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A Least Squares Method for Ensemble-based Multi-objective Oil Production Optimization ⋆

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Abstract: Despite a significant potential to improve industrial standards, practical applications of production optimization are impeded by geological uncertainty. As a mean to handle the associated financial risks, the oil literature has devised a range of ensemble-based strategies that seek to optimize proper combinations of sample-estimated risk measures to balance the opposing objectives of risk and reward. Many of the associated optimization problems are naturally formulated in terms of multi-objective optimization (MOO). Ideally, MOO problems should be solved by generating an approximation to the efficient frontier of optimal trade-offs between risk and return. However, the large-scale nature of real-life oil reservoirs implies that formation of the frontier often becomes computationally intractable in practice. To meet this challenge, this paper introduces a generalized least squares (LS) approach that provides an efficient and unified solution strategy for ensemble-based multi-objective optimization problems. At its core, the LS method uses an *a priori* characterization of desirable trade-offs that allows the method to focus on a single Pareto optimal point. Consequently, the LS approach avoids the need to generate a representative of the efficient frontier. In turn, this significantly reduces computational complexity compared to most MOO methods. As a result, the LS method poses a practical alternative to conventional strategies when the efficient frontier is unknown and computationally intractable to generate.

Keywords: Optimal control, Model-based control, Production control, Risk, Stochastic modelling.

1. INTRODUCTION

In the oil literature, production optimization refers to the use of nonlinear model predictive control (NMPC) to enhance the process of oil field water-flooding (Jansen et al., 2009). In particular, by combining mathematical reservoir models with advanced gradient-based optimization tools, production optimization seeks to determine the set of well configurations that maximize a performance index such as the cumulative oil recovery or the financial measure of life-cycle net present value (NPV). While numerical case studies have demonstrated a significant potential of production optimization to improve industrial practices, commercialization of the technology remains challenged by inherent geological uncertainties. To address the challenges of uncertainty, the oil literature has mainly considered ensemble-based methods (Van Essen et al. (2009)). Such methods seek to represent geological uncertainty by an ensemble of equally probable model realizations. Effectively, the ensemble is used to generate a discrete approximation to the continuous profit distribution. To minimize risk and promote profits, the idea is to formulate optimization problems that manipulate the discrete profit distribution to balance risk and reward according to appropriate measures of risk. Prevalent ensemble-based risk mitigation strategies include mean-variance optimization (MVO) (Bailey et al. (2005), Capolei et al. (2015b)) and conditional value-at-risk optimization (CVaRO) (Capolei et al. (2015a), Siraj et al. (2015b), Codas et al. (2016)). While the strategies differ by the way they quantify risk, the associated optimization problems naturally fall into the category of multi-objective optimization (MMO). In addition, production optimization problems that seek to balance short-term and long-term profits can also be classified in this way (Liu and Reynolds, 2015). This common structure of production optimization problems allows for a unified approach to their solution in terms of MOO methods. Typically, MOO methods seek to generate a representative of the Pareto front, i.e., the set of all optimal trade-offs between risk and reward. By generating the Pareto front, MOO methods present management with the opportunity to locate the trade-off that provides the best fit to the given application. As a drawback, the large scale nature of production optimization makes the approach computationally demanding to the point where many practical applications become intractable. This issue is particularly pronounced in the case of ensemble-based methods that rely on a large number of reservoir simu-

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lations. Recently, Christiansen et al. (2017) introduced a least squares (LS) method for efficient short-term versus long-term optimization. The method specifically targets problems where the Pareto front is computationally intractable to generate. Concretely, the LS approach relies on an *a priori* characterization of trade-offs. This allows the method to narrow it's focus to a single Pareto optimal point of pre-specified attractive features. As a drawback, the LS method assumes the reservoir description to be exactly known, i.e., the problem formulation does not allow for uncertain model parameters. To this end, this paper generalizes the LS framework to account for geological uncertainty. Effectively, this broadens applications of the LS methodology to ensemble-based risk mitigation strategies and provides a unified approach to the efficient and reliable solution of the underlying class of MOO problems, including MVO and CVaRO. Further, the computational complexity of the generalized LS method scales linearly with the number of objectives. This implies that the generalized LS approach provides a convenient and efficient way to trade-off a large number of risk-related objectives simultaneously. As a results, the LS method provides increased flexibility compared to conventional *a posteriori* methods, where the curse of dimensionality has limited previous applications to bi-criteria objective functions. To establish proof-of-concept, a numerical case study applies the generalized LS method to efficiently solve a mean-variance problem for a 2D synthetic black-oil reservoir with an ensemble of 10 permeability realizations. The results demonstrate the ability of the generalized LS method to efficiently trade-off risk and reward at significantly reduced computational cost relative to a conventional bi-criteria MVO approach.

