Preprints of the 18th IFAC World Congress
Milano (Italy) August 28 - September 2, 2011

Fault estimation and MRC-based active FTC

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Abstract: For systems that have no unique linearization equilibria, for example multi-link robot systems, the classical "direct" methods of Fault Tolerant Control (FTC) via fault estimation/compensation cannot easily be achieved via a linear time-invariant systems approach. This paper proposes an FTC strategy using an active fault estimator based on model reference control (MRC). The novelty lies in the combined use of on-line fault estimation and FTC design applied to a model reference system. The reference model is designed via pole-placement and the estimator design parameters are synthesized via a Linear Matrix Inequalities (LMIs) approach. An example of a non-linear two-link Manipulator (TLM) system is described to illustrate the design procedure.

Keywords: Model Reference Control, Fault Tolerant Control, Fault Estimation, LMI, Robot Manipulator Systems

1. INTRODUCTION

A wide range of system applications exist in which fault estimation can be used to compensate faults within the control system (Patton, 1997); (Blanke *et al*, 2003); (Gao and Ding, 2007); (Gao *et al*, 2010); (Khedher *et al*, 2010), (Patton and Klinkhieo, 2009), (Patton and Klinkhieo, 2010); (Patton, Putra and Klinkhieo, 2010b) subject to fault-tolerance stability requirements. This class of systems belongs to the domain of *active* or *direct* FTC in which the combined problems of fault estimation and control compensation are frequently based on the use of linear system models.

For all active fault compensation approaches in FTC on-line fault estimation is essential and a number of suitable estimation methods are well known. For example (Wang and Daley, 1996) proposed the adaptive observer approach. (Edwards, Spurgeon and Patton, 2000) use sliding mode fault estimation. (Zhang, Jiang and Cocquempot, 2008) developed the so-called fast fault estimation method and polytopic LPV fault estimation was proposed by (Patton and Chen, 2010).

Linear models are traditionally used for both estimation and control within the framework of robustness analysis and design. In a classical way the joint performance of the FTC estimation and control compensation of such systems may only be acceptable in a region of operation close to the defined equilibria and numerous studies have emerged focused on robust fault detection and isolation (FDI), robust fault estimation and robust FTC, based on this limitation.

However, many real system applications have no unique linearization equilibria e.g. advanced aircraft and various forms of robotic systems, which present a significant challenge to the use of linear modelling methods. Within the wider field of control the problem of feedback design for such systems has been of considerable interest in the literature, see for example (Marino and Tomei, 1997); (Astolfi *et al*., 2007).

For systems of this class a practicable control design approach for fault compensation systems is the use of MRC which forces the plant variables to follow those of a suitable reference model (Duan, Wang and Huang, 2004) (Qu and Dawson, 1994). This study is concerned with the development of an active FTC scheme involving a fault estimation and fault compensation/control strategy, based on MRC, for time-varying affine systems of the form:

$$\dot{x} = A(x)x + B(x)u + Ef \tag{1}$$

$E \in R^{nxq}$ is a full column rank fault weighting matrix and $f \in R^{q}$ denotes the bounded q system faults. Note that Ef could represent either an additive or a multiplicative fault. $A(x) \in R^{n,n}$, $B(x) \in R^{n,m}$ are smooth, continuous and controllable functions of the state vector $x \in R^{n}$ (supposed all can be measured) whilst $u \in R^{m}$ is a stabilising state feedback control vector. It is further assumed that $B(x)$ is full column rank for all states x defined by (1).

The main contribution is to design an on-line active fault estimator and fault compensator to achieve FTC purpose based only on the reference model and not on the plant dynamics. This leads to the use of a simpler parameterization in the fault estimator LMI computation compared with the estimator approach developed by Zhang, Jiang and Cocquempot (2008).

Section 2 summarises briefly the MRC approach to feedback design. Section 3 outlines the structure of the combined fault estimation and control compensation scheme, based on the MRC approach. Section 4 describes a non-linear system example of a TLM, illustrating the MRC strategy for a scenario when two actuator faults act independently on each joint. Concluding comments are given in Section5.

2. MODEL REFERENCE CONTROL SYSTEM

Consider the problem of developing an associated linear time-invariant (LTI) "open-loop" MRC reference model (2) for the linear time-varying multivariable plant in (1), with $f = 0$ as follows:

$$\dot{w} = A_m w + B_m ref \tag{2}$$

$w \in R^n$ is the reference model state vector and $ref \in R^r$ is a time-varying input signal designed to achieve a required reference trajectory for the states w . (A_m, B_m) is a compatibly dimensioned controllable pair with stable A_m.

