Preprints of the 18th IFAC World Congress
Milano (Italy) August 28 - September 2, 2011

Revisiting some practical issues in the implementation of model-free control

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Abstract: This paper simplifies several aspects of the practical implementation of the newly introduced *model-free* control and of the corresponding *intelligent* PID controllers (M. Fliess, C. Join, "Model-free control and intelligent PID controllers: towards a possible trivialization of nonlinear control?," 15^{th} IFAC Symp. System Identif, Saint-Malo, 2009). Four examples with their computer simulations permit to test our techniques.

Keywords: Model-free control; intelligent PID controllers; estimation; identification.

1. INTRODUCTION

Let us start with a brief review of the general principles of *model-free* control, introduced by Fliess & Join [2008, 2009], and of the corresponding *intelligent* PID controllers (see d'Andréa-Novel *et al.* [2010b] for the connections with "classic" PIDs), which already led to a number of exciting applications in various fields: d'Andréa-Novel *et al.* [2010a], Choi *et al.* [2009], Formentin *et al.* [2010], Gédouin *et al.* [2011], Join *et al.* [2008, 2010], Michel *et al.* [2010], Villagra *et al.* [2009, 2010, 2011]. For simplicity's sake we are restricting ourselves to single-input single-output systems. The input-output behavior of the plant is assumed to be well approximated within its operating range by an ordinary differential equation $E(y, \dot{y}, \cdots, y^{(a)}, u, \dot{u}, \cdots, u^{(b)}) = 0$, which is nonlinear in general and unknown, or at least poorly known. Replace it by the "ultra-local" model, which is continuously updated,

$$y^{(n)}(t) = F(t) + \alpha u(t) \quad (1)$$

where

- the order n, $1 \leq n \leq a$, of derivation has always been chosen to be equal to 1 or 2, and 1 in all concrete situations, [1]
- the constant coefficient α is chosen by the practitioner, such that αu and $y^{(n)}$ are of the same order of magnitude,
- the time-varying function $F(t)$, which is estimated thanks to the knowledge of u and y, subsumes the structural properties of the unknown system.

Close the loop, if $n = 2$ in Equation (1), with an *intelligent* PID controller, or i-PID,

$$u = -\frac{F + \ddot{y}^\star + K_P e + K_I \int e + K_D \dot{e}}{\alpha} \quad (2)$$

where

- $y^\star$ is the output reference trajectory,
- $e = y - y^\star$ is the tracking error,
- K_P, K_I and K_D are the usual tuning gains.

The above control strategy was put into practice until now via an estimate of the n^{th}-order derivative $y^{(n)}$ in Equation (1), which yields an estimate of F in Equations (1) and (2). In spite of recent significant advances on the numerical differentiation of noisy signals by Mboup *et al.* [2009], this task remains quite complex and sometimes difficult to implement. We replace it by recent, but quite simple, algebraic and non-asymptotic techniques (Fliess & Sira-Ramírez [2003, 2008]) for online parameter identification of linear systems. They have been utilized in several concrete case-studies (see, *e.g.*, Abouaïssa *et al.* [2008], Becedas *et al.* [2009], Pereira *et al.* [2009], Trapero *et al.* [2007]). Let us summarize this new viewpoint by considering the equation

$$L\left(\frac{d}{dt}\right) z = \phi + \alpha u \quad (3)$$

where

- $\phi \in \mathbb{R}$ is an unknown constant,
- $\frac{d^\nu z}{dt^\nu} = y$, for some $\nu \geq 0$,
- $L(\frac{d}{dt}) \in \mathbb{R}[\frac{d}{dt}]$ is a linear differential operator with constant coefficients.

ϕ is *linearly identifiable* according to Fliess & Sira-Ramírez [2003, 2008]. We thus approximate an unknown function like F by a piecewise constant one. We show that this new setting, which is easier to grasp and to implement, possesses excellent robustness properties.

[1] See Fliess *et al.* [2011] for an explanation.