The paper is organized as follows. Section 2 introduces ensemble-based oil production optimization. Section 3 describes risk mitigation in the context of multi-objective optimization and Section 4 presents the generalized LS method. Numerical results are presented in Section 5 and conclusions are made in Section 6.

2. OIL PRODUCTION OPTIMIZATION

Life-cycle oil production optimization seeks to enhance the recovery stage of oil field water-flooding by solution of an optimal control problem (Brouwer and Jansen, 2004; Sarma et al., 2005; Nævdal et al., 2006; Foss and Jensen, 2011; Völcker et al., 2011; Capolei et al., 2013):

$$\max_{u \in \mathcal{U}} \psi(u;\theta) \tag{1}$$

For a specific set of geological and petrophysical model parameters, $\theta \in \mathbb{R}^n$, the goal is to determine the optimal well settings, $u \in \mathcal{U}$, that maximize the cumulative NPV, ψ, defined by

$$\begin{aligned} \psi(u;\theta) = \sum_{k=0}^{N-1} \Big[& \sum_{i \in P} (r_g q_{g,i} + r_o q_{o,i} - r_{wp} q_{w,i}) \\ & - \sum_{l \in I} r_{wi} q_{w,l} + r_{gi} q_{g,l} \Big] \Delta t_k \end{aligned} \tag{2}$$

Here r_o, r_g, r_{wp}, r_{gi} and r_{wi} denote the gas price, the oil price, the water separation cost, the gas injection cost and the water injection cost, respectively; $q_{g,i}$, $q_{w,i}$ and $q_{o,i}$ are the volumetric gas, water and oil flow rates at producer i; $q_{w,l}$ and $q_{g,l}$ are the volumetric well injection rates at injector l; N is the number of control steps and $\Delta_{t_k} = t_{k+1} - t_k$ denotes the length of the time step. Well flow rates are computed using the Peaceman well model (Peaceman, 1983). For each time-step, t_k, the state-space variables, $x_k = x(t_k)$, denote reservoir pressures and fluid saturations whereas $u_k = u(t_k)$ represents a zero-order-hold parametrization of the well controls. The states x_k are computed by a black-oil model based on mass conservation and Darcy's law for porous media. Relative permeabilities are based on tables that mimic empirical data. See e.g. Aziz and Settari (1979); Chen et al. (2006); Chen (2007).

2.1 Production optimization under uncertainty

Simulation-based studies show that production optimization has a significant potential to improve real-world dominating practices. However, the transition to industrial applications relies on proper mathematical treatment of the largely unknown geological features of offshore oil fields. In particular, to be of practical relevance, the optimal control problem (1) must account for geological uncertainty in the model parameters, $\theta \in \mathbb{R}^n$. To this end, it has become common practice to use ensemble-based strategies. Ensemble-based methods seek to represent geological uncertainty by a collection of model realizations that all fit the available geological data equally well:

$$\theta_{n_d} = \{\theta^1, \theta^2, ..., \theta^{n_d}\} = \{\theta^i\}_{i=1}^{n_d}. \tag{3}$$

To quantify risk, ensemble-based methods approximate the continuous NPV probability distribution by the discrete set of profit realizations

$$\psi_{n_d} = \{\psi^i\}_{i=1}^{n_d}, \text{ where } \psi^i = \psi(u;\theta^i), \ 1 \le i \le n_d. \tag{4}$$

