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Stick-slip Oscillations in Oillwell Drilstrings: Distributed Parameter and Neutral Type Retarded Model Approaches

Belem Saldivar [*,**] **Sabine Mondié** [*] **Jean-Jacques Loiseau** [**] **Vladimir Rasvan** [***]

[*] *Departamento de Control Automático , Cinvestav, IPN, México D.F., (e-mails: msaldivar@ctrl.cinvestav.mx, smondie@ctrl.cinvestav.mx).* [⋆]
[**] *Institut de Recherche en Communications et Cybernétique de Nantes, Nantes, France, (e-mail: jean-jacques.loiseau@irccyn.ec-nantes.fr).*
[***] *University of Craiova, Craiova, Romania (e-mail: rasvan@automation.ucv.ro).*

Abstract: The drill pipe model is described by the wave equation with mixed boundary conditions in which a sliding velocity is considered at the top end. The proposal of an energy function for the distributed model allows to find a control law that ensures the energy dissipation during the drilling operation. The distributed parameter model is reduced through the d'Alembert transformation to a difference equation model. Torsional drillstring vibrations, also known as "stick-slip" oscillations appearing in oilwell drillstrings are a source of failures which reduce penetration rates and increase drilling operation costs. Some experience-based control strategies are evaluated in order to reduce stick-slip oscillations. The use of the angular velocity at the drillstring upper part, the torque on the bit and the weight on the bit is shown to have a key effect in the reduction of drillstring torsional vibrations.

Keywords: Drilling systems, stick-slip phenomenon, distributed parameter systems, time-delay systems, model validation.

1. INTRODUCTION

Oilwell drillstrings are mechanisms that play a key role in the petroleum extraction industry. Failures in drillstrings can be significant in the total cost of the perforation process. These devices are complex dynamic systems with many unknown and varying parameters due to the fact that drillstring characteristics change as the drilling operation makes progress. The drillstring interaction with the borehole gives rise to a wide variety of non-desired oscillations which are classified depending on the direction they appear. Three main types of vibrations can be distinguished: torsional (stick-slip oscillations), axial (bit bouncing phenomenon) and lateral (whirl motion due to the out-of-balance of the drillstring). Torsional drillstring vibrations appears due to downhole conditions, such as significant drag, tight hole, and formation characteristics. It can cause the bit to stall in the formation while the rotary table continues to rotate. When the trapped torsional energy (similar to a wound-up spring) reaches a level that the bit can no longer resist, the bit suddenly comes loose, rotating and whipping at very high speeds. This stick-slip behavior can generate a torsional wave that travels up the drillstring to the rotary top system. Because of the high inertia of the rotary table, it acts like a fixed end to the drillstring and reflects the torsional wave back down the drillstring to the bit. The bit may stall again, and the torsional wave cycle repeats (Navarro & Suarez [2004]). The whipping and high speed rotations of the bit in the slip phase can generate both severe axial and lateral vibrations at the bottom-hole assembly (BHA). The vibrations can originate problems such as drill pipe fatigue problems, drillstring components failures, wellbore instability (Kriesels et al. [1999]). They contribute to drillpipe fatigue and are detrimental to bit life.

A full simulation covering all relevant phenomena is not reasonably practical and authors generally study vibration mechanisms individually. We focus on the problem of reducing stick-slip vibrations by mean of the alignment of different drilling parameters, such as: rotary speed, drilling torque and weight-on-bit. This approach captures qualitatively the driller's expertise. In practice, the drilling operator typically controls the surface-controlled drilling parameters, such as the weight-on-bit, drilling fluid flow through the drill pipe, the drillstring rotational speed and the density and viscosity of the drilling fluid to optimize the drilling operations.

The great practical significance of oilwell drillstrings has interested some researchers. Several approaches have been used to treat the modelling and control problems. Most of them deal with the torsional behavior and the suppression of stick-slip oscillations. Some classical control techniques for suppressing the stick-slip phenomenon are: the introduction of a soft torque rotary system (STRS) at the top of

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the drillstring (Sananikone [1993]), where the underlying idea is making the top rotary system behave in a "soft" manner rather than as a fixed heavy flywheel so that the torsional waves arriving at the surface are absorbed breaking the harmful cycling motion, the introduction of a vibration absorber at the top of the drillstring (Jansen & van den Steen [1995]), the introduction of a PID controller at the surface in order to control the rotary speed and the introduction of an additional friction at the bit (Pavone & Desplans [1994]).

This contribution is focused on the drillstring analysis and the main goal is to give some operating recommendations in order to maintain optimal drilling conditions since the introduction of a automatic controlled drilling system can be unfeasible due to the complexity of oilwell drillstrings and drilling practices.

