Preprints of the 18th IFAC World Congress
Milano (Italy) August 28 - September 2, 2011

A distributed motion planning and distributed filtering approach for target tracking in mobile sensor networks

Gerasimos G. Rigatos *

* *Unit of Industrial Automation, Industrial Systems Institute, 26504, Rion Patras, Greece (e-mail: grigat@isi.gr).*

Abstract: The paper studies the problem of tracking of a target by a multi-robot system assuming that the target's state vector is not directly measurable and has to be estimated by distributed filtering based on the target's cartesian coordinates and bearing measurements obtained by the individual mobile robots. The objective is to make the robots converge in a synchronized manner towards the target, while avoiding collisions between them and avoiding collisions with obstacles in the motion plane. To solve the overall problem, the following steps are followed: (i) distributed filtering, so as to obtain an accurate estimation of the target's state vector. This estimate provides the desirable state vector to be tracked by each one of the mobile robots, (ii) motion planning and control that enables convergence of the vehicles to the goal position and also maintains the cohesion of the vehicles swarm. The efficiency of the proposed distributed filtering and distributed motion planning scheme is tested through simulation experiments.

Keywords: target tracking, multi-robot system, mobile sensor networks, distributed motion planning, distributed filtering.

1. INTRODUCTION

The problem treated in this research work is as follows: there are N mobile robots (unmanned ground vehicles) which pursue a moving target. The vehicles emanate from random positions in their motion plane. Each vehicle can be equipped with various sensors, such as odometric sensors, cameras and non-imaging sensors such as sonar, radar and thermal signature sensors. These vehicles can be considered as *mobile sensors* while the ensemble of the autonomous vehicles constitutes a *mobile sensor network* [Rigatos (2010)],[Olfati-Saber (2007),[Franchi et al. (2009)]. At each time instant each vehicle can obtain a measurement of the target's cartesian coordinates and orientation. Additionally, each autonomous vehicle is aware of the target's distance from a reference surface measured in a cartesian coordinates system. Finally, each vehicle can be aware of the positions of the rest $N-1$ vehicles. The objective is to make the unmanned vehicles converge in a synchronized manner towards the target, while avoiding collisions between them and avoiding collisions with obstacles in the motion plane. To solve the overall problem, the following steps are necessary: (i) to perform distributed filtering, so as to obtain an estimate of the target's state vector. This estimate provides the desirable state vector to be tracked by each one of the unmanned vehicles, (ii) to design a suitable control law for the unmanned vehicles that will enable not only convergence of the vehicles to the goal position but will also maintain the cohesion of the vehicles ensemble (see Fig. 1).

Regarding the implementation of the control law that will allow the mobile robots to converge to the target in a coordinated manner, this can be based on a distributed gradient algorithm and its convergence can be proved using Lyapunov stability theory and particularly LaSalle's theorem [Rigatos (2008a)],[Rigatos (2008b)]. Regarding the implementation of distributed filtering, the Extended Information Filter and the Unscented Information Filter are suitable approaches. The Extended Information Filter performs fusion of the local state vector estimates which are provided by the local Extended Kalman Filters (EKFs), using the *Information matrix* and the *Information state vector* [Lee (2008a)],[Vercauteren and Wang (2005),[Manyika and Durrant-Whyte (1994)], [Rigatos and Zhang (2009)]. The Unscented Information Filter is a derivative-free distributed filtering approach which permits to calculate an aggregate estimate of the target's state vector by fusing the state estimates provided by Unscented Kalman Filters (UKFs) running at the mobile robots. Using distributed EKFs and fusion through the Extended Information Filter or distributed UKFs through the Unscented Information Filter provides more robust state estimates comparing to the centralized Extended Kalman Filter, or similarly the centralized Unscented Kalman Filter.

2. DISTRIBUTED MOTION PLANNING FOR THE MULTI-ROBOT SYSTEM

2.1 Kinematic model of the multi-robot system

The continuous-time target's kinematic model is assumed to be that of a unicycle robot and is given by $\dot{x}(t) = v(t)cos(\theta(t))$, $\dot{y}(t) = v(t)sin(\theta(t))$, $\dot{\theta}(t) = \omega(t)$. The target is steered by a dynamic feedback linearization control algorithm which is based on flatness-based control : $u_1 = \ddot{x}_d + K_{p_1}(x_d - x) + K_{d_1}(\dot{x}_d - \dot{x})$, $u_2 = \ddot{y}_d + K_{p_2}(y_d - y) + K_{d_2}(\dot{y}_d - \dot{y})$, $\dot{\xi} = u_1 cos(\theta) + u_2 sin(\theta)$, $v = \xi$, $\omega = u_2 cos(\theta) - u_1 sin(\theta)/\xi$. The dynamics of the tracking error is given by $\ddot{e}_x + K_{d_1}\dot{e}_x + K_{p_1}e_x = 0$, $\ddot{e_y} + K_{d_2}\dot{e}_x + K_{p_2}e_y = 0$, where $e_x = x - x_d$ and $e_y = y - y_d$. The proportional-derivative (PD) gains are chosen as K_{p_1} and K_{d_1}, for $i = 1, 2$ [Villagra et al. (2007)],[Oriolo et al (2002)].

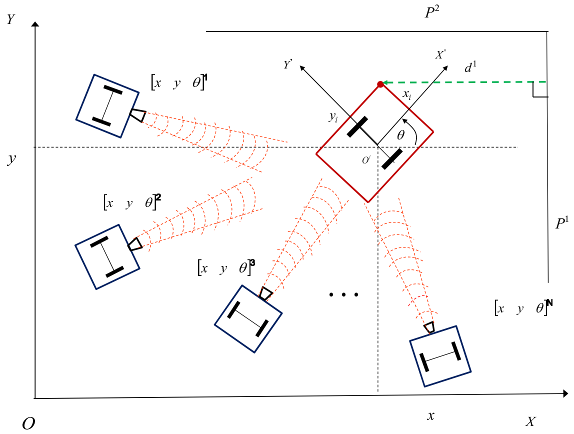


Fig. 1. Mobile robot providing estimates of the target's state vector, and the associated inertial and local coordinates reference frames

The position of each mobile robot in the 2-D space is described by the vector $x^i \in R^2$ (see Fig. 1). The motion of the robots is synchronous, without time delays, and it is assumed that at every time instant each robot i is aware about the position and the velocity of the other $N-1$ robots. The interaction between the i-th and the j-th mobile robot is $g(x^i - x^j) = -(x^i - x^j)[g_a(||x^i - x^j||) - g_r(||x^i - x^j||)]$, where $g_a()$ denotes the attraction term and is dominant for large values of $||x^i - x^j||$, while $g_r()$ denotes the repulsion term and is dominant for small values of $||x^i - x^j||$. Function $g_a()$ can be associated with an attraction potential, i.e. $\nabla_{x_i}V_a(||x^i - x^j||) = (x^i - x^j)g_a(||x^i - x^j||)$. Function $g_r()$ can be associated with a repulsion potential, i.e. $\nabla_{x_i}V_r(||x^i - x^j||) = (x^i - x^j)g_r(||x^i - x^j||)$. A suitable function $g()$ that describes the interaction between the robots is given by $g(x^i - x^j) = -(x^i - x^j)(a - be^{\frac{||x^i - x^j||^2}{\sigma^2}})$ where the parameters a, b and c are suitably tuned [Gazi and Passino (2004)]. It holds that $g_a(x^i - x^j) = -a$, i.e. attraction has a linear behavior (spring-mass system) $||x^i - x^j||g_a(x^i - x^j)$. Moreover, $g_r(x^i - x^j) = be^{\frac{-||x^i - x^j||^2}{\sigma^2}}$ which means that $g_r(x^i - x^j)||x^i - x^j|| \leq b$ is bounded. Applying Newton's laws to the i-th robot yields $\dot{x}^i = v^i$, $m^i\dot{v}^i = U^i$ where the aggregate force is $U^i = f^i + F^i$. The term $f^i = -K_v v^i$ denotes friction, while the term F^i is the propulsion. Assuming zero acceleration $\dot{v}^i = 0$ one gets $F^i = K_v v^i$, which for $K_v = 1$ and $m^i = 1$ gives $F^i = v^i$. Thus an approximate kinematic model for each mobile robot is