For a given operating profile, $u \in \mathcal{U}$, the set of discrete profits (4) provides a complete picture of how revenue is distributed over the range of model realizations. Using this information, ensemble-based methods seek to mitigate risk by reducing the probability of low profit outcomes. In practice, this is accomplished by minimizing an appropriate risk measure, $\mathcal{R} : \psi_{n_d} \to \mathbb{R}$:

$$\min_{u \in \mathcal{U}} \mathcal{R}(\psi(u;\theta_{n_d})). \tag{5}$$

In the oil literature, widely used risk measures, $\mathcal{R}$, include expected return ($\mathbb{E}$) (Van Essen et al., 2009), the profit variance ($\mathbb{V}$) (Capolei et al., 2015b)) and Conditional value-at-risk ($\mathbb{C}_\alpha$) (Siraj et al., 2015b) :

$$\mathbb{E} := -\frac{1}{n_d} \sum_{i=1}^{n_d} \psi^i, \tag{6}$$

$$\mathbb{V} := \frac{1}{n_d - 1} \sum_{i=1}^{n_d} (\psi^i - \mathbb{E})^2, \tag{7}$$

$$\mathbb{C}_\alpha := -\frac{1}{n_\alpha} \sum_{i=1}^{n_\alpha} \tilde{\psi}^i. \tag{8}$$

Each risk measure provides a quantification of risk by capturing different aspects of the discrete profit distribution, ψ_{n_d}. In particular, $\mathbb{E}$ aims to promote high profits by maximizing the mean value whereas $\mathbb{V}$ seeks to localize the distribution to avoid volatility and large profit deviations. In turn, $\mathbb{C}_\alpha$ targets the tail of the distribution by maximizing the expected return over the $\alpha \cdot 100\%$ lowest profit realizations, $\{\tilde{\psi}^i\}_{i=1}^{n_\alpha}$, for a given $\alpha \in (0,1)$. See Capolei et al. (2015a) for a comprehensive survey on risk measures in oil production optimization.

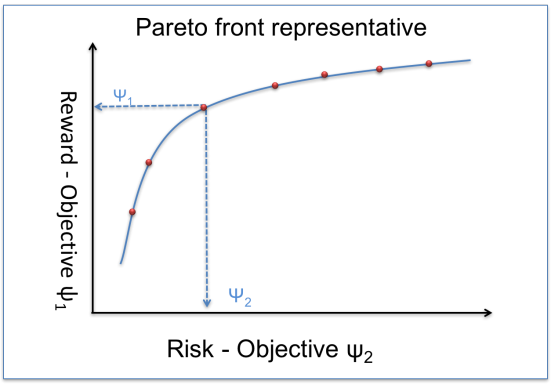

Fig. 1. Illustrative Pareto front of optimal trade-offs in the case of two objectives, e.g. risk and reward.

3. ENSEMBLE-BASED MULTI-OBJECTIVE OPTIMIZATION

To provide a proper quantification of risk, it is often necessary to include multiple features of the discrete profit distribution, ψ_{n_d}. To this end, many ensemble-based methods seek to optimize appropriate combinations of the fundamental risk measures (6)-(8). As a challenge, the individual risk-related objectives are often in mutual conflict. For example, the desire to increase returns are often bound to higher risks and vice versa. Consequently, many ensemble-based strategies, including MVO and CVaRO, are naturally formulated as multi-objective optimization problems that take the general form

$$\min_{u \in \mathcal{U}} \quad f(u) = \left(\mathcal{R}_1(u), \mathcal{R}_2(u), ..., \mathcal{R}_m(u)\right)^T. \tag{9}$$

Since the individual objectives are in mutual conflict, there does not exists a single optimal operating profile, $u \in \mathcal{U}$, that simultaneously minimizes all components of the objective vector, f. Instead, an optimal solution to (9) is characterized by the property that no objective, $\mathcal{R}_i$, can be further minimized without increasing at least one other objective, $\mathcal{R}_j$. Such a solution is said to be a *Pareto optimal trade-off* (Pareto, 1971). In practice, there is typically an infinite number of optimal trade-offs. Together they constitute the Pareto optimal set, $\mathcal{O}$. Each element $u \in \mathcal{O}$ gives rise to a different optimal risk scenario that, from a mathematical point of view, provides a satisfactory solution to the risk mitigation problem (9). The goal of *a posteriori* MOO methods is to generate a representative collection of these trade-offs in order for management to decide which scenario fits the application the best. In particular, *a posteriori* MOO methods seek to generate an approximation to the Pareto front of optimal trade-offs:

$$\mathcal{F} = \{f(u) = (\mathcal{R}_1(u), \mathcal{R}_2(u), ..., \mathcal{R}_m(u)) \mid u \in \mathcal{O}\}. \tag{10}$$