The paper is organized as follows: In Section II we present a distributed parameter model describing the drilling system coupled with a model for the torque on the bit which describes stick-slip oscillations. The proposal of an energy function for the distributed model provides a control law $\Omega(t)$ that ensures the energy dissipation during the drilling process. The distributed parameter model is transformed to a neutral type retarded equation in order to simplify the analysis and simulations. The formal stability of the neutral type delay model is determined. In Section III we analyze the effects of the weight on the bit and the rotational speed at the surface on the bit behavior. In Section IV two known strategies to reduce stick-slip oscillations are tested in the neutral type delay model. First, a variation law for the weight on the bit, second, the increment of the damping at the bottom extremity. The control law obtained in Section II is shown to have a key effect in the reduction of stick-slip oscillations. Conclusions are presented in the last section.

2. DRILLING SYSTEM MODEL

A sketch of a simplified drillstring system is shown on Fig.1.

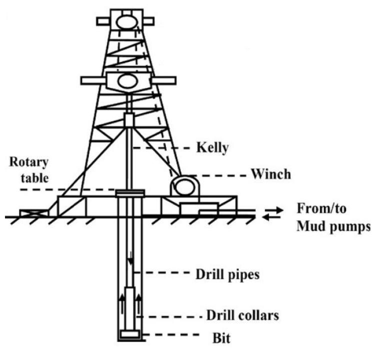

Fig. 1. The drilling system.

The main process during well drilling for oil is the creation of borehole by a rock-cutting tool called a bit. The drillstring consists of the BHA (bottom hole assembly) and drillpipes screwed end to end to each other to form a long pipe. The BHA comprises the bit, stabilizers (at least two spaced apart) which prevent the drillstring from unbalancing, and a series of pipe sections which are relatively heavy known as drill collars. While the length of the BHA remains constant, the total length of the drill pipes increases as the borehole depth does. An important element of the process is the drilling mud or fluid which among others, has the function of cleaning, cooling and lubricating the bit. The drillstring is rotated from the surface by an electrical motor. The rotating mechanism can be of two types: a rotary table or a top drive.

2.1 Distributed parameter model

A distributed parameter model describing the drilling system is first presented. The drill pipe is considered as a beam in torsion. A lumped inertia I_B is chosen to represent the assembly at the bottom hole and a damping $\beta \geq 0$ which includes the viscous and structural damping, is assumed along the structure. The drillstring is rotated from the surface ($\xi = 0$) by an electrical motor, Ω is the angular velocity coming from the rotor that does not match the rotational speed of the load $\frac{\partial \theta}{\partial t}(0,t)$. This sliding speed results in the local torsion of the drillstring. The other extremity ($\xi = L$), is subject to a torque $T\left(\frac{\partial \theta}{\partial t}(L,t)\right)$, which is a function of the bit speed. The mechanical system is described by the following partial differential equations:

$$GJ\frac{\partial^2 \theta}{\partial \xi^2}(\xi,t) - I\frac{\partial^2 \theta}{\partial t^2}(\xi,t) - \beta\frac{\partial \theta}{\partial t}(\xi,t) = 0, \qquad (1)$$

$$\xi \in (0,L),\ t > 0$$

with boundary conditions

$$\begin{aligned} GJ\frac{\partial \theta}{\partial \xi}(0,t) &= c_a\left(\frac{\partial \theta}{\partial t}(0,t) - \Omega(t)\right), \\ GJ\frac{\partial \theta}{\partial \xi}(L,t) + I_B\frac{\partial^2 \theta}{\partial t^2}(L,t) &= -T\left(\frac{\partial \theta}{\partial t}(L,t)\right) \end{aligned} \qquad (2)$$

where $\theta(\xi,t)$ is the angle of rotation, I is the inertia, G is the shear modulus and J is the geometrical moment of inertia.