The error state vector, $e \in R^n$ is defined as:

$$e = x - w \tag{3}$$

The error system dynamics are determined from (1), (2) as:

$$\dot{e} = A_m e + (A(x) - A_m)x + B(x)u - B_m ref \tag{4}$$

The restriction arising from use of a MRC design strategy is given in terms of the well-known *perfect model matching conditions* of Erzberger (1968) and Chen (1968) as:

$$rank[B: A_m - A] = rank[B] = rank[B: B_m], \tag{5}$$

For real applications, these conditions are easy to satisfy as the reference model can be chosen by the designer. The plant control signal can be designed as:

$$u = B(x)^{++}(-K_1(x)x + K_2 ref) \tag{6}$$

$B(x)^{++} \in R^{mxn}$ is a suitable pseudo-inverse matrix of the full column rank $B(x)$ and $K_1 \in R^{nxn}$, $K_2 \in R^{nxr}$ are feedback matrices given by:

$$K_1(x) = (A(x) - A_m) \tag{7}$$

$$K_2 = B_m \tag{8}$$

Hence, (4) can be reduced to:

$$\dot{e} = A_m e \tag{9}$$

Assuming the matching condition above, the error e tends asymptotically to zero at a rate determined by the placement of the eigenvalues of A_m in the open left plane.

3. MODEL REFERENCE FTC STRATEGY

An FTC design strategy is required to compensate for the effects of the faults acting in (1). The current study is based on the estimator proposed by (Zhang, Jiang and Cocquempot, 2008). However, the estimation is applied within an FTC fault compensation mechanism that makes use of a MRC structure.

Now consider the system (1) for the case $f \neq 0$. In order to proceed to the fault estimator design the following *two* assumptions A1 & A2 must be satisfied:

A1. Rank $(E) = q$.

A2. The invariant zeros of (A_m, I, E) lie in the open left half-plane (LHP) (Kudva, Viswanadham and Ramakrishna, 1980).

Assume that the fault estimator has the following dynamics:

$$\dot{\hat{f}} = L(\dot{e} + e) \tag{10}$$

where $\dot{e}$ can be achieved via (9), $\hat{f} \in R^q$ is the fault estimate and $L \in R^{qxn}$ is a suitable estimator gain determined using a suitable LMI calculation as described below.

A similar estimation structure is the *augmented state observer* (ASO) by (Patton and Klinkhieo, 2009). However, (10) has a more general proportional-plus-integral (P-I) structure (compared with the proportional only ASO approach). The estimation error is $e_f = f - \hat{f}$, and the error dynamics are given by:

$$\dot{e}_f = \dot{f} - \dot{\hat{f}} = \dot{f} - L(\dot{e} + e) \tag{11}$$

The proportional $L\dot{e}$ plus integral Le action on the error system in (11) provides degrees of design freedom to shape the estimator tracking performance.

Here it is proposed that the fault estimate signal $\hat{f}$ can be added to the control signal in (6) to compensate the fault signal f, according to the MRC structure of Fig.1.

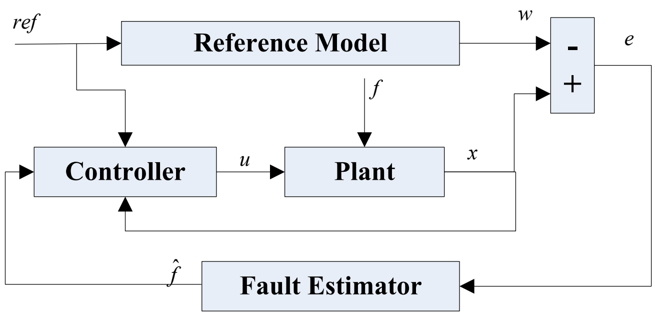

Fig.1. Model Reference FTC scheme

The fault-tolerant performance of this system depends on the robustness of the fault estimation applied to the control input:

$$u = B(x)^{++}(-K_1(x)x + K_2 ref - E\hat{f}) \tag{12}$$

It is further assumed that $B(x)^{++}E \neq 0$. From (1)-(12), it can be shown that:

$$\dot{e} = A_m e + E\, e_f \tag{13}$$

The existence of suitable positive definite (s.p.d.) Lyapunov matrices to guarantee stability of the error system (13) is determined as follows (Zhang, Jiang and Cocquempot, 2008):

<u>Theorem</u>: under Assumption (1)-(2), if there exist s.p.d. matrices $P \in R^{nxn}, G \in R^{qxq}, L \in R^{qxp}$ and if the following *two* conditions hold:

$$E^T P = L\ ; \tag{14}$$

$$\begin{bmatrix} PA_m + A_m{}^T P & -A_m{}^T PE \\ -E^T PA_m & -2LE + G \end{bmatrix} < 0, \tag{15}$$

then the fault estimation error can be guaranteed to remain in a bounded range. The proof is taken from (Zhang, Jiang and Cocquempot, 2008) but is given here for completeness as this approach is modified for the MRC design.