$$\dot{x}^i = F^i \tag{1}$$

According to the Euler-Langrange principle, the propulsion F^i is equal to the derivative of each robot's total potential, i.e. $F^i = -\nabla_{x^i}\{V^i(x^i) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}[V_a(||x^i - x^j|| + V_r(||x^i - x^j||)]\} \Rightarrow F^i = -\nabla_{x^i}\{V^i(x^i)\} + \sum_{j=1,j\neq i}^{N}[-\nabla_{x^i}V_a(||x^i - x^j||) - \nabla_{x^i}V_r(||x^i - x^j||)] \Rightarrow F^i = -\nabla_{x^i}\{V^i(x^i)\} + \sum_{j=1,j\neq i}^{N}[-(x^i - x^j)g_a(||x^i - x^j||) - (x^i - x^j)g_r(||x^i - x^j||)] \Rightarrow F^i = -\nabla_{x^i}\{V^i(x^i)\} - \sum_{j=1,j\neq i}^{N} g(x^i - x^j)$. Substituting in Eq. (1) one gets in discrete-time form

$$\begin{array}{c} x^i(k+1) = x^i(k) + \gamma^i(k)[h(x^i(k)) + e^i(k)] + \\ \sum_{j=1,j\neq i}^{N} g(x^i - x^j),\ i = 1, 2, \cdots, M \end{array} \tag{2}$$

The term $h(x(k)^i) = -\nabla_{x^i}V^i(x^i)$ indicates a local gradient algorithm, i.e. motion in the direction of decrease of the cost function $V^i(x^i) = \frac{1}{2}e^i(t)^T e^i(t)$. The term $\gamma^i(k)$ is the algorithm's step while the stochastic disturbance $e^i(k)$ enables the algorithm to escape from local minima. The term $\sum_{j=1,j\neq i}^{N} g(x^i - x^j)$ describes the interaction between the i-th and the rest $N-1$ stochastic gradient algorithms [Duflo (1996)],[Comets and Meyre (2006)],[Benveniste et al. (1990)].

2.2 *Cohesion of the multi-robot system*

The behavior of the multi-robot system is determined by the behavior of its center (mean of the vectors x^i) and of the position of each robot with respect to this center. The center of the multi-robot system is given by $\bar{x} = E(x^i) = \frac{1}{N}\sum_{i=1}^{N} x^i \Rightarrow \dot{\bar{x}} = \frac{1}{N}\sum_{i=1}^{N}\dot{x}^i \Rightarrow \dot{\bar{x}} = \frac{1}{N}\sum_{i=1}^{N}[-\nabla_{x^i}V^i(x^i) - \sum_{j=1,j\neq i}^{N}(g(x^i - x^j))]$. Using that $g(x^i - x^j) = -g(x^j - x^i)$, i.e. that $g()$ is an odd function, one obtains $\frac{1}{N}(\sum_{j=1,j\neq i}^{N} g(x^i - x^j)) = 0$, and

$$\dot{\bar{x}} = \frac{1}{N}\sum_{i=1}^{N}[-\nabla_{x^i}V^i(x^i)] \tag{3}$$

Denoting the target's position by x^* , and the distance between the i-th mobile robot and the mean position of the multi-robot system by $e^i(t) = x^i(t) - \bar{x}$ the objective of distributed gradient for robot motion planning can be summarized as follows: (i) $lim_{t\to\infty}\bar{x} = x^*$, i.e. the center of the multi-robot system converges to the target's position, (ii) $lim_{t\to\infty}x^i = \bar{x}$, i.e. the i-th robot converges to the center of the multi-robot system, (iii) $lim_{t\to\infty}\dot{\bar{x}} = \dot{x}^*$, i.e. the center of the multi-robot system stabilizes at the target's position.

If conditions (i) and (ii) hold then $lim_{t\to\infty}x^i = x^*$. Furthermore, if condition (iii) also holds then all robots will stabilize close to the target's position. It is known that the stability of local gradient algorithms can be proved with the use of Lyapunov theory [Benveniste et al. (1990)]. A similar approach can be followed in the case of the distributed gradient algorithms given by Eq. (2). The following simple Lyapunov function is considered for each gradient algorithm [Gazi and Passino (2004)]:

$$V_i = \frac{1}{2}{e^i}^T e^i \Rightarrow V_i = \frac{1}{2}||e_i||^2 \quad (4)$$

Thus, one gets $\dot{V}^i = {e^i}^T \dot{e}^i \Rightarrow \dot{V}^i = (\dot{x}^i - \dot{\bar{x}})e^i \Rightarrow \dot{V}^i = [-\nabla_{x^i} V^i(x^i) - \sum_{j=1,j\neq i}^N g(x^i - x^j) + \frac{1}{M}\sum_{j=1}^N \nabla_{x^j} V^j(x^j)]e^i$. Substituting $g(x^i - x^j)$ yields $\dot{V}_i = [-\nabla_{x^i} V^i(x^i) - \sum_{j=1,j\neq i}^N (x^i - x^j)a + \sum_{j=1,j\neq i}^N (x^i - x^j)g_r(||x^i - x^j||) + \frac{1}{N}\sum_{j=1}^N \nabla_{x^j} V^j(x^j)]e^i$, which gives, $\dot{V}_i = -a[\sum_{j=1,j\neq i}^N (x^i - x^j)]e^i + \sum_{j=1,j\neq i}^N g_r(||x^i - x^j||)(x^i - x^j)^T e^i - [\nabla_{x^i} V^i(x^i) - \frac{1}{N}\sum_{j=1}^M \nabla_{x^j} V^j(x^j)]^T e^i$. It holds that $\sum_{j=1}^N (x^i - x^j) = Nx^i - N\frac{1}{N}\sum_{j=1}^N x^j = Nx^i - N\bar{x} = N(x^i - \bar{x}) = Ne^i$, therefore $\dot{V}_i = -aN||e^i||^2 + \sum_{j=1,j\neq i}^N g_r(||x^i - x^j||)(x^i - x^j)^T e^i - [\nabla_{x^i} V^i(x^i) - \frac{1}{N}\sum_{j=1}^N \nabla_{x^j} V^j(x^j)]^T e^i$. It is assumed that for all x^i there is a constant $\bar{\sigma}$ such that $||\nabla_{x^i} V^i(x^i)|| \leq \bar{\sigma}$. Such an assumption is reasonable since for a mobile robot moving on a 2-D plane, the gradient of the cost function $\nabla_{x^i} V^i(x^i)$ is expected to be bounded. Moreover it is known that the following inequality holds: $\sum_{j=1,j\neq i}^N g_r(x^i - x^j)^T e^i \leq \sum_{j=1,j\neq i}^N b e^i \leq \sum_{j=1,j\neq i}^N b||e^i||$.

