, represented by the 10th and 90th percentiles. Figure 2 shows a seven-day probabilisic forecast of the global horizontal irradiance (GHI) in Bavaria, Germany, as well as the actual measured GHI.

Based on these probabilistic predictions, we generate the scenarios of the global solar irradiance (GHI) as follows. First, we fit a four-parameter beta distribution to the predicted mean and the 10th and 90th percentiles at each time point $t \in T$. The probability density function (pdf) of the four-parameter beta distribution is

$$f(y;\alpha,\beta,a,c) = \frac{f(x;\alpha,\beta)}{c-a}, \tag{8}$$

where $f(x;\alpha,\beta)$ is the pdf of the two-parameter beta distribution with parameters α and β and variable $x = (y-a)/(c-a) \in (0,1)$. a and c are scaling parameters, defining the lower and upper bound of the distribution, respectively. For the sake of robustness, we use the following bounds: $\alpha, \beta \geq 1$.

Second, we generate the scenarios of the hourly GHI $s^{\omega}_{\mathrm{GHI}}, t \in T$ by the statistical method by Pinson et al. [2009], which takes into account the temporal dependency of the realized solar power generation. The dependency is determined based on historical data of past predictions and realizations. We further assume that the photovoltaic panels are installed horizontally. The solar power generation scenarios are then

$$s^{\omega}_t = \eta_s A s^{\omega}_{\mathrm{GHI},t} \qquad t \in T, \omega \in \Omega \tag{9}$$

where $s^{\omega}_{\mathrm{GHI},t}$ is the GHI value obtained by the above-described procedure, A is the total area of the photovoltaic panels, and η_s is their efficiency. Figure 3 shows two sets of 100 daily GHI scenarios. Figure 3a is generated based on the probabilistic prediction shown in Fig. 2 and Fig. 3b on a similar prediction obtained on the following day.

Multi-period optimization

In order to evaluate the valuation methods for the terminal storage level valuation, we deploy the two-stage stochastic programming model in the following multi-period optimization procedure:

1. Solve the Stochastic programming model for the current optimization period with N scenarios, generated using the methods described in Section Scenario creation.
2. Fix the first stage decision variables $y_t, t \in T$.
3. Solve the seconds stage of the Stochastic programming model for one scenario $\omega = \omega'$ that represents the actual measured GHI.
4. Move to the next period. Set the initial stored energy x_0 of the new period to be the terminal stored energy $x^{\omega'}_{|T|+1}$ of the previous period. Return to Step 1.

Examples of the actual measured GHI are shown by dashed lines in Fig. 3. Finally, we note that our optimization procedure assumes the realized solar power $s^{\omega'}_t, t \in T$ during the entire period to be available before making the second-stage decisions. This simplification is made to avoid a multi-stage stochastic programming formulation.

Results

We test the methods, described in the previous section, on a 14-day time window from July 31 to August 13, 2022. The forecasted and actual GHI are obtained from Solcast (https://solcast.com/). The dependency in the realized GHI at the different time points in the prediction horizon (with respect to the predictions) is modeled from the data recorded from June 20 to July 30, 2022. The day-ahead electricity price data are from the German Federal Network Agency (https://www.smard.de/en). We use electricity prices from February 1 to July 30, 2022, as the training data for the electricity price prediction models. Figure 4 shows the test data of electricity prices during the 14-day time window. The results are generated on a laptop with an Intel i5-8365U processing unit. The stochastic programming problems problems are solved by Gurobi 9.1.0.

The parameters describing the energy storage unit are listed in Table 2. We investigate the same time window with electricity storage units that have three different maximum charge levels $\overline{X} = \{1000, 2000, 3000\}$, such that the minimum charge level $\underline{X}$ is always 20% of $\overline{X}$. The value for charge and discharge costs, C_c and C_d, are calculated for a lithium ion battery, having an estimated acquisition cost of \$132/kWh [Bloomberg, 2021] (we use a currency conversion rate of \$1 = 0.96 EUR), expected lifetime of 3000 cycles, and operation and maintenance costs of 2.1e-3 EUR/kWh [Zakeri and Syri, 2015]. We define the number of scenarios $N = 100$, the time interval $\tau = 1$ h, and photovoltaic panels to have an area of $A = 4000$ m^2 and efficiency of $\eta_s = 0.25$. Here, the penalty of not providing the promised power at hour t is $C_p = 10R_t$. In the valuation method 'small value', we choose $V = 0.03$ EUR/kWh, which is slightly greater than C_c (see Table 1). In the beginning of the first 24h period, the energy