Another important modification with respect to Fliess & Join [2008, 2009] is related to the case of a partially known model. If this partially known model happens to be *flat* (Fliess *et al.* [1995], Lévine [2009], Sira-Ramírez & Agrawal [2004]), it might facilitate the choice of a reference trajectory and of a corresponding nominal control. The stabilization around this reference is nevertheless achieved in Section 3.4 in a straightforward way, thanks to the model-free i-PID (2).

Section 2 explains our identification procedure. Four examples accompanied by several computer simulations are discussed in Section 3 in order to test our implementation, even with quite noisy measurements. Some concluding remarks are presented in Section 4

2. ONLINE PARAMETER IDENTIFICATION

2.1 Linear identifiability

Rewrite Equation (3) via the classic rules of operational calculus (see, *e.g.*, Yosida [1984])

$$L(s)Z = \frac{\phi}{s} + \alpha U + I(s) \tag{4}$$

where $I \in \mathbb{R}[s]$ is a polynomial associated to the initial conditions. For $N \geq 1$ sufficiently large, $\frac{d^N I}{ds^N} \equiv 0$. Multiplying both sides of Equation (4) by $\frac{d^N}{ds^N}$ permits to get rid of the initial conditions. It yields the *linear identifiability* (Fliess & Sira-Ramírez [2003, 2008]) of ϕ thanks to the formula

$$\frac{d^N}{ds^N} L(s)Z = \frac{(-1)^N N!}{s^{N+1}}\phi + \alpha \frac{d^N U}{ds^N} \tag{5}$$

Multiplying both sides of Equation (5) by s^{-M}, where $M \geq 0$ is sufficiently large, permits to get rid of positive powers of s, *i.e.*, of derivatives with respect to time. The corresponding formulae in the time domain are easily deduced thanks to the correspondence between $\frac{d}{ds}$ and the multiplication by $-t$ in the time domain.

2.2 Identification scheme

Assume that $n = 1$ in Equation (1). Close the loop, like in d'Andréa-Novel *et al.* [2010a], via an i-P, *i.e.*, an i-PID (2) where $K_I = K_D = 0$. Assume that F is approximatively constant during the short time window $[T - \delta, T]$. The above algebraic manipulations lead to the following estimate of F:

$$F \approx \frac{1}{\delta}\int_{T-\delta}^{T} \dot{y}^\star - \frac{\alpha}{\delta}\int_{T-\delta}^{T} u - \frac{K_P}{\delta}\int_{T-\delta}^{T} e. \tag{6}$$

The robustness with respect to noises is ensured by the integrals which are the simplest low-pass filters.[2] Note moreover that the estimator (6) may be easily implemented in the form of a discrete linear filter.

3. FOUR COMPUTER SIMULATIONS

3.1 A perturbed pendulum

Bring and maintain an actuated simple pendulum around its upright unstable equilibrium position. Numerical simulations are performed via

$$J\ddot{\theta} = mgl\sin\theta + \tau - k\text{sign}(\dot{\theta}) - c\dot{\theta} \tag{7}$$

where θ is the angular coordinate, and $m = 0.2$, $g = 10$, $l = 0.7$, $k = 0.01$, $c = 0.4$ and $J = ml^2$ are physical parameters. The control torque is τ. Take $n = 1$ in Equation (1) and estimate F by a procedure similar to the one in Section 2.2. Figures 1 and 2 show a successful test where the pendulum starts near to its downward position and rejoin its upright position. Noises are added in Figures 3 and 4. The control input, displayed in Figure 4, reflects the excellent filtering provided by the estimator of F especially when the pendulum is close to its upright (zero) position where the signal to noise ratio is very small (negative in dB). The control input is actualized each 10^{-3} second and a Runge-Kutta method is used in order to simulate Equation (7) during each time increment of 10^{-3} second.