Thus, for the derivative of the Lyapunov function holds: $\dot{V}^i \leq aN||e^i||^2 + \sum_{j=1,j\neq i}^N g_r(||x^i - x^j||)||x^i - x^j|| \cdot ||e^i|| + ||\nabla_{x^i} V^i(x^i) - \frac{1}{N}\sum_{j=1}^M \nabla_{x^j} V^j(x^j)|| ||e^i|| \Rightarrow \dot{V}^i \leq aN||e^i||^2 + b(N-1)||e^i|| + 2\bar{\sigma}||e^i||$ where it holds that $\sum_{j=1,j\neq i}^N g_r(||x^i - x^j||)^T ||e^i|| \leq \sum_{j=1,j\neq i}^N b||e^i|| = b(N-1)||e^i||$. Thus, one finally obtains that $||\nabla_{x^i} V^i(x^i) - \frac{1}{N}\sum_{j=1}^N \nabla_{x^i} V^j(x^j)|| \leq ||\nabla_{x^i} V^i(x^i)|| + \frac{1}{N}||\sum_{j=1}^N \nabla_{x^i} V^j(x^j)|| \leq \bar{\sigma} + \frac{1}{N}N\bar{\sigma} \leq 2\bar{\sigma}$.

Consequently, one gets $\dot{V}^i \leq aN||e^i|| \cdot [||e^i|| - \frac{b(N-1)}{aN} - 2\frac{\bar{\sigma}}{aN}]$. The following bound ϵ is defined: $\epsilon = \frac{b(N-1)}{aN} + \frac{2\bar{\sigma}}{aN} = \frac{1}{aN}(b(N-1) + 2\bar{\sigma})$. Thus, when $||e^i|| > \epsilon$, $\dot{V}_i$ will become negative and consequently the error $e^i = x^i - \bar{x}$ will decrease. Therefore the tracking error e^i will remain in an area of radius ϵ i.e. the position x^i of the i-th robot will stay in the cycle with center $\bar{x}$ and radius ϵ.

2.3 Convergence of the multi-robot system to the target

The case of a convex quadratic cost function is examined

$$V^i(x^i) = \frac{A}{2}||x^i - x^*||^2 = \frac{A}{2}(x^i - x^*)^T(x^i - x^*) \quad (5)$$

where $x^* \in R^2$ denotes the target's position, while the associated Lyapunov function has a minimum at the target's position x^*, i.e. $V^i(x^i = x^*) = 0$. The distributed gradient algorithm is expected to converge to x^*. The robotic vehicles will follow different trajectories on the 2-D plane and will end at the target's position. Using Eq.(5) yields $\nabla_{x^i} V^i(x^i) = A(x^i - x^*)$. Moreover, the assumption $\nabla_{x^i} V^i(x^i) \leq \bar{\sigma}$ can be used, since the gradient of the cost function remains bounded. The robotic vehicles will concentrate round $\bar{x}$ and will stay in a radius ϵ. The motion of the mean position $\bar{x}$ of the vehicles is $\dot{\bar{x}} = -\frac{1}{N}\sum_{i=1}^N \nabla_{x^i} V^i(x^i) \Rightarrow \dot{\bar{x}} = -\frac{A}{N}\sum_{i=1}^N (x^i - x^*) \Rightarrow \dot{\bar{x}} = -\frac{A}{N}\sum_{i=1}^N x^i + \frac{A}{N}Nx^* \Rightarrow \dot{\bar{x}} - \dot{x}^* = -A(\bar{x} - x^*) - \dot{x}^*$. The variable $e_\sigma = \bar{x} - x^*$ is defined, and consequently

$$\dot{e}_\sigma = -Ae_\sigma - \dot{x}^* \quad (6)$$

The following cases can be distinguished: (i) The target is not moving, i.e. $\dot{x}^* = 0$. In that case Eq. (6) results in an homogeneous differential equation, the solution of which is given by $\epsilon_\sigma(t) = \epsilon_\sigma(0)e^{-At}$. Knowing that $A > 0$ results into $lim_{t\to\infty} e_\sigma(t) = 0$, thus $lim_{t\to\infty}\bar{x}(t) = x^*$. (ii) the target is moving at constant velocity, i.e. $\dot{x^*} = a$, where $a > 0$ is a constant parameter. Then the error between the mean position of the multi-robot formation and the target becomes $\epsilon_\sigma(t) = [\epsilon_\sigma(0) + \frac{a}{A}]e^{-At} - \frac{a}{A}$ where the exponential term vanishes as $t\to\infty$. (iii) the target's velocity is described by a sinusoidal signal or a superposition of sinusoidal signals, as in the case of function approximation by Fourier series expansion. For instance consider the case that $\dot{x^*} = b \cdot sin(at)$, where $a, b > 0$ are constant parameters. Then the nonhomogeneous differential equation Eq. (6) accepts a sinusoidal solution. Therefore the distance $\epsilon_\sigma(t)$ between the center of the multi-robot formation $\bar{x}(t)$ and the target's position $x^*(t)$ will be also a bounded sinusoidal signal.

2.4 Stability analysis using La Salle's theorem

The Lyapunov function given by Eq. (4) is negative semi-definite, therefore asymptotic stability cannot be guaranteed. It remains to make precise the area of convergence of each robot in the cycle C of center $\bar{x}$ and radius ϵ. To this end, La Salle's theorem can be employed [Rigatos (2008a)],[Gazi and Passino (2004)],[Khalil (1996)].

La Salle's Theorem: Assume the autonomous system $\dot{x} = f(x)$ where $f : D \to R^n$. Assume $C \subset D$ a compact set which is positively invariant with respect to $\dot{x} = f(x)$, i.e. if $x(0) \in C \Rightarrow x(t) \in C\ \forall\ t$. Assume that $V(x) : D \to R$ is a continuous and differentiable Lyapunov function such that $\dot{V}(x) \leq 0$ for $x \in C$, i.e. $V(x)$ is negative semi-definite in C. Denote by E the set of all points in C such that $\dot{V}(x) = 0$. Denote by M the largest invariant set in E and its boundary by L^+, i.e. for $x(t) \in E : lim_{t\to\infty} x(t) = L^+$, or in other words L^+ is the positive limit set of E. Then every solution $x(t) \in C$ will converge to M as $t \to \infty$.

La Salle's theorem is applicable in the case of the multi-robot system and helps to describe more precisely the area round $\bar{x}$ to which the robot trajectories x^i will converge. A generalized Lyapunov function is defined as $V(x) = \sum_{i=1}^N V^i(x^i) + \frac{1}{2}\sum_{i=1}^N \sum_{j=1,j\neq i}^N \{V_a(||x^i - x^j|| - V_r(||x^i - x^j||)\}$. This gives $V(x) = \sum_{i=1}^N V^i(x^i) + \frac{1}{2}\sum_{i=1}^N \sum_{j=1,j\neq i}^N \{a||x^i - x^j|| - V_r(||x^i - x^j||)$, and $\nabla_{x^i} V(x) = [\sum_{i=1}^N \nabla_{x^i} V^i(x^i)] + \frac{1}{2}\sum_{i=1}^N \sum_{j=1,j\neq i}^N \nabla_{x^i}\{a||x^i - x^j|| - V_r(||x^i - x^j||)\} \Rightarrow \nabla_{x^i} V(x) = [\sum_{i=1}^N \nabla_{x^i} V^i(x^i)] + \sum_{j=1,j\neq i}^N (x^i - x^j)\{g_a(||x^i - x^j||) - g_r(||x^i - x^j||)\} \Rightarrow \nabla_{x^i} V(x) = [\sum_{i=1}^N \nabla_{x^i} V^i(x^i)] + \sum_{j=1,j\neq i}^N (x^i - x^j)\{a - g_r(||x^i - x^j||)\}$. Then using Eq. (2) with $\gamma^i(t) = 1$ yields $\nabla_{x^i} V(x) = -\dot{x}^i$, and $\dot{V}(x) = \nabla_x V(x)^T \dot{x} = \sum_{i=1}^N \nabla_{x^i} V(x)^T \dot{x}^i \Rightarrow \dot{V}(x) = -\sum_{i=1}^N ||\dot{x}^i||^2 \leq 0$. Therefore it holds $V(x) > 0$ and $\dot{V}(x) \leq 0$ and the set $C = \{x : V(x(t)) \leq V(x(0))\}$ is