<u>Proof</u>: The Lyapunov function can be considered as:

$$V = e^T Pe + e_f{}^T e_f \tag{16}$$

after differentiation of V with respect to time (16) becomes:

$$\dot{V} = \dot{e}^T Pe + e^T P\dot{e} + 2{e_f}^T \dot{e}_f$$
$$= \left(A_m e + E\, e_f\right)^T Pe + e^T P\left(A_m e + E\, e_f\right)$$
$$+2{e_f}^T\left[\dot{f} - L(\dot{e} + e)\right]$$
$$= e^T {A_m}^T Pe + {e_f}^T E^T Pe + e^T PA_m e + e^T PEe_f$$
$$+2{e_f}^T \dot{f} - 2{e_f}^T L(\dot{e} + e)$$
$$= e^T\left({A_m}^T P + PA_m\right)e + 2{e_f}^T E^T Pe$$
$$-2{e_f}^T L\left(A_m e + E\, e_f + e\right) + 2{e_f}^T \dot{f} \quad (17)$$

From (17), (16) can be re-written in the form:

$$\dot{V} = e^T\left({A_m}^T P + PA_m\right)e - 2e^T {A_m}^T PEe_f$$
$$-2{e_f}^T LE\, e_f + 2{e_f}^T \dot{f} \quad (18)$$

Lemma 1 (Jiang, Wang and Soh, 2002): given a scalar $a > 0$ and a s.p.d. matrix Q, for which the following inequality holds:

$$2x^T y \le \frac{1}{a} x^T Qx + ay^T Q^{-1} y \qquad x, y \in R^n. \quad (19)$$

Then (18) satisfies:

$$\dot{V} \le e^T\left({A_m}^T P + PA_m\right)e - 2e^T {A_m}^T PEe_f$$
$$-2{e_f}^T LE\, e_f + {e_f}^T Ge_f + \dot{f}^T G^{-1} \dot{f}$$
$$\le e^T\left({A_m}^T P + PA_m\right)e - 2e^T {A_m}^T PEe_f$$
$$-2{e_f}^T LE\, e_f + {e_f}^T Ge_f + \|\dot{f}\|^2 \lambda_{max}(G^{-1}) \quad (20)$$

By defining a vector Θ as $\Theta = \begin{bmatrix} e \\ e_f \end{bmatrix}$,

(20) can now be re-written more succinctly as:

$$\dot{V} \le \Theta^T \Psi \Theta + \Upsilon \quad (21)$$

Where:

$$\Psi = \begin{bmatrix} PA_m + {A_m}^T P & -{A_m}^T PE \\ -E^T PA_m & -2LE + G \end{bmatrix}, \text{ and } \Upsilon = \|\dot{f}\|^2 \lambda_{max}(G^{-1})$$

Applying the Rayleigh Principle to (21) it can be shown that:

$$\dot{V} < -\lambda_{min}(-\Psi)\|\Theta\|^2 + \Upsilon \quad (22)$$

where $\lambda_{max}(.)$ and $\lambda_{min}(.)$ are the largest and smallest eigenvalues of the matrix (.), respectively. (22) shows that when $-\lambda_{min}(-\Psi)\|\Theta\|^2 > \Upsilon$, $\dot{V} < 0$, which is satisfied for $\|\Theta\|^2 < \frac{\Upsilon}{-\lambda_{min}(-\Psi)}$, which implies that $\begin{bmatrix} e \\ e_f \end{bmatrix}$ is bounded within a small finite range determined by the derivative of the fault $\dot{f}$. Furthermore, the lower the value of the scalar $\|\dot{f}\|^2$ the faster will be the fault estimation speed, i.e. when $\|\dot{f}\|^2 = 0$ (the fault is constant with the error vector $\Theta = 0$) and the fault estimator achieves perfect tracking. Q.E.D.