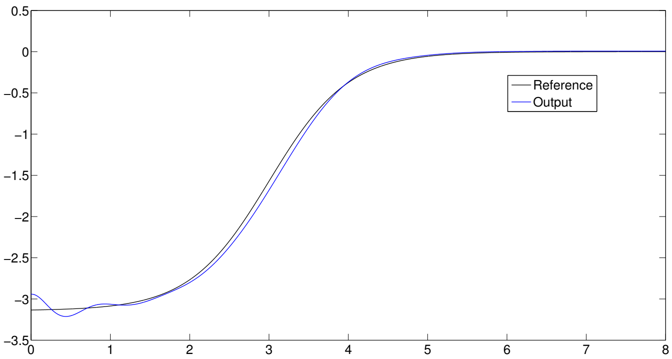

Fig. 1. Pendulum angular position: θ

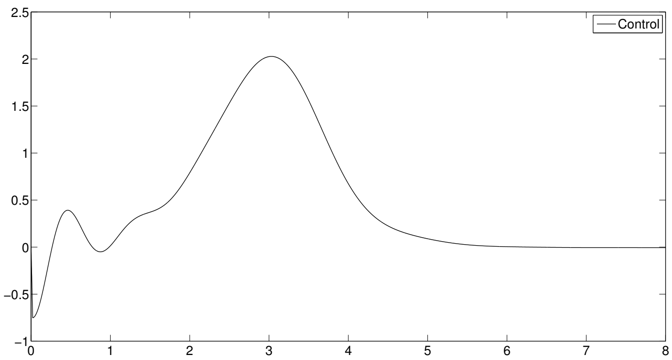

Fig. 2. Control for the noise-free simulation: τ

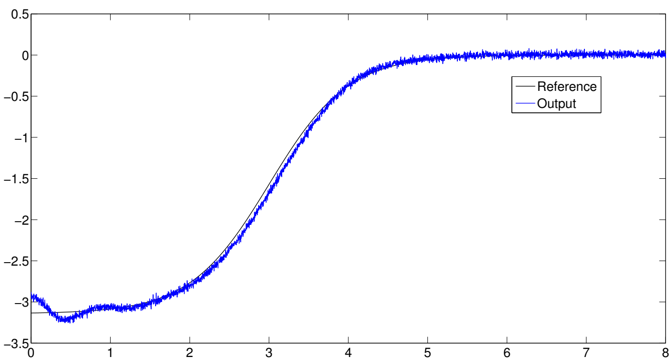

Fig. 3. Pendulum angular position in the presence of noise

[2] See the explanations in Fliess [2006].

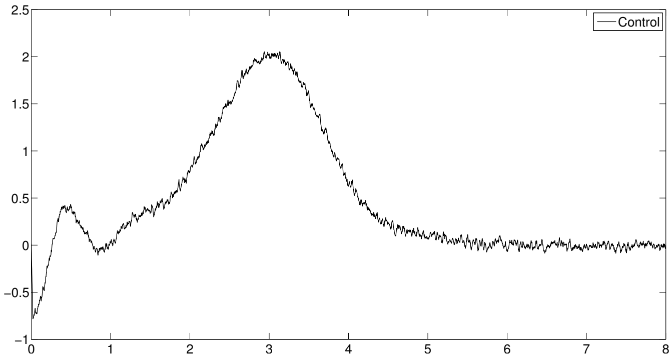

Fig. 4. Control action in the presence of noise

3.2 A DC motor

Consider a 800 watts DC motor, with a desired sinusoidal angular velocity $\sin 2t + 2$ in spite of perturbations. Numerical simulations are performed via

$$\dot{\omega} = \frac{kI - Cr + p_1(t)}{J}$$
$$\dot{I} = \frac{u - RI - k\omega + p_2(t)}{L}$$

where $R = 1.8$, $L = 0.016$, $k = 0.3$, $J = 0.005$, and

$$p_1(t) = \sin(t) + 0.1\text{sign}(\omega) - 200$$
$$p_2(t) = \cos(2t) + 0.1\text{sign}(\cos(4t))$$

are the perturbations. Take as before $n = 1$ in Equation (1). The model-free controller provides very good tracking performances in the absence of noise (see Figures 5 and 6), as well as in a noisy situation as displayed in Figures 7 and 8.