Model for the torque on the bit The following switched equation is introduced in Navarro & Corts [2007], it allows to approximate the physical phenomenon at the bottom hole

$$T = c_b\frac{\partial \theta}{\partial t}(L,t) + W_{ob}R_b\mu_b\left(\frac{\partial \theta}{\partial t}(L,t)\right)sgn\left(\frac{\partial \theta}{\partial t}(L,t)\right) (3)$$

the term $c_b\frac{\partial \theta}{\partial t}(L,t)$ is a viscous damping torque at the bit which approximates the influence of the mud drilling and the term $W_{ob}R_b\mu_b sgn(\frac{\partial \theta}{\partial t}(L,t))$ is a dry friction torque modelling the bit-rock contact. $R_b > 0$ is the bit radius and $W_{ob} > 0$ the weight on the bit. The bit dry friction coefficient $\mu_b(\frac{\partial \theta}{\partial t}(L,t))$ is modeled as

$$\mu_b\left(\frac{\partial \theta}{\partial t}(L,t)\right) = \mu_{cb} + (\mu_{sb} - \mu_{cb})e^{-\frac{\gamma_b}{v_f}\left|\frac{\partial \theta}{\partial t}(L,t)\right|} \qquad (4)$$

where $\mu_{sb}, \mu_{cb} \in (0,1)$ are the static and Coulomb friction coefficients and $0 < \gamma_b < 1$ is a constant defining the velocity decrease rate. The constant velocity $v_f > 0$ is introduced in order to have appropriate units.

The friction torque (3)-(4) leads to a decreasing torque-on-bit with increasing bit angular velocity for low velocities which acts as a negative damping (Stribeck effect) and is the cause of stick-slip self-excited vibrations. The exponential decaying behavior of $T\left(\frac{\partial\theta}{\partial t}(L,t)\right)$ coincides with experimental torque values.

Dissipativity analysis Substituting the model for the torque on the bit (3)-(4) into (1), the normalized drilling system is:

$$\begin{array}{l} z_{tt}(\sigma,t) = a z_{\sigma\sigma}(\sigma,t) - d z_t(\sigma,t) \\ t>0,\ 0<\sigma<1 \end{array} \tag{5}$$

where $a = \frac{GJ}{IL^2}$ and $d = \frac{\beta}{I}$ coupled to the mixed boundary conditions

$$z_\sigma(0,t) = g\left(z_t(0,t) - \Omega(t)\right), \sigma \in (0,1), \quad t>0, \tag{6}$$
$$z_\sigma(1,t) = -k z_t(1,t) - q\mu_b(z_t(1,t)) sgn(z_t(1,t)) - h z_{tt}(1,t),$$

where $g = \frac{c_a L}{GJ}$, $k = \frac{c_b L}{GJ}$, $q = \frac{W_{ob} R_b L}{GJ}$ and $h = \frac{I_B L}{GJ}$.

Following the ideas introduced in Fridman [2010] we consider the energy function

$$E(t) = \int_0^1 a z_\sigma^2(\sigma,t)d\sigma + \int_0^1 z_t^2(\sigma,t)d\sigma + ahz_t(1,t)^2. \tag{7}$$

Differentiating $E(t)$ yields

$$\frac{d}{dt}E(t) = 2\int_0^1 a z_\sigma(\sigma,t) z_{t\sigma}(\sigma,t)d\sigma + 2\int_0^1 z_t(\sigma,t) z_{tt}(\sigma,t)d\sigma$$
$$+2ahz_t(1,t)z_{tt}(1,t),$$

$$\frac{d}{dt}E(t) = 2a\int_0^1 \left[z_\sigma(\sigma,t)z_{t\sigma}(\sigma,t) \ + z_t(\sigma,t)z_{\sigma\sigma}(\sigma,t)\right]d\sigma$$
$$-2d\int_0^1 z_t(\sigma,t)z_t(\sigma,t)d\sigma + 2ahz_t(1,t)z_{tt}(1,t).$$

Integrating by parts and substituting the boundary conditions (6) gives

$$\int_0^1 z_t(\sigma,t)z_{\sigma\sigma}(\sigma,t)d\sigma$$
$$= z_t(\sigma,t)z_\sigma(\sigma,t)|_0^1 - \int_0^1 z_\sigma(\sigma,t)z_{t\sigma}(\sigma,t)d\sigma$$
$$= z_t(1,t)(-kz_t(1,t) - q\mu_b(z_t(1,t))sgn(z_t(1,t)) - hz_{tt}(1,t))$$
$$-gz_t(0,t)\left(z_t(0,t)-\Omega(t)\right) - \int_0^1 z_\sigma(\sigma,t)z_{t\sigma}(\sigma,t)d\sigma.$$