In order to solve the LMI (15) subject to (14) a well known procedure of (Corless and Tu, 1998) can be used to transfer (14) into a (convex optimization) LMI problem. For this case and based on the selected reference model of (13), a new LMI must be solved, as follows (Zhang, Jiang and Cocquempot, 2008):

$$\begin{bmatrix} -\text{J}I & L - E^T P \\ PE - L^T & -\text{J}I \end{bmatrix} < 0 \quad (23)$$

The LMIs (15) and (23) are solved simultaneously to determine the matrices P, G and hence L so that the model-reference estimator can be determined.

4. TWO-LINK MANIPULATOR EXAMPLE

A two-link planar manipulator model (TLM) based on the work of (Hassen, *et al*., 2000) and modified by (Klinkhieo, 2009) is used to illustrate the application of the MRC fault compensation as shown in Fig.2.

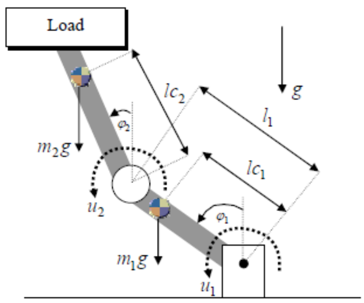

Fig.2. Two link planar manipulator structure

The Euler-Lagrange dynamic model of the TLM is given in state space notation as:

$$\left(m_1 {lc_1}^2 + m_2 {l_1}^2 + I_1\right)\ddot{\varphi}_1$$
$$+m_2 l_1 lc_2 \cos(\varphi_1 - \varphi_2)\,\ddot{\varphi}_2 + m_2 l_1 lc_2\, sin(\varphi_1 - \varphi_2)\,{\dot{\varphi}_2}^2$$
$$-(m_1 lc_1 + m_2 l_1) g sin\varphi_1 = u_1 \quad (24)$$
$$m_2 l_1 lc_2\, cos(\varphi_1 - \varphi_2)\,\ddot{\varphi}_1$$
$$+(m_2 {lc_2}^2 + I_2)\ddot{\varphi}_2 - m_2 l_1 lc_2\, sin(\varphi_1 - \varphi_2)\,{\dot{\varphi}_1}^2$$
$$-m_2 g lc_2 sin\varphi_2 = u_2 \quad (25)$$

where:

I_1: Inertia of arm-1 and load
I_2: Inertia of arm-2
l_1: Distance between joint-1 and joint-2
lc_1: Distance of joint-1 from centre of mass arm-1
lc_2: Distance of joint-2 from centre of mass arm-2
m_1: Mass of arm-1 and load
m_2: Mass of arm-2

$\varphi_1, \dot{\varphi}_1$ and $\varphi_2, \dot{\varphi}_2$ are state variables representing the angle and angular velocity of Link-1 and Link-2, respectively. u_1 and u_2 are the control signals. The associated parameters are given in Table.1.

Table.1. Parameters of TLM

Parameters	I_1	I_2	l_1	lc_1	lc_2	m_1	m_2	g
Values	0.833	0.417	1.0	0.5	0.5	10.0	5.0	9.80
Units	$Kg * m^2$	$Kg * m^2$	m	m	m	Kg	Kg	m/s^2

4.1 Simplification of the TLM system

To facilitate the development of the MRC design the controlled non-linear TLM dynamics can be simplified (noting that the angles φ_1, φ_2 are measured) and using the following notation:

$$a_1 = m_1 lc_1{}^2 + m_2 l_1{}^2 + I_1\,,\ a_2 = m_2 l_1 lc_2,$$

$$a_3 = (m_1 lc_1 + m_2 l_1)g,\ a_4 = m_2 lc_2{}^2 + I_2\,,$$

$$a_5 = m_2 g lc_2$$

The notation can be simplified as follows:

$$M_{(32)} = a_2\, sin(\varphi_1 - \varphi_2)\,\dot{\varphi}_2, \quad M_{(33)} = a_1,$$

$$M_{(34)} = a_2 cos\,(\varphi_1 - \varphi_2), M_{(41)} = -a_2\, sin(\varphi_1 - \varphi_2)\,\dot{\varphi}_1,$$

$$M_{(43)} = M_{(34)}, \quad M_{(44)} = a_4.$$

Now define:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & M_{(32)} & M_{(33)} & M_{(34)} \\ M_{(41)} & 0 & M_{(43)} & M_{(44)} \end{bmatrix}, x = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix},$$