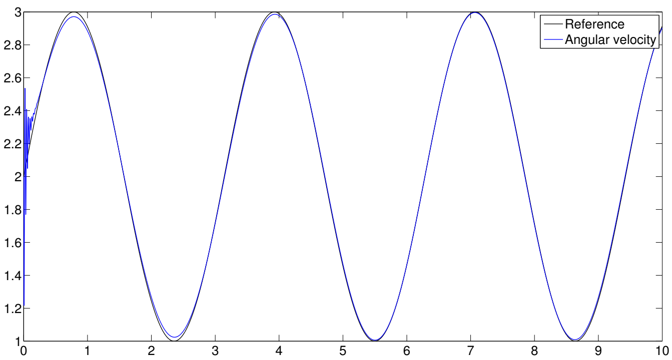

Fig. 5. Output & reference angular velocities.

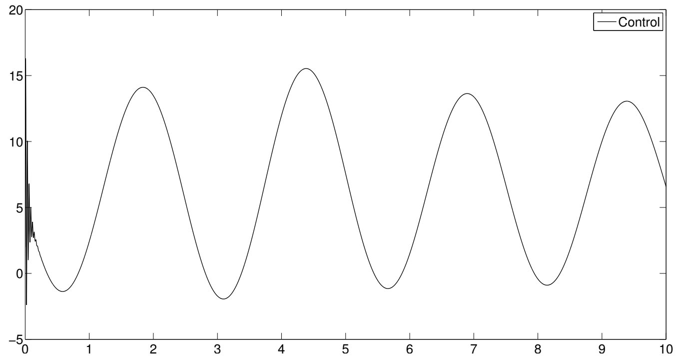

Fig. 6. Control input for the DC motor

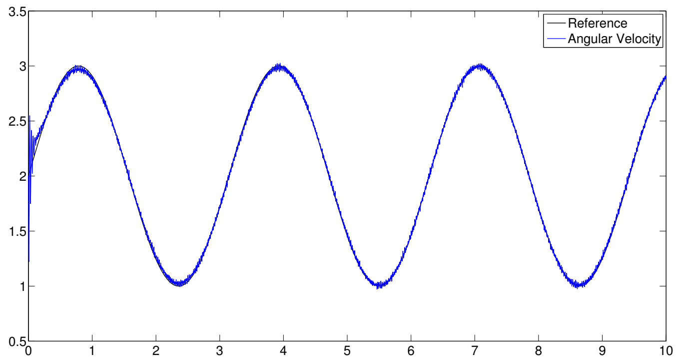

Fig. 7. Output & reference angular velocities in noisy case

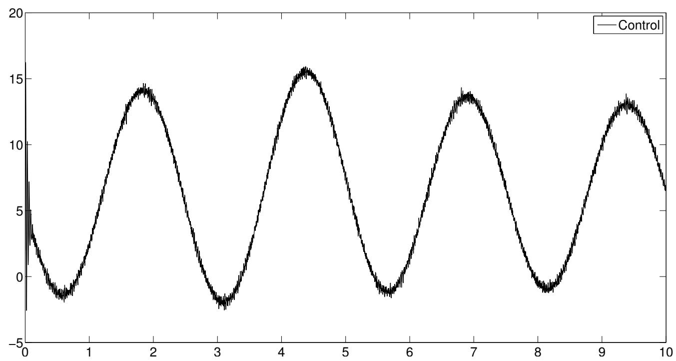

Fig. 8. Control input for DC motor in noisy case

3.3 A comparison with a sliding mode controller

Control via sliding modes of perturbed nonlinear, systems where the classical matching condition of Drazenovic [1969] is not satisfied, is a challenging problem. It has been the subject of many studies especially by Estrada & Fridman [2008, 2010], where a particular class of nonlinear single-input systems of the form

$$\dot{x}_1 = f_1(x_1, t) + B_1(x_1, t)x_2 + \omega_1(x_1, t)$$
$$\dot{x}_i = f_i(\bar{x}_i, t) + B_i(\bar{x}_i, t)x_{i+1} + \omega_i(\bar{x}_i, t), \ i = 2, \cdots, m-1$$
$$\dot{x}_m = f_m(x, t) + B_m(x, t)u + \omega_m(x, t)$$