In the general case of m objectives, the Pareto front defines a hyper-surface in the objective space. Fig. 1 illustrates the Pareto front in the concrete case of two objectives. For a more elaborate introduction to multi-objective oil production optimization, see e.g. Liu and Reynolds (2015); Christiansen et al. (2017).

Remark 1. In addition to *a posteriori* methods, the oil literature has proposed a number of *a priori* MOO methods, where only a single trade-off is generated (Van Essen et al., 2011; Chen et al., 2012; Fonseca et al., 2014; Siraj et al., 2015a). For a discussion on these methods in the context of multi-objective optimization, see e.g. Christiansen et al. (2017).

3.1 Multi-objective risk mitigation by weighted sums

In the oil literature, the weighted sum method (WS) represents one of the most widely used *a posteriori* MOO methods. To approximate the Pareto front, the WS method reformulates the MOOP as a sequence of single objective optimization problems by assigning a non-negative weight, w_i, to each objective, $\mathcal{R}_i$, such as to minimize the wighted sum of objectives

$$\min_{u \in \mathcal{U}} \quad \sum_{i=1}^{m} w_i \mathcal{R}_i(\psi(u; \theta_{n_d})). \tag{11}$$

Each choice of positive weights, $\{w_i\}_{i=1}^m$, leads to a Pareto optimal trade-off (Miettinen (1999), Thm. 3.1.2). Consequently, by iterating over different weight constellations, it is possible to generate a representative of the efficient frontier. As a drawback, the large scale nature of oil production optimization makes the need for repeated optimizations time-consuming and computationally demanding. Further, it is a non-trivial task to determine the weights, $\{w_i\}_{i=1}^m$, that provide a proper representation of the frontier. Ultimately, these computational challenges prevent industrial applications.

Example 2. Mean-variance optimization (MVO) (Capolei et al., 2015b) uses the WS method to locate the optimal balance between the objectives of risk, $\mathbb{V}$, and reward, $\mathbb{E}$, by solving a sequence of m bi-criteria optimization problems for different choices of the weight parameter, λ_i :

$$\max_{u \in \mathcal{U}} \quad \lambda_i \mathbb{E}(\psi(u; \theta_{n_d})) - (1 - \lambda_i)\mathbb{V}(\psi(u; \theta_{n_d})), \; 1 \leq i \leq m. \tag{12}$$

Each iteration of the optimization algorithm relies on n_d reservoir simulations to calculate $\mathbb{E}(\psi(u; \theta_{n_d})$ and $\mathbb{V}(\psi(u; \theta_{n_d}))$. The highly non-linear nature of oil production optimization implies that iteration counts usually exceed 100. Hence, it is not uncommon that ensemble-based risk mitigation strategies like MVO require more than $100 \cdot n_d \cdot m$ reservoir simulations to generate a representation of the Pareto front.

4. A GENERALIZED LEAST SQUARES APPROACH

To meet the computational challenges of *a posteriori* MOO methods, the following introduces a new least squares (LS) approach for efficient risk-related multi-objective decision-making in oil production optimization. The LS method extends the work of Christiansen et al. (2017) to account for uncertainty in the model parameters, $\theta \in \mathbb{R}^n$. In this way, the LS approach provides a unified and computationally attractive approach to the solution of multi-objective ensemble-based risk mitigation problems.