Hence,

$$\frac{d}{dt}E(t) = 2a\int_0^1 z_\sigma(\sigma,t)z_{t\sigma}(\sigma,t)d\sigma$$
$$+2az_t(1,t)(-kz_t(1,t) - q\mu_b(z_t(1,t))sgn(z_t(1,t))$$
$$-hz_{tt}(1,t)) - 2agz_t(0,t)\left(z_t(0,t)-\Omega(t)\right)$$
$$-2a\int_0^1 z_\sigma(\sigma,t)z_{t\sigma}(\sigma,t)d\sigma$$
$$-2d\int_0^1 z_t(\sigma,t)z_t(\sigma,t)d\sigma + 2ahz_t(1,t)z_{tt}(1,t),$$

since $\mu_b(z_t(1,t))sgn(z_t(1,t))z_t(1,t) = \mu_b(z_t(1,t))\,|z_t(1,t)|$, we have

$$\frac{d}{dt}E(t) = -2aq\mu_b(z_t(1,t))\,|z_t(1,t)| - 2d\int_0^1 z_t^2(\sigma,t)d\sigma \tag{8}$$
$$-2agz_t(0,t)\left(z_t(0,t)-\Omega(t)\right) - 2akz_t^2(1,t).$$

In order to ensure the dissipativity of the system, the control law $\Omega(t)$ should allow the negativity of (8). Choosing as a control law the following expression

$$\begin{array}{c} \Omega(t) = c_1 z_t(0,t) + 2\sqrt{\left(k(1-c_1)\right)/g}\, z_t(1,t) \\ +2\sqrt{c_2(1-c_1)}z_t^{ref}(1,t) - c_2\left(z_t^{ref}(1,t)\right)^2/z_t(0,t) \\ -2\sqrt{(c_2 k)/g}\; z_t(1,t)z_t^{ref}(1,t)/z_t(0,t) \end{array} \tag{9}$$

where $z_t^{ref}(1,t)$ is the angular velocity to be achieved and c_1, c_2 are free design parameters satisfying $c_1 < 1, c_2 > 0$, we obtain that

$$\frac{d}{dt}E(t) = -2aq\mu_b(z_t(1,t))\,|z_t(1,t)| - 2d\int_0^1 z_t^2(\sigma,t)d\sigma$$
$$-2ag\Big[z_t^2(0,t) - c_1 z^2(0,t) - 2\sqrt{\left(k(1-c_1)\right)/g}\,z_t(0,t)z_t(1,t)$$
$$-2\sqrt{c_2(1-c_1)}z_t(0,t)z_t^{ref}(1,t) + c_2\left(z_t^{ref}(1,t)\right)^2$$
$$+2\sqrt{(c_2 k)/g}\,z_t(1,t)z_t^{ref}(1,t) + \frac{k}{g}z_t^2(1,t)\Big]$$

equivalently,

$$\frac{d}{dt}E(t) = -2aq\mu_b(z_t(1,t))\,|z_t(1,t)| - 2d\int_0^1 z_t^2(\sigma,t)d\sigma$$
$$-2ag\left(\sqrt{1-c_1}z_t(0,t) - \sqrt{k/g}z_t(1,t) - \sqrt{c_2}z_t^{ref}(1,t)\right)^2$$

Taking into account that $\mu_b(z_t(1,t)) > 0$ and a, q, d and g are positive constants we find that $\frac{d}{dt}E(t) \le 0$. The non-growth of the energy of the drilling system (which reflects the oscillatory behavior of the system) is established:

Proposition 1. For all solutions of (5) under the boundary conditions (6), the energy given by (7) does not grow if the control law (9) is applied.

2.2 Neutral type model

In the sequel the damping β is assumed to be zero, hence the distributed parameter model reduces to the unidimensional wave equation

$$\frac{\partial^2\theta}{\partial\xi^2}(\xi,t) = p^2\frac{\partial^2\theta}{\partial t^2}(\xi,t),\ \xi\in(0,L),\ p = \sqrt{\frac{I}{GJ}}, \tag{10}$$

with boundary conditions (2). The general solution of (10) can be written as:

$$\theta(\xi,t) = \phi(t+p\xi) + \psi(t-p\xi),$$

where ϕ, ψ are continuously differentiable real-valued functions of one variable.

The boundary conditions can be rewritten as

$$p\frac{\partial\phi}{\partial\gamma}(t) - p\frac{\partial\psi}{\partial\eta}(t) = \frac{c_a}{GJ}\left(\frac{\partial\phi}{\partial\gamma}(t) + \frac{\partial\psi}{\partial\eta}(t) - \Omega(t)\right) \tag{11}$$

$$\begin{array}{c} p\left(\frac{\partial\phi}{\partial\gamma}(t+\Gamma) - \frac{\partial\psi}{\partial\eta}(t-\Gamma)\right) = -\frac{I_B}{GJ}\frac{\partial^2\phi}{\partial\gamma^2}(t+\Gamma) \\ -\frac{I_B}{GJ}\frac{\partial^2\psi}{\partial\eta^2}(t-\Gamma) - \frac{1}{GJ}T\left(\frac{\partial\phi}{\partial\gamma}(t+\Gamma) + \frac{\partial\psi}{\partial\eta}(t-\Gamma)\right) \end{array} \tag{12}$$

where $\Gamma = pL$. Let $w(t)$, $y(t)$ be the displacements from the unexcited position at the bottom and top extremities respectively:

$$\begin{array}{l} w(t) = \theta(L,t) = \phi(t+\Gamma) + \psi(t-\Gamma), \\ y(t) = \theta(0,t) = \phi(t) + \psi(t). \end{array} \qquad (13)$$