has been considered where $\bar{x}_i = (x_1, \cdots, x_i)$. The scalar functions f_i and B_i are smooth. The unknown perturbations ω_i are bounded and at least $n - i$ differentiable. The controller is a cascaded structure where the state x_i is controlled through the virtual input x_{i+1}. Since the states $x_2, \cdots, x_m$, taken as virtual inputs, cannot be discontinuous, higher order sliding mode algorithms were used for each x_i, $i = 2 \cdots, m - 1$.

We consider the simulation examples treated in Estrada & Fridman [2008, 2010] and test our model-free controller. We start first by the example treated in Estrada & Fridman [2010] which is given by

$$\begin{aligned} \dot{x}_1 &= 2\sin(x_1) + 1.5x_2 + g_1(x_1, t) \\ \dot{x}_2 &= 0.8x_1x_2 + x_3 + g_2(x_1, x_2, t) \\ \dot{x}_3 &= -x_3^2 + 2u + g_3(x_1, x_2, x_3, t) \end{aligned} \quad (8)$$

$$g_1(x_1, t) = 0.2\sin(t) + 0.1x_1 + 0.12$$
$$g_2(x_1, x_2, t) = 0.3\sin(2t) + 0.2x_1 + 0.2x_2 - 0.4$$
$$g_3(x_1, x_2, x_3, t) = 0.2\sin(2t) + 0.2x_1 + 0.3x_2 + 0.2x_3 + 0.3$$

The goal is to track $y^\star(t) = 2\sin(0.15t) + 4\cos(0.1t) - 4$. The simulations of Estrada & Fridman [2010] are reproduced in Figures 9 and 10. For the model-free controller, $n = 2$ is used in Equation (1) to design an i-PD. Estimation of F is computed by adapting the method described in Section 2. The initial conditions are $(0.2, 0, 0)^T$. For the sake of comparisons with the simulations in Estrada & Fridman [2010], noise-free simulations are also displayed.

The output tracking, which is quite similar to the sliding mode one shown in Figure 9, is not shown again. Besides, the control input in Figure 11 is smoother than the one in 10. It seems that our controller provide two advantages with respect to the sliding mode based controller:

- There is only one parameter, α in Equation (1), to be tuned. The choice of K_P and K_D is trivial.
- Smoothness of the control input, *i.e.*, no chattering.

Remark 1. The chattering in the controller of Estrada & Fridman [2010] could have been removed by considering a new input $\dot{u} = v$. But this would have induced more complications in the controller synthesis.

Remark 2. We point out that the local model utilized to synthesize the i-PD is of the second order while the relative degree of (8) with respect to $e = y - y^\star$ is three.