4.1 Characterization of desirable Pareto points

Ideally, *a posteriori methods* should be used to generate a representative of the Pareto front to support informed decision making. However, for large-scale applications, the Pareto front is often computationally unavailable. This poses the natural question of how to preselect desirable

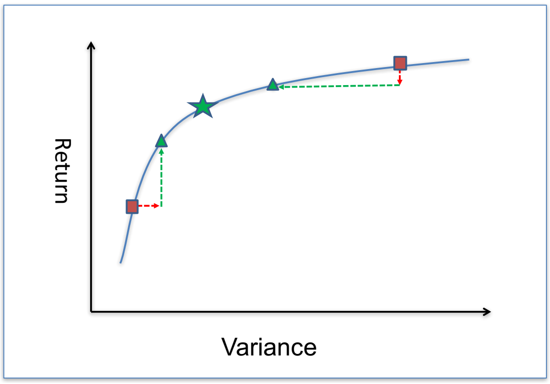

Fig. 2. Illustration of the key idea behind the LS method and the characterization of desirable trade-offs. By minimizing the Euclidian distance to the utopian risk scenario, the LS method locates the trade-off at the bend of the frontier ($\star$). By moving towards the bend, from $\square$ to Δ, the LS method properly balances risk and reward by promoting each objectives as much as possible without compromising the other in the process.

Pareto points without explicit knowledge of the efficient frontier. Christiansen et al. (2017) have recently proposed an answer to this question using an *a priori* characterization of desirable Pareto points in terms of the *utopian* risk scenario:

Definition 3. The utopian risk scenario is defined as the vector $\mathcal{R}^* := (\mathcal{R}_1^*, \mathcal{R}_2^*, ..., \mathcal{R}_m^*)$, where $\mathcal{R}_i^*$, $1 \leq i \leq m$ denotes the solution to the optimization problem

$$\min_{u \in \mathcal{U}} \quad \mathcal{R}_i(\psi(u; \theta_{n_d})). \tag{13}$$

The utopian risk scenario serves as a natural mean to characterize desirable trade-offs. The key idea is to favor Pareto points that remain close to the utopian profit scenario as measured in terms of deviations from the ideal, $\mathcal{R}^*$:

$$\Delta \mathcal{R}_i := \psi^* - \psi_i, \ i \in \{1, 2, ..., m\}. \tag{14}$$

In particular, a Pareto point is considered desirable if it remains close to the utopian risk scenario in the sense of least squares:

Definition 4. The Pareto point $\Psi = (\mathcal{R}_1, \mathcal{R}_2, ..., \mathcal{R}_m)$ is considered preferable to the Pareto point $\varrho = (\rho_1, \rho_2, ..., \rho_m)$, provided that

$$\sum_{i=1}^{m} (\mathcal{R}_i - \mathcal{R}_i^*)^2 \leq \sum_{i=1}^{m} (\rho_i - \mathcal{R}_i^*)^2, \tag{15}$$

where $\mathcal{R}^* = (\mathcal{R}_1^*, \mathcal{R}_2^*, ..., \mathcal{R}_m^*)$ denotes the utopian risk scenario. A Pareto point, $\mathcal{P}$, that is preferable to all other Pareto points is considered optimal.

In essence, the classification of Definition 4 ensures that desirable optimal Pareto points only deviate slightly from the utopian risk scenario as measured by the Euclidian distance. In this way, the classification screens out trade-offs where a given objective, $\mathcal{R}_i$, is severely compromised at the expense of promoting others. For example, the classification naturally disregards the unattractive trade-offs that only favor high profits and neglect the associated risks. As a result, the classification narrows the search to trade-offs that provide a proper balance between risk and reward. Fig. 2 illustrates the key idea.

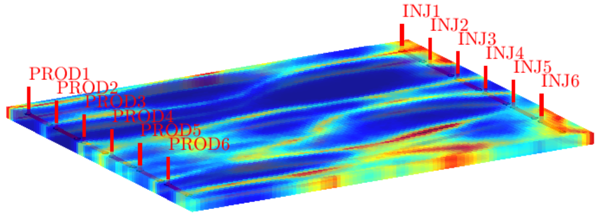

Fig. 3. Well setup and permeability field.