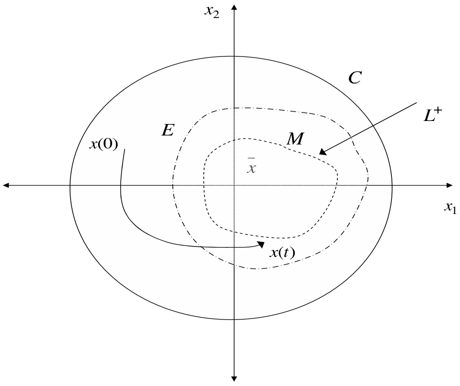


Fig. 2. LaSalle's theorem: C: invariant set, $E \subset C$: invariant set which satisfies $\dot{V}(x) = 0$, $M \subset E$: invariant set, which satisfies $\dot{V}(x) = 0$, and which contains the limit points of $x(t) \in E$, L^+ the set of limit points of $x(t) \in E$

compact and positively invariant. Thus, by applying La Salle's theorem one can show the convergence of $x(t)$ to the set $M \subset C$, $M = \{x : \dot{V}(x) = 0\} \Rightarrow M = \{x : \dot{x} = 0\}$.

3. DISTRIBUTED STATE ESTIMATION USING THE EXTENDED INFORMATION FILTER

3.1 Extended Kalman Filtering at local processing units

The distributed Extended Kalman Filter, also know as Extended Information Filter, performs fusion of the state estimates which are provided by local Extended Kalman Filters. Thus, the functioning of the local Extended Kalman Filters should be analyzed first. The following nonlinear state-space model is considered:

$$\begin{aligned} x(k+1) &= \phi(x(k)) + L(k)u(k) + w(k) \\ z(k) &= \gamma(x(k)) + v(k) \end{aligned} \tag{7}$$

where $x \in R^{m\times 1}$ is the system's state vector and $z \in R^{p\times 1}$ is the system's output, while $w(k)$ and $v(k)$ are uncorrelated, Gaussian zero-mean noise processes with covariance matrices $Q(k)$ and $R(k)$ respectively. The operators $\phi(x)$ and $\gamma(x)$ are $\phi(x) = [\phi_1(x), \phi_2(x), \cdots, \phi_m(x)]^T$, and $\gamma(x) = [\gamma_1(x), \gamma_2(x), \cdots, \gamma_p(x)]^T$, respectively. It is assumed that ϕ and γ are sufficiently smooth in x so that each one has a valid series Taylor expansion. Following a linearization procedure, ϕ is expanded into Taylor series about $\hat{x}$, i.e. $\phi(x(k)) = \phi(\hat{x}(k)) + J_\phi(\hat{x}(k))[x(k) - \hat{x}(k)] + \cdots$ where $J_\phi(x)$ is the Jacobian of ϕ calculated at $\hat{x}(k)$. Likewise, γ is expanded about $\hat{x}^-(k)$, i.e. $\gamma(x(k)) = \gamma(\hat{x}^-(k)) + J_\gamma[x(k) - \hat{x}^-(k)] + \cdots$ where $\hat{x}^-(k)$ is the estimation of the state vector $x(k)$ before measurement at the k-th instant to be received and $\hat{x}(k)$ is the updated estimation of the state vector after measurement at the k-th instant has been received. $J_\gamma(x)$ is the Jacobian of γ calculated at $\hat{x}(k)$. The resulting expressions create first order approximations of ϕ and γ. Thus the linearized version of the system is obtained: $x(k+1) = \phi(\hat{x}(k)) + J_\phi(\hat{x}(k))[x(k) - \hat{x}(k)] + w(k)$, $z(k) = \gamma(\hat{x}^-(k)) + J_\gamma(\hat{x}^-(k))[x(k) - \hat{x}^-(k)] + v(k)$./ Now, the EKF recursion is as follows: First the time update is considered: by $\hat{x}(k)$ the estimation of the state vector at instant k is denoted. Given initial conditions $\hat{x}^-(0)$ and $P^-(0)$ the recursion proceeds as:

Measurement update. Acquire $z(k)$ and compute: $K(k) = P^-(k)J_\gamma^T(\hat{x}^-(k)) \cdot [J_\gamma(\hat{x}^-(k))P^-(k)J_\gamma^T(\hat{x}^-(k)) + R(k)]^{-1}$, $\hat{x}(k) = \hat{x}^-(k) + K(k)[z(k) - \gamma(\hat{x}^-(k))]$, $P(k) = P^-(k) - K(k)J_\gamma(\hat{x}^-(k))P^-(k)$

Time update. Compute: $P^-(k+1) = J_\phi(\hat{x}(k))P(k)J_\phi^T(\hat{x}(k)) + Q(k)$, $\hat{x}^-(k+1) = \phi(\hat{x}(k)) + L(k)u(k)$,

3.2 Calculation of local estimations in EIF terms

Again the discrete-time nonlinear system of Eq. (7) is considered. The Extended Information Filter (EIF) performs fusion of the local state vector estimates which are provided by the local Extended Kalman Filters, using the *Information matrix* and the *Information state vector* [Lee (2008a)], [Vercauteren and Wang (2005)], [Manyika and Durrant-Whyte (1994)]. The Information Matrix is the inverse of the state vector covariance matrix, and can be also associated to the Fisher Information matrix [Rigatos and Zhang (2009)]. The Information state vector is the product between the Information matrix and the local state vector estimate

$$\begin{aligned} Y(k) &= P^{-1}(k) = I(k) \\ \hat{y}(k) &= P^-(k)^{-1}\hat{x}(k) = Y(k)\hat{x}(k) \end{aligned} \tag{8}$$