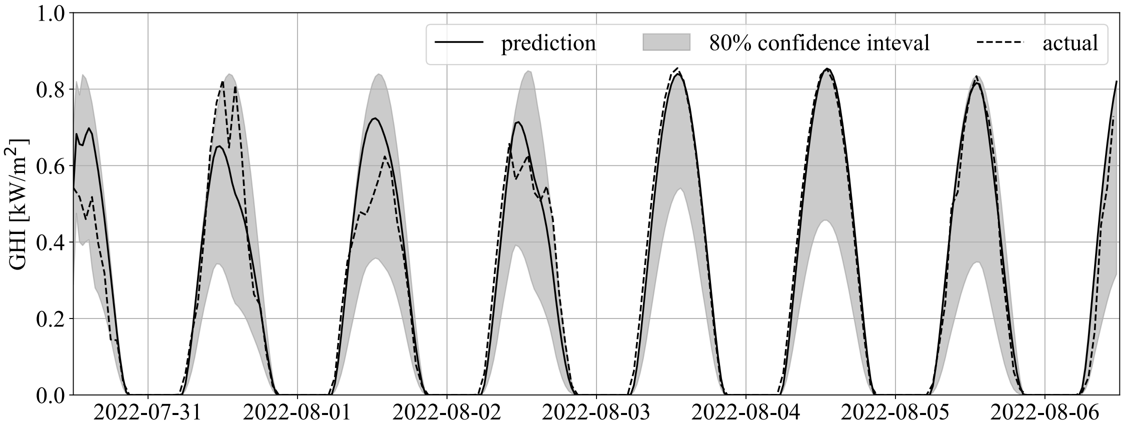
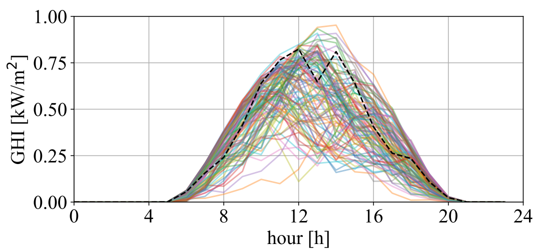
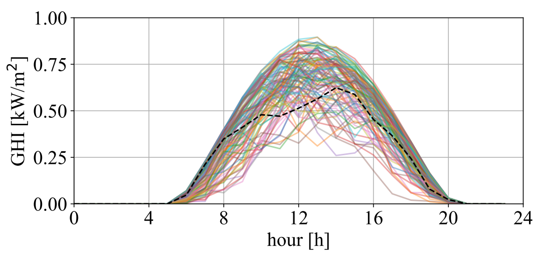


Figure 2: Probabilistic prediction of global horizontal irradiance (GHI) in Bavaria, Germany (49.158°N, 11.433°E), obtained by Solcast [Solcast, 2019] on July 31, 2022, at noon.



(a) Day 1



(b) Day 2

Figure 3: 100 scenarios of global horizontal irradiance (GHI), generated based on probabilistic predictions (colorful lines). The dashed line shows the actual measured GHI.

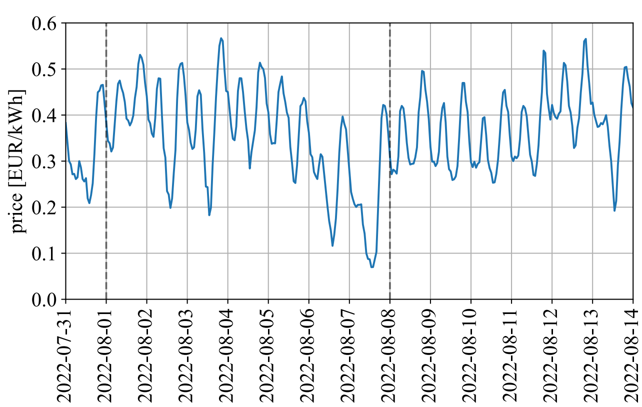


Figure 4: Electricity price during the tested 14-day time window. The start of a week is shown by vertical dashed lines.

storage unit is at the minimum charge level, $x_1^\omega = \underline{X}, \omega \in \Omega$. For the sake of a fair comparison, the last period is optimized using the 'no value' valuation method.