4.2 The LS optimization problem

To locate desirable trade-offs, $\mathcal{P}$, this paper introduces the generalized least-squares approach:

$$\min_{u \in \mathcal{U}} \quad \psi_{\text{LS}} = \frac{1}{2} \sum_{i=1}^{m} (\mathcal{R}_i(\psi(u; \theta_{n_d})) - \mathcal{R}_i^*)^2. \tag{16}$$

The LS method (16) is guaranteed to locate a (local) Pareto optimal solution (Miettinen (1999), Thm. 2.1.1). By design, this Pareto optimal trade-off minimizes the Euclidian distance to the utopian risk scenario. In this way, the LS method generates the optimal trade-off that increases each objective as much as possible without severely compromising other objectives in the process. As a result, the method obtains a proper balance between risk and reward without having to generate the efficient frontier. Further, as opposed to *a posteriori* methods, the computational complexity of the LS method scales linearly with the number of objectives. Altogether this makes the approach a computationally attractive alternative to conventional MOO methods in situations where the efficient frontier is unavailable.

5. NUMERICAL RESULTS

To demonstrate the LS method's ability to properly balance the objectives of risk and reward, the following case study uses (16) to solve the mean-variance multi-objective optimization problem

$$\min_{u \in \mathcal{U}} \quad f(u) = [-\mathbb{E}(\psi(u; \theta_{n_d})), \mathbb{V}(\psi(u; \theta_{n_d}))] \,. \tag{17}$$

As a base case reference, the LS solution is compared to the Pareto front generated by a bi-criteria MVO approach (12) with weights $\{\lambda_i\}_{i=0}^{10} = \{\frac{i}{10}\}$. In all computations, the variance is scaled by $w_v := 10^{-7}$ to ensure comparable magnitudes of the objectives.

5.1 Case study description

The case study uses a 2D synthetic black oil reservoir model of dimensions 800 m $\times$ 1000 m $\times$ 10 m, which by spatial discretization has been divided into $80 \times 100 \times 1$ cell blocks. To represent geological uncertainty, the case study uses an ensemble of 10 channeled isotropic permeability fields, where the permeability ranges between 0-1200mD. The reservoir is produced for 3600 days under water flooding conditions. It contains six water injectors and six producers. All wells are horizontal. Fig. 3 shows the well setup. The bhps of the producer wells are kept fixed

Table 1. Reservoir data.

description	symbol	value	Metric
physical dim	(x, y, y)	$(800, 1000, 10)$	[m]
grid-cell dim	$(\Delta x, \Delta y, \Delta z)$	$(10, 10, 10)$	[m]
porosity, uniform	ϕ	0.2	-
water comp	c_w	1.45e-5	[bar^{-1}]
rock comp	c_r	4.35E-10	[bar^{-1}]
capillary pressure	P_c	0	[bar]
pore volume	V_{pore}	1.6e6	[m^3]
oil in place	V_{oip}	1.28e6	[m^3]
permeability range	(k_x, k_y)	$[0, 1200]$	[mD]
bubble-point pressure	P_b	153.67	[bar]
datum press	P_r	130.00	[bar]
datum depth	d_r	1500.00	[m]
oil water contact	OWC	2000.00	[m]
gas-oil contact	GOC	1000.00	[m]
initial water saturation	S_{wi}	0.2	-

at 125 bar and the water injection rates are subject to control with a sample time of 30 days. The water injection rates are bound to be in the interval $[0.1, 250]$ m^3/day, with a rate of movement constraint of ± 30m^3/day. The total injection rate is constrained to a maximum of 750 m^3/day. The initial water injection rates, u_{init}, are set to 62.5 m^3/day for each well. All simulations are performed using the Eclipse E300 black oil reservoir simulator. Table 1 shows petrophysical simulation parameters. Solutions to the optimization problems, (12) and (16), are found by MATLABs build-in function `fmincon` with an interior-point algorithm and a tolerance of $\varepsilon = 10^{-8}$. The current best, but non-optimal iterate, is returned if the iteration count exceeds 100 and the objective function violates sufficient decrease conditions. Gradients required by the optimization algorithm are computed efficiently by the adjoint method (Jørgensen, 2007; Völcker et al., 2011; Capolei et al., 2012a,b; Jansen, 2011; Sarma et al., 2005; Suwartadi et al., 2012).