The update equation for the Information Matrix and the Information state vector are given by $Y(k) = P^-(k)^{-1} + J_\gamma^T(k)R^{-1}(k)J_\gamma(k) = Y^-(k) + I(k)$ and $\hat{y}(k) = \hat{y}^-(k) + J_\gamma^T R(k)^{-1}[z(k) - \gamma(x(k)) + J_\gamma\hat{x}^-(k)] = \hat{y}^-(k) + i(k)$, where $I(k) = J_\gamma^T(k)R(k)^{-1}J_\gamma(k)$ is the associated information matrix and, $i(k) = J_\gamma^T R(k)^{-1}[(z(k) - \gamma(x(k))) + J_\gamma\hat{x}^-(k)]$ is the information state contribution. The predicted information state vector and Information matrix are obtained from $\hat{y}^-(k) = P^-(k)^{-1}\hat{x}^-(k)$, $Y^-(k) = P^-(k)^{-1} = [J_\phi(k)P^-(k)J_\phi(k)^T + Q(k)]^{-1}$. It is assumed that an observation vector $z^i(k)$ is available for the N different sensor sites (mobile robots) $i = 1, 2, \cdots, N$ and each robot observes the target according to the local observation model, expressed by $z^i(k) = \gamma(x(k)) + v^i(k)$, $i = 1, 2, \cdots, N$, where the local noise vector $v^i(k) \sim N(0, R^i)$ is assumed to be white Gaussian and uncorrelated between sensors. The variance of a composite observation noise vector v_k is expressed in terms of the block diagonal matrix $R(k) = diag[R(k)^1, \cdots, R^N(k)]^T$. The information contribution can be expressed by a linear combination of each local information state contribution i^i and the associated information matrix I^i at the i-th sensor site $i(k) = \sum_{i=1}^N {J_\gamma^i}^T(k)R^i(k)^{-1}[z^i(k) - \gamma^i(x(k)) + J_\gamma^i(k)\hat{x}^-(k)]$ $I(k) = \sum_{i=1}^N {J_\gamma^i}^T(k)R^i(k)^{-1}J_\gamma^i(k)$. Thus, the update equations for fusing the local state estimates become $\hat{y}(k) = \hat{y}^-(k) + \sum_{i=1}^N {J_\gamma^i}^T(k)R^i(k)^{-1}[z^i(k) - \gamma^i(x(k)) + J_\gamma^i(k)\hat{x}^-(k)]$, $Y(k) = Y^-(k) + \sum_{i=1}^N {J_\gamma^i}^T(k)R^i(k)^{-1}J_\gamma^i(k)$. It is noted that in the Extended Information Filter an aggregation (master) fusion filter produces a global estimate by using the local sensor information provided by each local filter. As in the case of the Extended Kalman Filter the local filters which constitute the Extended information

Filter can be written in terms of *time update* and a *measurement update* equation.

Measurement update: Acquire $z(k)$ and compute $Y(k) = P^-(k)^{-1} + J_\gamma^T(k)R(k)^{-1}J_\gamma(k)$, or $Y(k) = Y^-(k) + I(k)$ where $I(k) = J_\gamma^T(k)R^{-1}(k)J_\gamma(k)$, and $\hat{y}(k) = \hat{y}^-(k) + J_\gamma^T(k)R(k)^{-1}[z(k) - \gamma(\hat{x}(k)) + J_\gamma\hat{x}^-(k)]$, or $\hat{y}(k) = \hat{y}^-(k) + i(k)$.

Time update: Compute $Y^-(k+1) = P^-(k+1)^{-1} = [J_\phi(k)P(k)J_\phi(k)^T + Q(k)]^{-1}$, and also $y^-(k+1) = P^-(k+1)^{-1}\hat{x}^-(k+1)$.

3.3 Extended Information Filtering for state estimates fusion

The outputs of the local filters are treated as measurements which are fed into the aggregation fusion filter [Lee (2008a)], [Vercauteren and Wang (2005)]. Then each local filter is expressed by its respective error covariance and estimate in terms of information contributions and is described by

$$\begin{array}{l} P_i^{-1}(k) = P_i^-(k)^{-1} + J_\gamma^T R^(k)^{-1}J_\gamma(k) \\ \hat{x}_i(k) = P_i(k)(P_i^-(k)^{-1}\hat{x}_i^-(k)) + \\ +J_\gamma^T R(k)^{-1}[z^i(k) - \gamma^i(x(k)) + J_\gamma^i(k)\hat{x}_i^-(k)]. \end{array} \quad (9)$$

It is noted that the local estimates are suboptimal and also conditionally independent given their own measurements. The global estimate and the associated error covariance for the aggregate fusion filter can be rewritten in terms of the computed estimates and covariances from the local filters using the relations $J_\gamma^T(k)R(k)^{-1}J_\gamma(k) = P_i(k)^{-1} - P_i^-(k)^{-1}$, $J_\gamma^T(k)R(k)^{-1}[z^i(k) - \gamma^i(x(k)) + J_\gamma^i(k)\hat{x}^-(k)] = P_i(k)^{-1}\hat{x}_i(k) - P_i(k)^{-1}\hat{x}_i(k-1)$. For the general case of N local filters $i = 1, \cdots, N$, the distributed filtering architecture is described by the following equations

$$\begin{array}{l} P(k)^{-1} = P^-(k)^{-1} + \sum_{i=1}^N [P_i(k)^{-1} - P_i^-(k)^{-1}] \\ \hat{x}(k) = P(k)[P^-(k)^{-1}\hat{x}^-(k) + \\ +\sum_{i=1}^N (P_i(k)^{-1}\hat{x}_i(k) - P_i^-(k)^{-1}\hat{x}_i^-(k))] \end{array} \quad (10)$$

The global state update equation in the above distributed filter can be written in terms of the information state vector and of the information matrix, i.e. $\hat{y}(k) = \hat{y}^-(k) + \sum_{i=1}^N(\hat{y}_i(k) - \hat{y}_i^-(k))$, and $\hat{Y}(k) = \hat{Y}^-(k) + \sum_{i=1}^N(\hat{Y}_i(k) - \hat{Y}_i^-(k))$. The local filters provide their own local estimates and repeat the cycle at step $k+1$. In turn the global filter can predict its global estimate and repeat the cycle at the next time step $k+1$ when the new state $\hat{x}(k+1)$ and the new global covariance matrix $P(k+1)$ are calculated. From Eq. (10) it can be seen that if a local filter (processing station) fails, then the local covariance matrices and the local state estimates provided by the rest of the filters will enable an accurate computation of the target's state vector.

4. DISTRIBUTED STATE ESTIMATION USING THE UNSCENTED INFORMATION FILTER

4.1 Unscented Information Filtering

The nonlinear state model of Eq. (7) is considered again. The Unscented Information Filter (UIF) can also perform fusion of the state vector estimates which are provided by local Unscented Kalman Filters running on the mobile robots, by weighting these estimates with local Information matrices (inverse of the local state vector covariance matrices), also known as Fisher information matrices [Rigatos and Zhang (2009)]. First, an augmented state vector $x_\alpha{}^-(k)$ is considered, along with the process noise vector, and the associated covariance matrix is introduced: $\hat{x}_\alpha^-(k) = [\hat{x}^-(k), \hat{w}^-(k)]^T$ and $P_\alpha{}^-(k) = diag\{P^-(k), Q^-(k\}$. As in the case of local (lumped) Unscented Kalman Filters, a set of weighted sigma points $X_\alpha^i{}^-(k)$ is generated as $X_{\alpha,0}^-(k) = \hat{x}_\alpha^-(k)$, $X_{\alpha,i}^-(k) = \hat{x}_\alpha^-(k) + [\sqrt{(n_\alpha+\lambda)P_\alpha^-(k-1)}]_i$, $i = 1, \cdots, n$, $X_{\alpha,i}^-(k) = \hat{x}_\alpha^-(k) - [\sqrt{(n_\alpha+\lambda)P_\alpha^-(k-1)}]_i$, $i = n+1, \cdots, 2n$, where $\lambda = \alpha^2(n_\alpha + \kappa) - n_\alpha$ is a scaling, while $0 \leq \alpha \leq 1$ and κ are constant parameters. The corresponding weights for the mean and covariance are defined as in the case of the lumped Unscented Kalman Filter $W_0^{(m)} = \frac{\lambda}{n_\alpha+\lambda}$, $W_0^{(c)} = \frac{\lambda}{(n_\alpha+\lambda)+(1-\alpha^2+\beta)}$, $W_i^{(m)} = \frac{1}{2(n_\alpha+\lambda)}$, $W_i^{(C)} = \frac{1}{2(n_\alpha+\lambda)}$, where $i = 1, \cdots, 2n_\alpha$ and β is again a constant parameter [Julier and Uhlmann (2004)]. The equations of the prediction stage (measurement update) of the information filter, i.e. the calculation of the information matrix and the information state vector are [Lee (2008a)],[Vercauteren and Wang (2005)]:

$$\begin{array}{c} \hat{y}^-(k) = Y^-(k)\sum_{i=0}^{2n_\alpha} W_i^{(m)} X_i^x(k) \\ Y^-(k) = P^-(k)^{-1} \end{array} \quad (11)$$

where X_i^x are the predicted state vectors when using the sigma point vectors X_i^w in the state equation $X_i^x(k+1) = \phi(X_i^{w-}(k)) + L(k)U(k)$. The predicted state covariance matrix is computed as $P^-(k) = \sum_{i=0}^{2n_\alpha} W_i^{(c)}[X_i^x(k) - \hat{x}^-(k)][X_i^x(k) - \hat{x}^-(k)]^T$. In the equations of the Unscented Kalman Filter (UKF) there is no linearization of the system dynamics, thus the UKF cannot be included directly into the Extended Information Filter (EIF) equations [Lee (2008a)]. Instead, it is assumed that the nonlinear measurement equation of the system given in Eq. (7) can be mapped into a linear function of its statistical mean and covariance, which makes possible to use the information update equations of the EIF. Denoting $Y_i(k) = \gamma(X_i^x(k))$ (i.e. the output of the system calculated through the propagation of the i-th sigma point X^i through the system's nonlinear equation) the observation covariance and its cross-covariance are approximated by $P_{YY}^-(k) = E[(z(k) - \hat{z}(k)^-)(z_k - \hat{z}(k)^-)^T] \simeq J_\gamma(k)P^-(k)J_\gamma(k)^T$, $P_{XY}^-(k) = E[(x_k - \hat{x}(k)^-)(z_k - \hat{z}(k)^-)^T] \simeq P(k)J_\gamma(k)^T$ where $z(k) = \gamma(x(k))$ and $J_\gamma(k)$ is the Jacobian of the output equation $\gamma(x(k))$. Next, multiplying the predicted covariance and its inverse term on the right side of the information matrix and replacing $P(k)J_\gamma(k)^T$ with $P_{XY}^-(k)$ gives the following representation of the information matrix [Lee (2008a)],[Vercauteren and Wang (2005)]:

$$\begin{array}{l} I(k) = J_\gamma(k)^T R(k)^{-1} J_\gamma(k) = \\ P^-(k)^{-1} P^-(k) J_\gamma(k)^T R(k)^{-1} J_\gamma^-(k) P^-(k)^T (P^-(k)^{-1})^T = \\ P^-(k)^{-1} P_{XY}(k) R(k)^{-1} P_{XY}(k)^T (P^-(k)^{-1})^T \end{array} \tag{12}$$

where the cross-correlation matrix $P_{XY}(k)$ is calculated from $P_{XY}^-(k) = \sum_{i=0}^{2n_\alpha} W_i^{(c)} [X_i^x(k) - \hat{x}^-(k)][Y_i(k) - \hat{z}^-(k)]^T$, where $Y_i(k) = \gamma(X_i^x(k))$ and the predicted measurement vector $\hat{z}^-(k)$ is obtained by $\hat{z}^-(k) = \sum_{i=0}^{2n_\alpha} W_i^{(m)} Y_i(k)$. Similarly, the information state vector $i(k)$ can be rewritten as $i(k) = J_\gamma(k)^T R(k)^{-1}[z(k) - \gamma(x(k)) + J_\gamma(k)^T \hat{x}^-(k)]$ i.e, $i(k) = P(k)^{-1} P_{XY}^-(k) R(k)^{-1}$ $[z(k) - \gamma(x(k)) + P_{XY}^-(k)(P^-(k)^{-1})^T \hat{x}^-(k)]$. A "measurement" matrix $H^T(k)$ is defined as

$$H(k)^T = P^-(k)^{-1} P_{XY}^-(k) \tag{13}$$

while the information contributions equations are written as $i(k) = H^T(k) R(k)^{-1} [z(k) - \gamma(x(k)) + H(k)\hat{x}^-(k)]$, $I(k) = H^T(k) R(k)^{-1} H(k)$. The above procedure leads to an implicit linearization in which the nonlinear measurement equation of the system given in Eq. (7) is approximated by the statistical error variance and its mean $z(k) = \gamma(x(k)) \simeq H(k)x(k) + \bar{u}(k)$, where $\bar{u}(k) = \gamma(\hat{x}^-(k)) - H(k)\hat{x}^-(k)$ is a measurement residual term.

4.2 Local estimations in UIF terms

It is assumed that the observation vector $\bar{z}_i(k+1)$ is available from N different sensors, and that each sensor observes a common state according to the local observation model, expressed by $\bar{z}_i(k) = H_i(k)x(k) + \bar{u}_i(k) + v_i(k)$, where the noise vector $v_i(k)$ is taken to be white Gaussian and uncorrelated between sensors. Then one can define the local information matrix $I_i(k)$ and the local information state vector $i_i(k)$ at the i-th sensor, as follows $i_i(k) = H_i^T(k) R_i(k)^{-1} [z_i(k) - \gamma_i(x(k)) + H_i(k)\hat{x}^-(k)]$, $I_i(k) = H_i^T(k) R_i(k)^{-1} H_i(k)$. The update equations for the multiple state estimation and data fusion are $\hat{y}(k) = \hat{y}^-(k) + \sum_{i=1}^N i_i(k)$, $Y(k) = Y^-(k) + \sum_{i=1}^N I_i(k)$. The Unscented Information Filter running at the i-th measurement processing unit can be written in terms of *measurement update* and *time update* equations:

Measurement update: Acquire measurement $z(k)$ and compute $Y(k) = P^-(k)^{-1} + H^T(k) R(k)^{-1} H(k)$, or $Y(k) = Y^-(k) + I(k)$ where $I(k) = H^T(k) R(k)^{-1} H(k)$, and also $\hat{y}(k) = \hat{y}^-(k) + H^T(k) R(k)^{-1} [z(k) - \gamma(\hat{x}(k)) + H(k)\hat{x}^-(k)]$, or $\hat{y}(k) = \hat{y}^-(k) + i(k)$.

Time update: Compute $Y^-(k+1) = (P^-(k+1))^{-1}$, where $P^-(k+1) = \sum_{i=0}^{2n_\alpha} W_i^{(c)} \cdot [X_i^x(k+1) - \hat{x}^-(k+1)][X_i^x(k+1) - \hat{x}^-(k+1)]^T$ and also $\hat{y}^-(k+1) = Y^-(k+1) \sum_{i=0}^{2n_\alpha} W_i^{(m)} X_i^x(k+1)$, where $X_i^x(k+1) = \phi(X_i^w(k)) + L(k)U(k)$