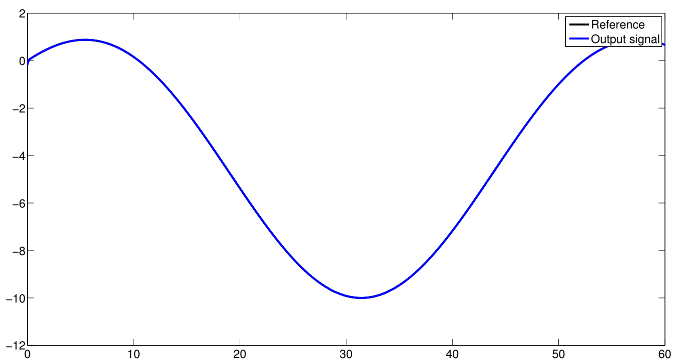

Fig. 9. output & reference

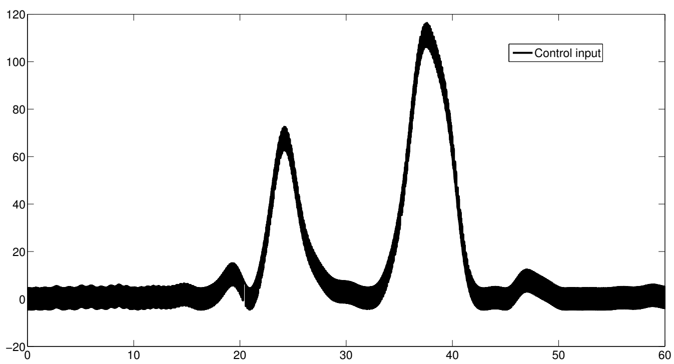

Fig. 10. Control input for the sliding mode controller

In Estrada & Fridman [2008], trajectory tracking of the system

$$\begin{aligned} \dot{x}_1 &= 2x_1 + 1.5x_2 + g_1(x_1, t) \\ \dot{x}_2 &= x_2 + x_3 + g_2(x_1, x_2, t) \\ \dot{x}_3 &= -1.5x_3 + 2u + g_3(x_1, x_2, x_3, t) \end{aligned} \quad (9)$$

was accomplished with the same sliding mode controller of Estrada & Fridman [2010] but with different parameters.

We tested our model-free controller based on the ultra-local model (1) with $n = 2$ and exactly the same parameters α, K_P and K_D previously used in (8). The control input can be seen in Figure 12. Note that the control input in Estrada & Fridman [2008] is similar to the one in Figure 12 but with a chattering of amplitude 1. Lack of space prevents us from reproducing it here.

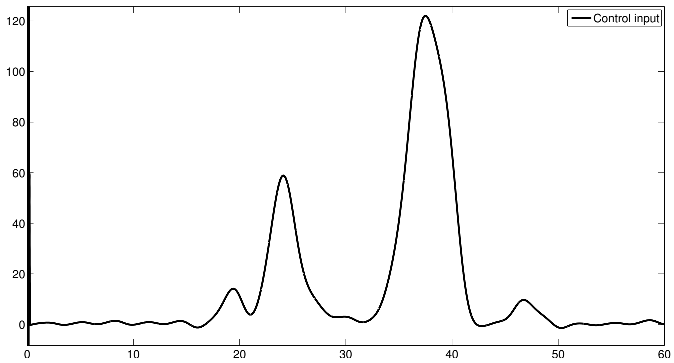

Fig. 11. Control input of the i-PD

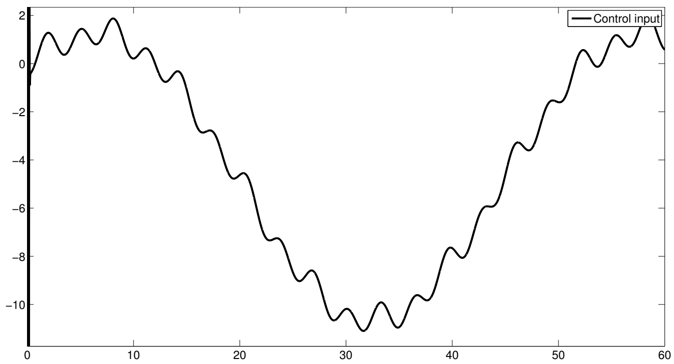

Fig. 12. Control input for the model free controller corresponding to system 9

3.4 A nonlinear spring with frictions

Consider as in Fliess & Join [2008, 2009] a nonlinear spring-mass system:

$$m\ddot{y} = -k_1 y - k_3 y^3 + \mathcal{F}(\dot{y}) - d\dot{y} + u \quad (10)$$

m is the mass, $-k_1 y - k_3 y^3$ the stiffness and

$$\mathcal{F}(\dot{y}) = \begin{cases} -0.3 - 0.4(\dot{y} + .25)^2 - d\dot{y} & \text{if} \quad \dot{y} > 0 \\ 0.3 + 0.4(\dot{y} + .25)^2 - d\dot{y} & \text{if} \quad \dot{y} < 0. \end{cases}$$

the discontinuous friction. Utilize

$$m\ddot{y} = -\hat{k}_1 y - \hat{k}_3 y^3 - \hat{d}\dot{y} + u \quad (11)$$

where $\hat{k}_1 = 2$, $\hat{k}_3 = 7$, and $\hat{d} = 2.5$ are estimates of $k_1 = 3$, $k_3 = 10$ and $d = 5$ respectively. The mass $m = 0.5$ is perfectly known. The flatness of System (11) permits via the flat output y to design a nominal open-loop control strategy:

$$u^\star = m\ddot{y}^\star + \hat{k}_1 y^\star + \hat{k}_3 (y^\star)^3 + \hat{d}\dot{y}^\star. \quad (12)$$