Under appropriate manipulations of the equations (11)-(13) we obtain a neutral type retarded expression for the behavior at the bottom and upper extremities of the drillstring:

$$\begin{array}{c} \ddot{w}(t) - \Upsilon\ddot{w}(t-2\Gamma) + \Psi\dot{w}(t) + \Psi\Upsilon\dot{w}(t-2\Gamma) = \\ -\frac{1}{I_B}T\left(\dot{w}(t)\right) + \frac{1}{I_B}\Upsilon T\left(\dot{w}(t-2\Gamma)\right) + 2\Psi(c_a/\Lambda)\Omega(t-\Gamma) \end{array} \qquad (14)$$

$$\begin{array}{l} \dot{y}(t) + \Upsilon\dot{y}(t-2\Gamma) = (c_a/\Lambda)\left[\Omega(t) + \Omega(t-2\Gamma)\right] \\ \quad -(2/\Lambda)\left[I_B\ddot{w}(t-\Gamma) + T\left(\cdot w(t-\Gamma)\right)\right] \end{array} \qquad (15)$$

where $\Upsilon = \frac{c_a - \sqrt{IGJ}}{c_a + \sqrt{IGJ}}$, $\Psi = \frac{\sqrt{IGJ}}{I_B}$, $\Lambda = c_a + \sqrt{IGJ}$ and

$$T\left(w(t)\right) = c_b\dot{w}(t) + W_{ob}R_b\mu_b(\dot{w}(t))sgn(\dot{w}(t)) \qquad (16)$$

$$\mu_b(\dot{w}(t)) = \mu_{cb} + (\mu_{sb} - \mu_{cb})e^{-\frac{\gamma_b}{v_f}|\dot{w}(t)|}.$$

Formal stability analysis The term *formal stability* was introduced in Byrnes et al. [1984]. According to Pontryagin [1942], it means that a neutral type delay system has only a finite number of zeros in the right half complex plane. The formal stability property corresponds to the stability of the difference operator associated with the neutral system. Its well known that the formal stability is a necessary condition to the stability and stabilization of such systems (Loiseau et al. [2002]).

We consider a system of the form

$$\begin{array}{l} \dot{x}(t) = \sum_{k=1}^{q} E_k\dot{x}(t-k\Gamma) + \sum_{k=0}^{q} A_k x(t-k\Gamma) \\ \quad + \sum_{k=0}^{q} B_k u(t-k\Gamma) + \sum_{k=o}^{q} C_k f(x(t-k\Gamma)) \end{array} \qquad (17)$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input and $f(\cdot)$ is a perturbation of the system depending on the state. Such system is said to be *formally stable* if

$$Rank(I_n - \hat{E}(s)) = n, \forall s\,|Re(s) \geq 0,$$

where

$$\hat{E}(s) = \sum_{k=1}^{q} E_k e^{-ks\theta} \qquad (18)$$

and $Re(s)$ denotes the real part of the complex number s.

Choosing $w_1 = w$, $w_2 = \dot{w}$ and $w_3 = y$ the neutral type model of the drilling system (14)-(15) can be rewritten in the form (17) with $x = (\,w_1\ w_2\ w_3\,)^T$, $q = 2$, $u = \Omega$, $f = T$ and constant matrices A_0, $A_1 = 0_{n\times n}$, A_2, B_0, B_1, B_2, C_0, C_1, C_2,

$$E_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{2I_B}{\Lambda} & 0 \end{pmatrix} \text{ and } E_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Upsilon & 0 \\ 0 & 0 & -\Upsilon \end{pmatrix} \qquad (19)$$

According to (18) we have that

$$\hat{E}(s) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Upsilon e^{-2\Gamma s} & 0 \\ 0 & -\frac{2I_B}{\Lambda}e^{-\Gamma s} & -\Upsilon e^{-2\Gamma s} \end{pmatrix}. \qquad (20)$$