4.3 Distributed UIF for state estimates fusion

It is assumed that the local Unscented Kalman Filters do not have access to each other row measurements and are allowed to communicate only their information matrices and their local information state vectors. Then Eq. (11) gives $P_i(k)^{-1} = P_i^-(k)^{-1} + H_i^T(k) R_i(k)^{-1} H_i(k)$, $\hat{x}_i = P_i(k)(P_i^-(k)\hat{x}_i^-(k) + H_i^T(k) R_i(k)^{-1} [z_i(k) - \gamma_i(x(k)) + H_i(k)\hat{x}^-(k)])$. Each local information state contribution i_i and its associated information matrix I_i at the i-th filter are rewritten in terms of the computed estimates and covariances of the local filters, i.e. $H_i^T(k) R_i k^{-1} H_i(k) = P_i^{-1}(k) - P_i^{-\,-1}(k)$, $H_i^T(k) R_i(k)^{-1} [z_i(k) - \gamma_i(x(k)) + H_i(k)\hat{x}^-(k)]) = P_i(k)^{-1}\hat{x}_i - (P_i^-(k)^{-1})\hat{x}_i^-(k)$, where according to Eq.(13) it holds $H_i(k) = P_i^-(k)^{-1} P_{XY,i}^-(k)$. Next, the aggregate estimates of the UIF, can be written in terms of covariances [Lee (2008a)],[Vercauteren and Wang (2005)]:

$$\begin{array}{l} P(k)^{-1} = P^-(k)^{-1} + \sum_{i=1}^N [P_i(k)^{-1} - P_i^-(k)^{-1}] \\ \hat{x}(k) = P(k)[P^-(k)^{-1}\hat{x}^-(k) + \\ + \sum_{i=1}^N (P_i(k)^{-1}\hat{x}_i(k) - P_i^-(k)^{-1}\hat{x}_i^-(k))] \end{array} \tag{14}$$

and also in terms of the information state vector and of the information matrix, i.e. $\hat{y}(k) = \hat{y}^-(k) + \sum_{i=1}^N (\hat{y}_i(k) - \hat{y}_i^-(k))$, $Y(k) = Y^-(k) + \sum_{i=1}^N [Y_i(k) - Y_i^-(k)]$.

5. SIMULATION TESTS

5.1 Estimation of target's position usign EIF

The number of mobile robots used for target tracking in the simulation experiments was $N = 10$. It is assumed that each mobile robot can obtain an estimation of the target's cartesian coordinates and bearing, i.e. the target's cartesian coordinates $[x, y]$ as well as the target's orientation θ. To improve the accuracy of the target's localization, the target's coordinates and bearing are fused with the distance of the target from a reference surface measured in an inertial coordinates system (see Fig. 1 and 3).

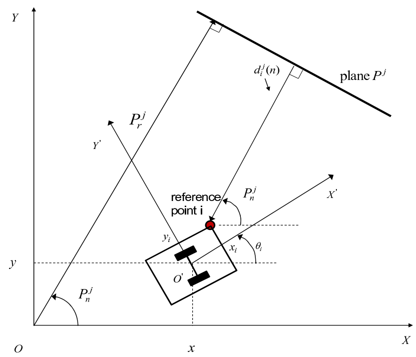


Fig. 3. Distance of the target's reference point i from the reference plane P^j, measured in the inertial coordinates system OXY

The inertial coordinates system OXY is defined. Furthermore the coordinates system $O'X'Y'$ is considered (Fig.

1). $O'X'Y'$ results from OXY if it is rotated by an angle θ (Fig. 1). The coordinates of the center of the wheels axis with respect to OXY are (x, y), while the coordinates of the reference point i that is mounted on the vehicle, with respect to $O'X'Y'$ are $x_i^{'}, y_i^{'}$. The orientation of the reference point with respect to $OX'Y'$ is $\theta_i^{'}$. Thus the coordinates of the reference point with respect to OXY are (x_i, y_i) and its orientation is θ_i, and are given by: $x_i(k) = x(k) + x_i^{'}sin(\theta(k)) + y_i^{'}cos(\theta(k))$, $y_i(k) = y(k) - x_i^{'}cos(\theta(k)) + y_i^{'}sin(\theta(k))$, $\theta_i(k) = \theta(k) + \theta_i$. Each plane P^j in the robot's environment can be represented by P_r^j and P_n^j (Fig. 3), where (i) P_r^j is the normal distance of the plane from the origin O, (ii) P_n^j is the angle between the normal line to the plane and the x-direction.

The target's reference point i is at position $x_i(k), y_i(k)$ with respect to the inertial coordinates system OXY and its orientation is $\theta_i(k)$. Using the above notation, the distance of the reference point i, from the plane P^j is represented by P_r^j, P_n^j (see Fig. 3): $d_i^j(k) = P_r^j - x_i(k)cos(P_n^j) - y_i(k)sin(P_n^j)$. Assuming a constant sampling period $\Delta t_k = T$ the measurement equation is $z(k+1) = \gamma(x(k)) + v(k)$, where $z(k)$ is the vector containing target's coordinates and bearing estimates obtained from a mobile sensor and the measurement of the target's distance to the reference surface, while $v(k)$ is a white noise sequence $\sim N(0, R(kT))$. To obtain the Extended Kalman Filter (EKF), the kinematic model of the target is discretized and written in a discrete-time state-space form [Rigatos (2009)]. The *measurement update* and *time update* are applied, where with reference to Eq. (7) matrix $L(k)$ is:

$$L(k) = \begin{pmatrix} Tcos(\theta(k)) & 0 \\ Tsin(\theta(k)) & 0 \\ 0 & T \end{pmatrix} \qquad (15)$$

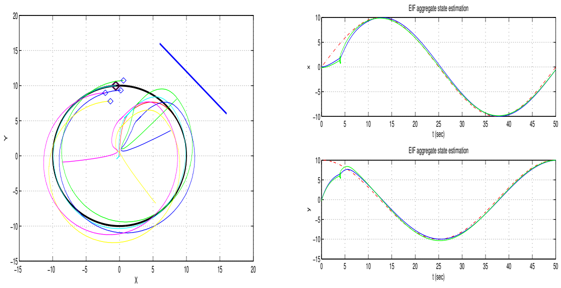
while $Q(k) = diag[\sigma^2(k), \sigma^2(k), \sigma^2(k)]$, with $\sigma^2(k)$ chosen to be 10^{-3} and $\phi(\hat{x}(k)) = [\hat{x}(k), \hat{y}(k), \hat{\theta}(k)]^T$, $\gamma(\hat{x}(k)) = [\hat{x}(k), \hat{y}(k), \hat{\theta}(k), d(k)]^T$, which is also written as $\gamma(\hat{x}(k)) = \left(\hat{x}(k)\ \hat{y}(k)\ \hat{\theta}(k)\ P_r^j - x_i(k))cos(P_n^j) - y_i(k)sin(P_n^j)\right)^T$.

The use of EKF for fusing the target's coordinates and bearing measured by each mobile robot with the target's distance from a reference surface measured in an inertial coordinates system provides an estimation of the target's state vector $[x(t), y(t), \theta(t)]$ and enables the successful tracking of the target's position $[\hat{x}^*, \hat{y}^*]$ by the individual mobile robots through the application of the motion planning algorithm analyzed in Section 2.

The tracking of the target's trajectory by the mobile robots ensemble as well as the accuracy of the sensor fusion-based estimation of the target's coordinates is depicted in Fig. 4 to Fig. 5. The target is marked as a thick-line rectangle and the associated reference trajectory is plotted as a thick line.