Table 3 lists the average daily profits, obtained using the four terminal state valuation methods, during the 14-day test time window. The value of the stored energy is not considered in the profit. Based on the results, the method 'small fixed value' yields 5.1 to 11.5% higher daily profits than the method 'no value', depending on the maximum charge level of the energy storage unit. The prediction strategy 2 yields the highest average daily profits, which are 11.7 to 18.7% higher than those of the method 'no value'. Valuing the terminal stored energy level based on the predicted maximum (prediction strategy 2) electricity price during the following optimization period seems to be a slightly better method than that based on the corresponding mean (prediction strategy 1). The obtained result is reasonable, as such energy systems typically have the flexibility of selling the stored energy at the most profitable time point.

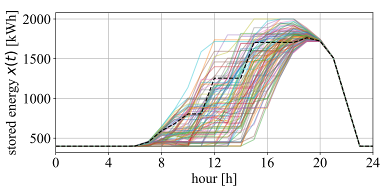
Finally, let us examine the behavior of the energy system when using different valuation methods. Figure 5 shows the stored energy $x_t^\omega, t \in T$ in each of the 100 scenarios $\omega \in \Omega$ (shown by colored continuous lines) during the first three days of the test time window. The dark dashed line shows the actual realized energy storage level. Notice that the starting level of this line (on Days 2 and 3) is the same as the the end level on the previous day. Figures 6 and 7 show the corresponding electricity prices and promised power $y_t, t \in T$ (i.e., the first stage decision variable) during the same days. The daily start and terminal energy levels are at $\underline{X}$ in all scenarios for the valuation method 'no value'. As the daily sum of the available solar energy varies between the scenarios, this means that in some scenarios all solar energy is not utilized. The terminal energy levels with the valuation method

Table 2: Parameter values of the energy storage system.

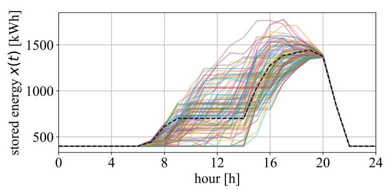
parameter	value
charge efficiency η_c	0.9
discharge efficiency η_d	0.9
charge cost C_c	0.02217 EUR/kWh
discharge cost C_d	0.02217 EUR/kWh
maximum charge power $\overline{P}$	500 kW
maximum discharge power $\overline{Q}$	500 kW
maximum charge level $\overline{X}$	$\{1000, 2000, 3000\}$ kWh
minimum charge level $\underline{X}$	$\{200, 400, 600\}$ kWh

Table 3: Numerical results of the different valuation methods for the terminal stored energy.

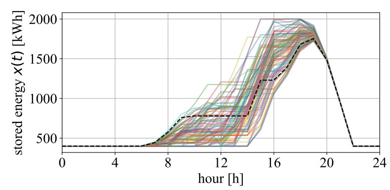
maximum charge level $\overline{X}$ [kWh]	terminal state valuation	average daily profit [EUR]	relative difference [%]
1000	no value	1652.93	0.0
	small fixed value	1737.82	+5.1
	prediction strategy 1	1841.44	+11.4
	prediction strategy 2	1846.58	+11.7
2000	no value	1738.33	0.0
	small fixed value	1873.02	+7.7
	prediction strategy 1	1987.20	+14.3
	prediction strategy 2	1995.72	+14.8
3000	no value	1773.32	0.0
	small fixed value	1977.29	+11.5
	prediction strategy 1	2092.83	+18.0
	prediction strategy 2	2104.38	+18.7