Then, we have that the neutral model of the drilling system is formally stable if

$$\det\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-\Upsilon e^{-2\Gamma s} & 0 \\ 0 & \frac{2I_B}{\Lambda}e^{-\Gamma s} & 1+\Upsilon e^{-2\Gamma s} \end{pmatrix} \neq 0 \quad \forall s|\,Re(s) \geq 0$$

i.e., $1-\ \Upsilon^2 e^{-4\Gamma s} \neq 0$ for all s such that $Re(s) \geq 0$. Since

$$e^{-4\Gamma s} \leq 1 \quad \forall s|\,Re(s) \geq 0$$

and

$$|\Upsilon| = \left|\frac{c_a - \sqrt{IGJ}}{c_a + \sqrt{IGJ}}\right| < 1 \text{ for } c_a > 0, \qquad (21)$$

we can conclude that the neutral type model of the drillstring is formally stable.

3. STICK-SLIP PHENOMENON

Equation (14) coupled to the model for the torque on the bit (16) describes the drillstring behavior and the occurrence of stick-slip oscillations. A simulation of this model with $W_{ob} = 97347N$ and $\Omega = 10rad/s$ is shown on Fig. 2. Notice that when the phenomenon occurs, as reported in real wells, important torque fluctuations appear Kriesels et al. [1999].

The model parameters used for the simulations presented in the sequel are:

$$G = 79.3x10^9 N/m^2, I = 0.095Kg\cdot m, L = 1172m,$$
$$J = 1.19x10^{-5}m^4, Rb = 0.155575, v_f = 1, c_a = 2000Nms$$
$$c_b = 0.03Nms/rad, \mu_{cb} = 0.5\mu_{sb} = 0.8, \gamma_b = 0.9.$$

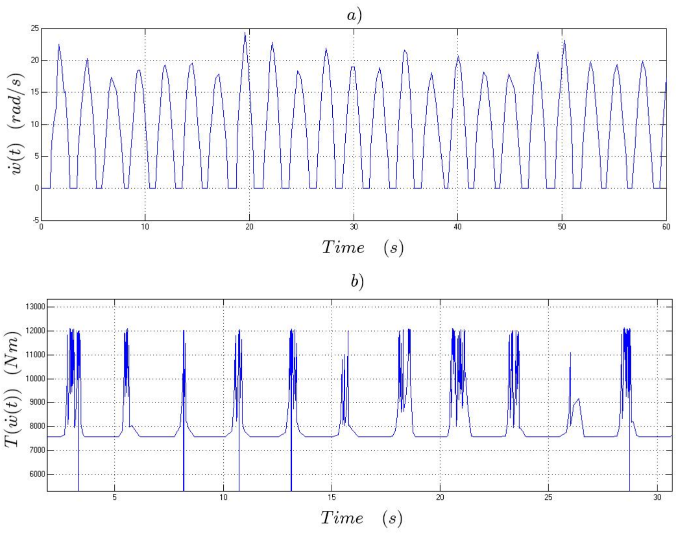

Fig. 2. Stick-slip phenomenon. *a*) Velocity at the bottom extremity. *b*) Torque on the bit.

According to drillers'experience, two practical strategies to avoid the stick-slip phenomenon are: the reduction of the weight on the bit W_{ob} and the increase of the velocity at the surface Ω. Simulations results of this model show an important reduction of the stick-slip oscillations by decreasing W_{ob} from $97347N$ to $31649N$ (Fig. 3) and by increasing Ω from $10rad/s$ to $20rad/s$ (Fig. 4).

As the above simulations reflect experimental results, the model (14)-(16) is validated.

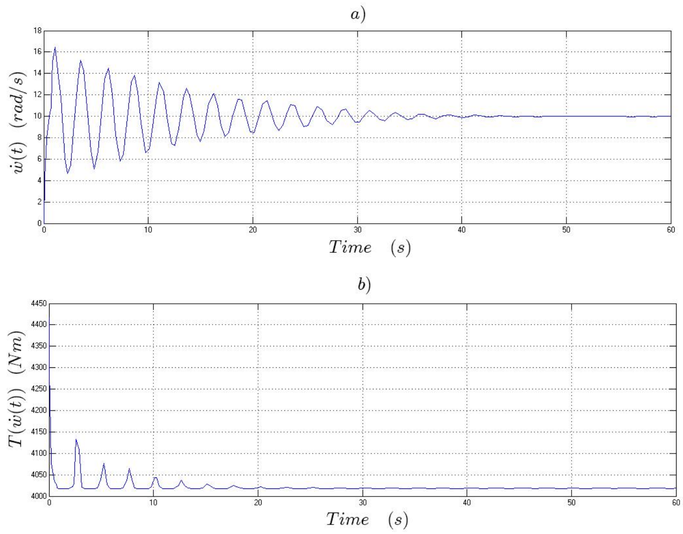

Fig. 3. Reduction of stick-slip phenomenon by decreasing W_{ob}.