The simulation result of the nominal controller are shown in Figures 13 and 14. In order to compensate the un-

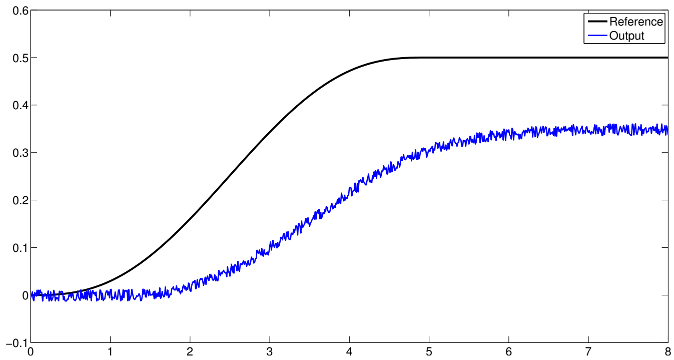

Fig. 13. Noisy output & reference trajectory for the spring-mass system with the nominal control.

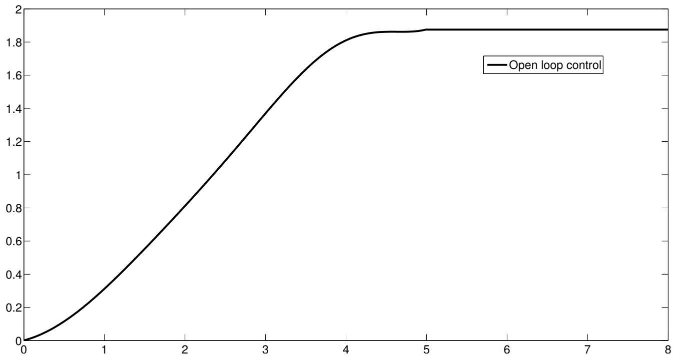

Fig. 14. Control input for the spring-mass system with the nominal control.

modeled part, set $e = y - y^\star$ and $u = u^\star + \Delta u$ and stabilize around $e = 0$ the system with input Δu via our model-free design. Choose again $n = 1$ in Equation (1). This is again achieved via an i-PI strategy. We impose a critically damped behavior by using the characteristic equation $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$, and choosing $\xi = 0.707$ and a settling time of 1 second. Only 20 samples are needed to get a good estimate of F in (1). A Runge-Kutta algorithm is used to simulate Equation (10) for each time increment of 0.01 second. The control input is updated at a rate of 0.01 second. The simulation results in the noise-free case are shown in Figures 15 and 16. Simulations with additive noise are shown in the Figures 17, 18 and 19.