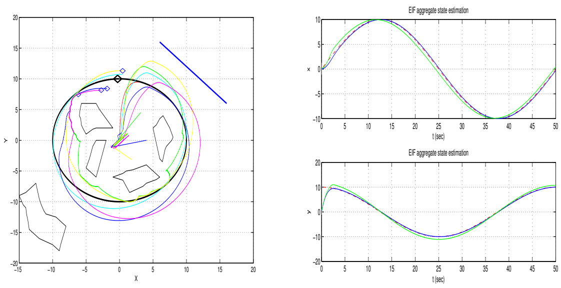
5.2 Estimation of target's position using UIF

Next, the estimation of the target's state vector was performed using the Unscented Information Filter. Again, the proposed distributed filtering enabled precise estimation of the target's state vector $[x, y, \theta]^T$ through the fusion of measurements of the target's coordinates and bearing obtained by each mobile sensor with the distance of the target from a reference surface measured in an inertial coordinates system. The state estimates of the local Unscented Kalman Filters running at each mobile sensor (autonomous vehicle) were aggregated into a single estimation by the Unscented Information Filter. The estimated coordinates $[\hat{x}^*, \hat{y}^*]$ of the target were used to generate the reference path which was followed by the mobile sensors. The tracking of the target's trajectory by the mobile robots ensemble as well as the accuracy of the two-level sensor fusion-based estimation of the target's position is shown in Fig. 6 to Fig. 7.



(a) (b)

Fig. 4. (a) Distributed target tracking for a circular trajectory in an obstacles-free motion space, (b) Aggregate estimation of the target's position with the use of EIF (continuous line) and target's reference path (dashed line).

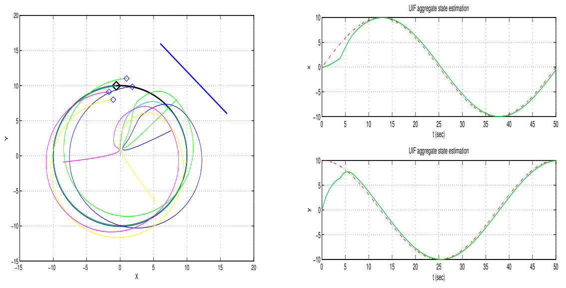


(a) (b)

Fig. 5. (a) Distributed target tracking for a circular path in a motion space with obstacles, (b) Aggregate estimation of the target's position with the use of EIF (continuous line) and target's reference path (dashed line).

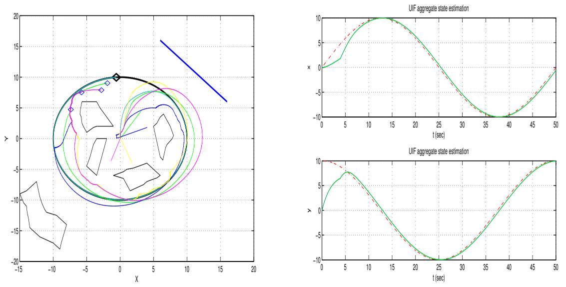
6. CONCLUSIONS

The paper has examined the problem of coordinated tracking of a target by an ensemble of mobile robots (unmanned ground vehicles). Each mobile robot was able to obtain measurements of the target's cartesian coordinates and orientation while a measurement of the target's distance from a reference surface in an inertial coordinates frame



(a) (b)

Fig. 6. (a) Distributed target tracking for a circular trajectory in an obstacles-free motion space, (b) Aggregate estimation of the target's position with the use of UIF (continuous line) and target's reference path (dashed line)



(a) (b)

Fig. 7. (a) Distributed target tracking for a circular path in a motion space with obstacles, (b) Aggregate estimation of the target's position with the use of UIF (continuous line) and target's reference path (dashed line).

was also communicated to the mobile robots. The state estimates of local Extended Kalman Filters running at each mobile robot were fused into one single estimate using the Extended Information Filter. Similarly, the state estimates of local Unscented Kalman Filters running at each mobile sensor were aggregated by the Unscented Information Filter into one single estimation. The estimated coordinates of the target $[\hat{x}^*, \hat{y}^*]$ were used to generate the reference path which was followed by the mobile robots. Next, a suitable motion planning algorithm was designed. The algorithm assured not only tracking of the reference path by the individual autonomous vehicles but also permitted (i) convergence of the autonomous vehicles to the target in a synchronized manner and (ii) avoidance of collisions with obstacles in the motion plane as well as avoidance of collisions between the autonomous vehicles. The performance of the distributed tracking algorithm was tested through simulation experiments.

REFERENCES

A. Benveniste, M. Métivier and P. Priouret, Adaptive algorithms and stochastic approximations, Springer, 1990.

F. Comets and T. Meyre, Calcul stochastique et modèles de diffusions, *Dunod*, 2006.

M. Duflo, Algorithmes stochastiques, Mathématiques et Applications vol. 23, *Springer*, 1996.

A. Franchi, L. Freda, G. Oriolo, and M. Vendittelli, The Sensor-based Random Graph Method for Cooperative Robot Exploration. IEEE/ASME Transactions on Mechatronics, vol. 14, no. 2, pp. 163-175, 2009.

V. Gazi and K. Passino, Stability analysis of social foraging swarms, IEEE Transactions on Systems, Man and Cybernetics - Part B: Cybernetics, vol. 34, no. 1, pp. 539-557, 2004.

S.J. Julier and J.K. Uhlmann, Unscented Filtering and Nonlinear Estimation, Proceedings of the IEEE, vol.92, pp. 401-422, 2004.

H. Khalil, Nonlinear Systems, Prentice Hall, 1996.

G. Oriolo, A. De Luca, and M. Vendittelli, WMR Control Via Dynamic Feedback Linearization: Design, Implementation and Experimental Validation, IEEE Transactions on Control Systems Technology, vol. 10, no.6, pp. 835-852, 2002.

D.J. Lee, Nonlinear estimation and multiple sensor fusion using unscented information filtering, IEEE Signal Processing Letters, vol. 15, pp. 861-864, 2008.

J. Manyika and H. Durrant-Whyte, Data fusion and sensor management: a decentralized information theoretic approach, Englewood Cliffs, NJ, Prentice Hall, 1994.

R. Olfati-Saber, Distributed tracking for mobile sensor networks with information-driven mobility, American Control Conference, ACC 2007, pp.4606-4612.

G. Oriolo, A. De Luca, and M. Vendittelli, WMR Control Via Dynamic Feedback Linearization: Design, Implementation and Experimental Validation, IEEE Transactions on Control Systems Technology, vol. 10, no.6, pp. 835-852, 2002.

G.G. Rigatos, Distributed gradient and particle swarm optimization for multi-robot motion planning, Robotica, Cambridge University Press, vol. 26, no 3, pp. 357-370, 2008.

G.G. Rigatos, Coordinated motion of autonomous vehicles with the use of a distributed gradient algorithm, Journal of Applied Mathematics and Computation, Elsevier, vol 199, n. 2, pp 494-503, 2008.

G.G. Rigatos, Particle Filtering for State Estimation in Nonlinear Industrial Systems, IEEE Transactions on Instrumentation and Measurement, vol. 58, no. 11, pp. 3885-3900, 2009.

G. Rigatos and Q. Zhang, Fuzzy model validation using the local statistical approach, Fuzzy Sets and Systems, Elsevier, vol. 60, no.7, pp. 882-904, 2009.

G.G. Rigatos, Distributed particle filtering over sensor networks for autonomous navigation of UAVs, in: "Robot Manipulators", SciYo Publications, Croatia, 2010.

T. Vercauteren and X. Wang, Decentralized Sigma-Point Information Filters for Target Tracking in Collaborative Sensor Networks, IEEE Transactions on Signal Processing, vol.53, no.8, pp. 2997-3009, 2005.

J. Villagra, B. d'Andrea-Novel, H. Mounier, M. Pengov, Flatness-based vehicle steering control strategy with SDRE feedback gains tuned via a sensitivity approach, IEEE Transactions on Control Systems Technology, vol. 15, 554-565, 2007 .