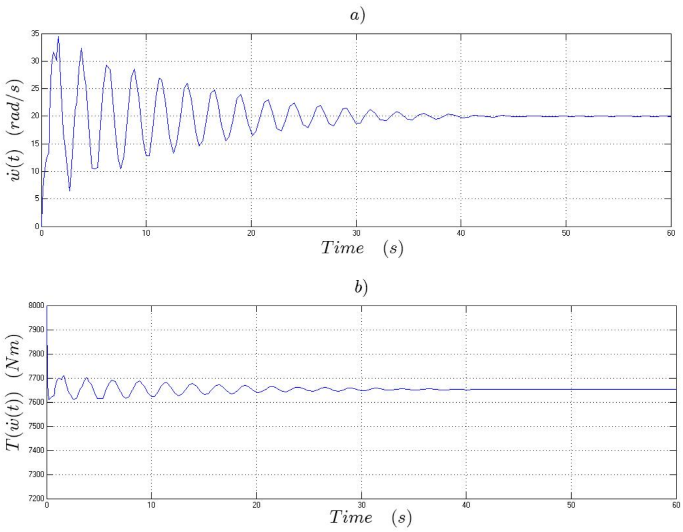

Fig. 4. Reduction of stick-slip phenomenon by increasing Ω.

4. STRATEGIES TO REDUCE STICK-SLIP OSCILLATIONS

In practice, the driller operator typically controls the surface-controlled drilling parameters, such as the weight on the bit, the drillstring rotational speed and the density and viscosity of the drilling fluid to optimize the drilling operations. Next, we present effective strategies to reduce stick-slip phenomenon.

4.1 Manipulation of the weight on the bit

From field data experience and from simulations of the model studied, it is concluded that the manipulation of the weight on the bit can be a solution for stick-slip oscillations even for low velocities at the surface Ω. Increasing velocities at the rotary top driving system may lead to lateral vibrations, that is why the manipulation of the weight on the bit can be an alternative solution in order to attenuate stick-slip oscillations. The variation of the weight on the bit is proposed in Navarro & Suarez [2004] as follows:

$$W_{ob}(\dot{w}) = K_w \, |\dot{w}(t)| + W_{ob0} \qquad (22)$$

with $W_{ob0} > 0$ and $W_{ob} > W_{ob0}$. Expression (22) captures the main characteristics of the weight on the bit. When $\dot{w}(t)$ decreases, W_{ob} decreases. As too low values of W_{ob} would make drilling stop, the weight on the bit must be maintained at a minimal value W_{ob0} to insure a desirable rate of penetration.

W_{ob} variation law (22) can be substituted into model (14)-(16) for a drilling parameters combination for which stick-slip oscillations are presented. This is the case of considering $\Omega = 10rad/s$ and $W_{ob0} = 97347N$ as it is shown on Fig. 5.

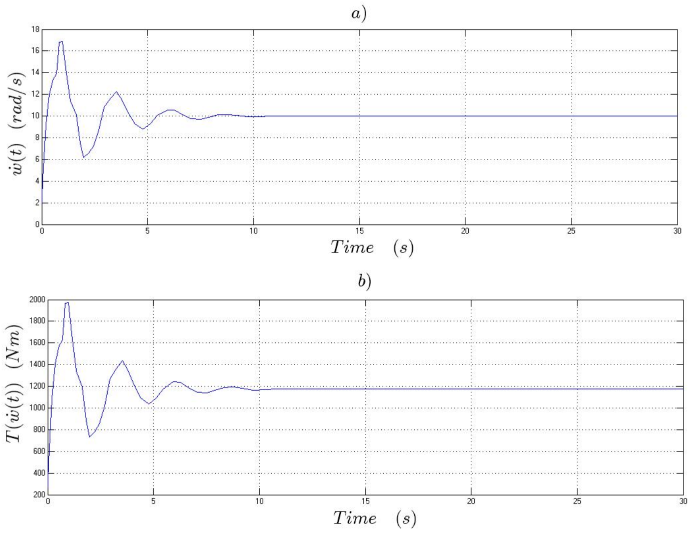

Fig. 5. Reduction of stick-slip oscillations by means of considering (22).