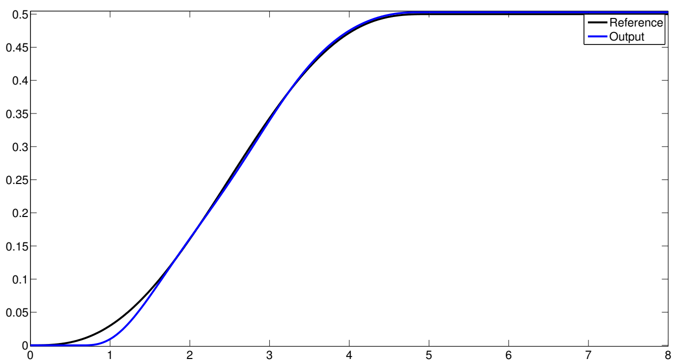

Fig. 15. Noise-free output & referene trajectory for the spring-mass system

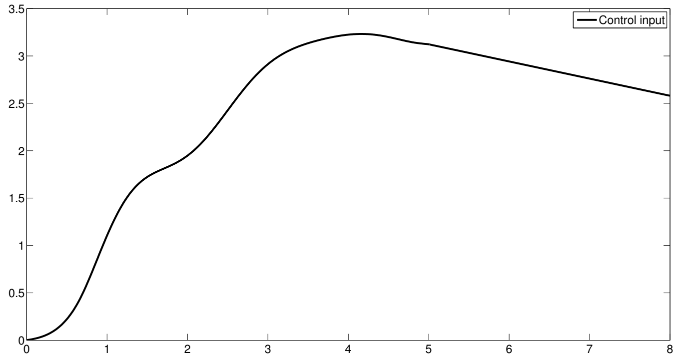

Fig. 16. Control input for the spring-mass system. Noise-free case.

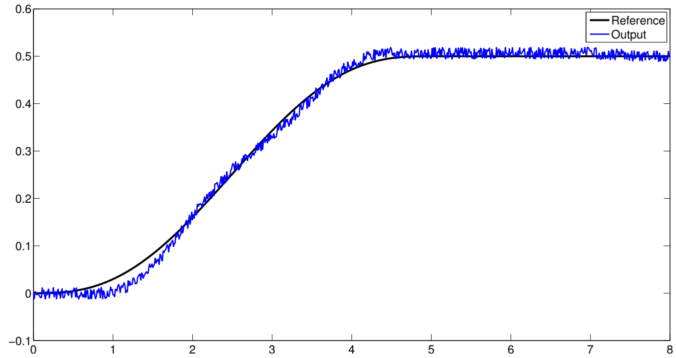

Fig. 17. Noisy output & reference trajectory for the spring-mass system.

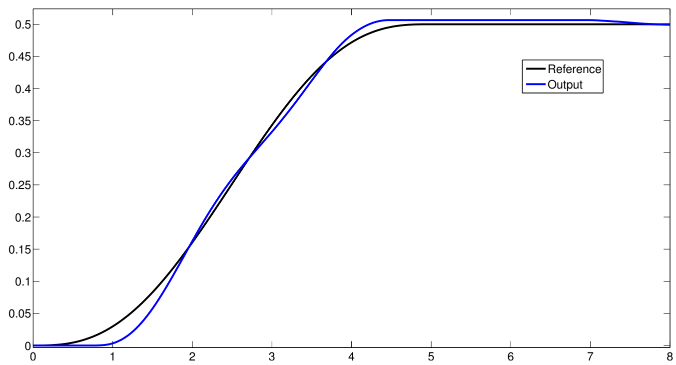

Fig. 18. Filtered noisy output & reference trajectory for the spring-mass system.

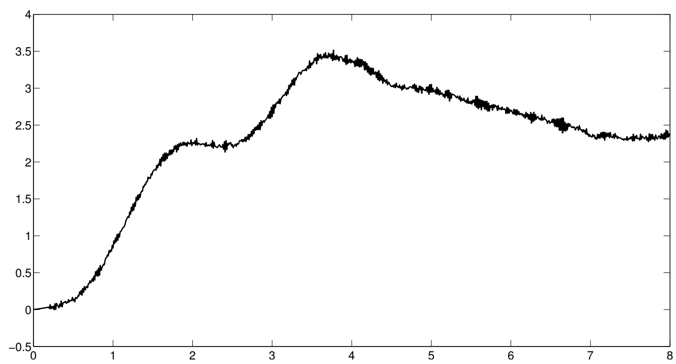

Fig. 19. Control input for the spring mass system. Noisy case.

4. CONCLUSION

Our model-free control design, which

- permitted to bypass the difficult task of mathematical modeling,
- leads to a straightforward gain tuning,

has been improved by

- replacing numerical differentiation of noisy signals by simple online parameter identification procedures,
- using the model-free i-PID (2) even if a submodel is partially known.

See Fliess *et al.* [2011] for an up to date survey of model-free control, the valisity of which has already been confirmed by several concrete applications.

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