5.2 Comparison of the LS method and MVO

Fig. 4 compares the LS solution to the Pareto front representative generated by MVO. Table 2 shows the corresponding Pareto optimal values of expected return ($\mathbb{E}$) and standard deviation ($\mathbb{S}$). The results show that the LS method manages to locate the trade-off at the bend of the efficient frontier. Compared to the extreme MVO case of $\lambda := 1$, the LS method produces a trade-off that maintains a high expected return, while reducing the risk of profit loss considerably. In particular, expected return is reduced by just 1.5 million USD, while the the standard deviation is reduced by approximately 3 million USD. Also, compared to the extreme MVO case of $\lambda := 0$, the LS method manages to drastically increase expected return from 76.1 million USD to 102.6 million USD. The cost is a slight increase in the standard deviation of approximately 3.2 million USD. Overall, the results show that the LS method manages to locate a trade-off that properly balances the objectives of risk and reward, without any knowledge of the shape of the efficient frontier. In turn, this makes the LS method a computational attractive alternative to MVO. In particular, Table 2 compares MVO and the LS method in terms of equivalent reservoir simulations required to meet the stopping criteria of `fmincon`. While MVO requires a total of 17150 reservoir simulations, the LS method needs only perform 3090. This

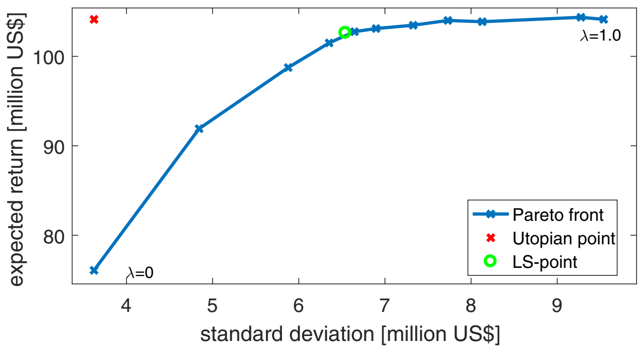

Fig. 4. Pareto front representative computed by MVO compared to the LS optimal trade-off. The Pareto optimal points are displayed in terms of expected return ($\mathbb{E}$) and standard deviation ($\mathbb{S}$).

Table 2. Comparison of MVO and the LS method.

Strategy	$\mathbb{E}$ [million USD]	$\mathbb{S}$ [million USD]	# Reservoir simulations
MVO (WS)	-	-	**17150**
$\lambda = 1$	104.1	9.53	1120
$\lambda = 0.9$	104.4	9.28	3550
$\lambda = 0.8$	103.9	8.13	2140
$\lambda = 0.7$	104	7.73	210
$\lambda = 0.6$	103.5	7.33	1930
$\lambda = 0.5$	103.1	6.90	1540
$\lambda = 0.4$	102.8	6.65	610
$\lambda = 0.3$	101.5	6.35	2110
$\lambda = 0.2$	98.7	5.88	1370
$\lambda = 0.1$	91.9	4.84	1510
$\lambda = 0$	76.1	3.36	1060
LS	102.6	6.54	3090

amounts to a reduction of the computational effort by approximately 82 %.

6. CONCLUSION

This paper has introduced a generalized least squares method for efficient and reliable ensemble-based multi-objective optimization. The LS approach extends the work of Christiansen et al. (2017) to handle geological uncertainties and it is designed specifically for situations where the efficient frontier is computationally intractable to generate. The extension paves the way for a unified approach to large-scale risk mitigation strategies in oil production optimization and serves a convenient mean to efficiently overcome the computational challenges of ensemble-based multi-objective optimization. In particular, the computational complexity of the LS approach scales linearly with the number of objectives. This allows for optimization of multiple risk-related objectives, whereas previous applications have been limited to bi-criteria problems. Using 10 realizations of a 2D synthetic black oil reservoir, numerical results have demonstrated the LS method's ability to properly balance risk and reward by solving a mean-variance optimization problem at significantly reduced computational effort relative to a conventional bi-criteria MVO approach. In this regard, the main computational workload tied to repeated reservoir simulations was reduced by approximately 82 %. Future work seeks to use this increased computationally flexibility of the LS method to investigate the potential benefits on the balance between risk and reward that comes from combining more than two risk-related objectives.

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