4.2 Manipulation of the damping at the bottom extremity

Another strategy for reducing stick-slip oscillations at the BHA is by increasing the damping at the down end of the drillstring. This can mainly be done in two ways: modifying the drilling fluid characteristics (it could be approximated by means of increasing the damping coefficient c_b) and with the inclusion of vibration absorbers at the BHA in order to dampen torsional vibrations generated at the bit and prevent them from travelling up and back down the drillstring Navarro & Suarez [2004].

The behavior of the velocity in the bottom extremity of the system (14) with the expression for the torque (16) for different values of c_b is shown on Fig. 6.

4.3 Application of a control law that ensures the dissipativity of the system

In Section II we have shown that the control law (9) ensure a dissipative behavior of the drilling system. This could lead to the idea that the system dissipates energy when the stick-slip phenomenon is not happening. A simulation of the system (14)-(15) coupled to (16) could assert this idea.

The simulation results of Fig. 7 show that the stick-slip vibrations are reduced by means of the application of the control law (9) depending on the angular velocity at the top and bottom extremities. The reference velocity considered in this simulation is $\dot{w}^{ref}(t) = 10rad/s$ and the constants c_1, c_2 are 0.3 and 2 respectively.

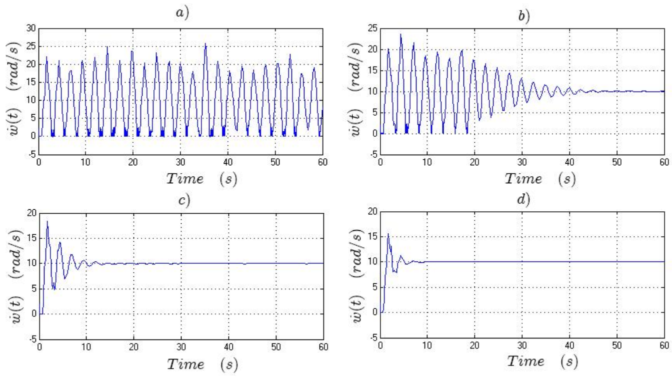

Fig. 6. Velocity at the bottom extremity for diferent values of c_b (Nms/rad): *a*)0.8, *b*)15, *c*)65, *d*)150.

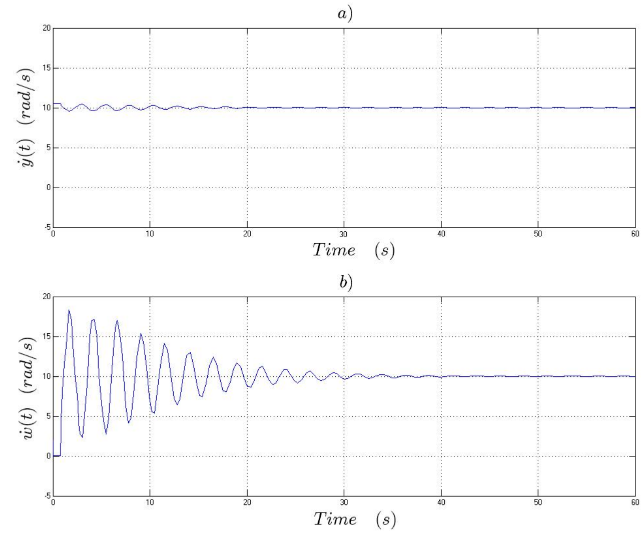

Fig. 7. Reduction of stick-slip oscillations by means of applying the control law (9)

Remark 1. There are several methods in order to estimate parameters at the bottom hole, such as, the angular velocity and the weight on the bit, see for example different methods used together with TRAFOR system designed in the Institut Francais du Pétrole (Perreau et al. [1998]). Measurement While Drilling (MWD) refers to a technique of making down hole measurements of borehole position, tool face orientation, formation parameters and drilling information using sensors located in the bottom hole assembly adjacent to the drill bit. These measurements are made during drilling and can be recorded down hole and/or transmitted to surface, see for example Close et al. [1988].

5. CONCLUSION

A distributed parameter model describing the torsional behavior of a vertical oilwell drillstring has been presented. We use the D'Alembert transformation and the Laplace transform to obtain a neutral type retarded model in order to simplify the analysis and simulations of the drillstring. We presented a model for the rock-bit interaction originating stick-slip self-excited oscillations.

Some strategies for reducing stick-slip oscillations are applied based on the manipulation of BHA characteristics. Two main solutions are highlighted, first, the variation of the weight on the bit, second, the increasing of the damping at the down end of the drillstring. These two solutions are in accordance with experimental field data.

The proposal of an energy function for the distributed model allowed us to find a control law that leads a dissipative drilling system. This control law is shown to have a key effect in the reduction of stick-slip oscillations.

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