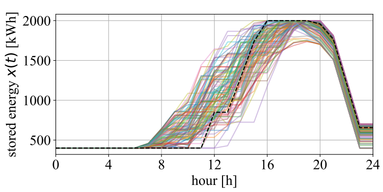
(a) Day 1: no value



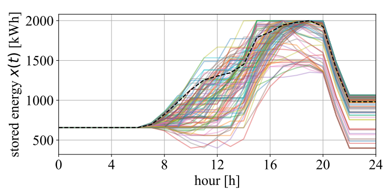
(b) Day 2: no value



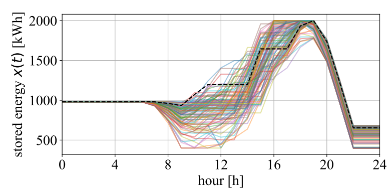
(c) Day 3: no value



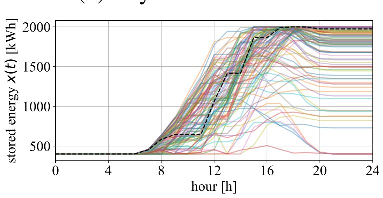
(d) Day 1: small value



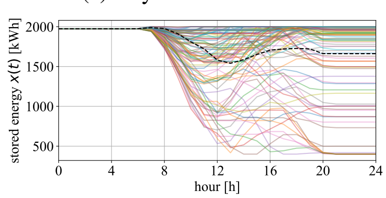
(e) Day 2: small value



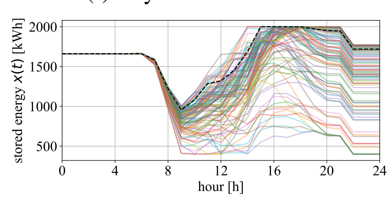
(f) Day 3: small value



(g) Day 1: prediction strategy 2

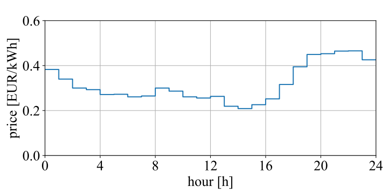


(h) Day 2: prediction strategy 2

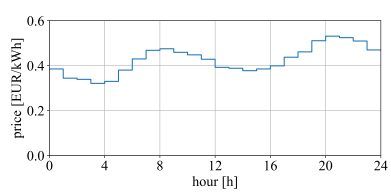


(i) Day 3: prediction strategy 2

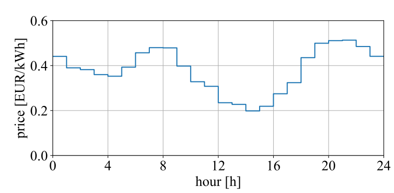
Figure 5: Stored energy $x(t)$ during first three days, when using no value (Subfigures a-c), a small fixed value (Subfigures d-f) or prediction strategy 2 (Subfigures g-i).



(a) Day 1



(b) Day 2

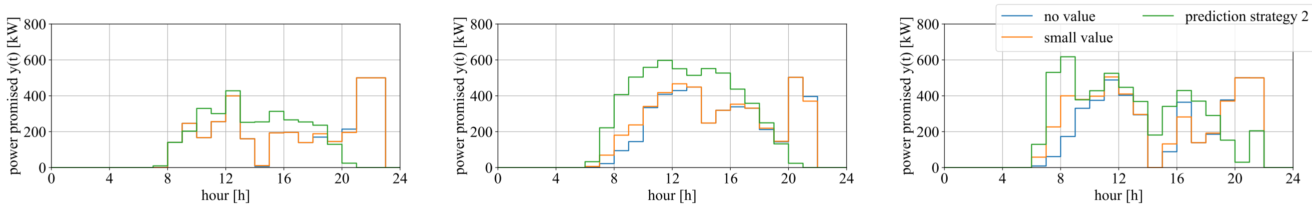


(c) Day 3

Figure 6: Actual electricity price during the first three days.

'small fixed value' have variation between the scenarios, as the residual solar energy is 'saved' to the energy storage unit. Figures 5(g)-(i) show that when using the prediction strategy 2 the energy system stores a large amount of energy on Day 1, some of which is then sold on the following days.

The improved profit of prediction strategies 1 and 2 is due to at least two reasons. The first is that the methods are far-sighted in comparison to the valuation method 'no value'. The second is that the methods lead, in general, to higher terminal stored energy levels. This improves the robustness of the system against unexpected reductions in the solar energy, reducing the risk of paying penalties for undelivered power.



(a) Day 1 (b) Day 2 (c) Day 3

Figure 7: Promised power during the first three days.

Conclusions

This work investigates valuation methods for stored energy at end of an optimization period in stochastic programming of an energy system. The studied energy system consists of photovoltaic power generation and an energy storage unit and sells electricity to the day-ahead market. We show that, if no value is given for the stored energy (i.e., a myopic method), the stochastic programming model does not necessarily utilize all of the available photovoltaic power in multi-period optimization. The best of the tested methods is to value the stored energy based on the predicted maximum electricity price during the next optimization period. On the studied 14-day period with three different electricity storage capacities, the method increases the profit by 11.7 to 18.7% in comparison to the myopic method, often used in the literature.

The future work will utilize state-of-the-art electricity price forecasting methods [Weron, 2014, Lago et al., 2021] to seek further improvements in the valuation methods. In addition, future work will investigate the valuation methods on more complex energy systems, involving also wind power and buying of electricity from the market.

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