Where $M_{(i,p)}, i, p \in N$ are the elements of the new matrix M. Then the TLM dynamical description (24), (25) can be written as:

$$M\dot{x} = \begin{bmatrix} 0 & I_2 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ I_2 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ a_3 sin\varphi_1 \\ a_5 sin\varphi_2 \end{bmatrix}, \qquad (26)$$

where I_2 is the identity matrix on R^2. For all multi-link manipulator systems, including the TLM system, the matrix M is full rank, so that M^{-1} exists. Hence, (26) can be transformed into:

$$\dot{x} = M^{-1} \begin{bmatrix} 0 & I_2 \\ 0 & 0 \end{bmatrix} x + M^{-1}\left(\begin{bmatrix} 0 \\ I_2 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ a_3 sin\varphi_1 \\ a_5 sin\varphi_2 \end{bmatrix} \right) \qquad (27)$$

As φ_1, φ_2 are measured angles, the MRC design strategy can be simplified by de-coupling the gravity terms $a_3 sin\varphi_1$ and $a_5 sin\varphi_2$ in (27), using as basic form of feedback linearization, with:

$$u = u_s - u_g \qquad (28)$$

where

$$u_s = \begin{bmatrix} u_{s1} \\ u_{s2} \end{bmatrix} \text{ and } u_g = \begin{bmatrix} a_3 sin\varphi_1 \\ a_5 sin\varphi_2 \end{bmatrix}$$

It follows that (27) is simplified to the structure:

$$\dot{v} = A(x)x + B(x)u_s \qquad (29)$$

where:

$$A(x) = M^{-1} \begin{bmatrix} 0 & I_2 \\ 0 & 0 \end{bmatrix}, \mathrm{B}(x) = M^{-1} \begin{bmatrix} 0 \\ I_2 \end{bmatrix},$$

This completes the TLM model system simplification procedure.

4.2. MRC-based FTC of TLM system

For this example, the matrices $A(x)$ and $\mathrm{B}(x)$ of the plant model have the following structure:

$$A(x) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ A_{31}(x) & A_{32}(x) & A_{33}(x) & A_{34}(x) \\ A_{41}(x) & A_{42}(x) & A_{43}(x) & A_{44}(x) \end{bmatrix},$$

$$B(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ B_{31}(x) & B_{32}(x) \\ B_{41}(x) & B_{42}(x) \end{bmatrix}$$

$A_{31}(x), \dots, A_{44}(x)$ and $B_{31}(x), \dots, B_{42}(x)$ represent the elements in matrices A and B as functions of the plant states. The controllable reference model is obtained from the structure of matrices $A(x)$ and $\mathrm{B}(x)$ in (29). Once, a candidate set of model parameters is selected (e.g. as a single point in the linearization). A suitable reference model for this system is assumed to retain the structure of $A(x)$ and $\mathrm{B}(x)$. A suitable pair $A_m\, B_m$ has been chosen with spectrum of $\rho(A_m) = [-4, -6, -8, -10]$ as follows:

$$A_m = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -44.1098 & 14.2476 & -13.7422 & 2.0281 \\ 13.5021 & -47.8890 & 1.9389 & -14.2578 \end{bmatrix},$$

$$B_m = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2 & 3 \\ 3 & 4 \end{bmatrix}$$

For this example, the control problem is that of moving the two links to constant reference angles (corresponding to φ_1, φ_2) of 20 deg and 15 deg, respectively. As a special case (regulator tracking) problem for this example the solution for ref is set to $\dot{w} = 0$ in (2). The initial values w_0 of the states w in (2) are also set to:

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \\ 0 \\ 0 \end{bmatrix}$$, which are also the equilibrium values w_e of w.

The required reference signal ref is thus given by:

$$ref = -B_m{}^{++} A_m w_e$$

with $B_m{}^{++} = \begin{bmatrix} 0 & 0 & -4 & 3 \\ 0 & 0 & 3 & -2 \end{bmatrix}$

The solution for ref is expressed in terms of the reference angles w_1 and w_2 as:

$$ref = [Q] \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} -217 & 201 \\ 159.334 & -138.521 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \qquad (30)$$

This reference model system is applied to the MRC-FTC design (10)-(13), and the control signal is given in terms of the pseudo-inverse $B(x)^{++}$of $B(x)$ as:

$$u = B(x)^{++}(-K_1(x)x + K_2 ref) - \hat{f} - u_g \qquad (31)$$

Now consider a vector of actuator faults $f(t)$ acting on the TLM system joints according to:

$$\dot{x} = A(x)x + B(x)u_s + B(x)f(t) \qquad (32)$$

where $f(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$, and

$$f_1(t) = \begin{cases} 0 & 0 \le t \le 5 \\ sint & 5 \le t \end{cases}, f_2(t) = \begin{cases} 0 & 0 \le t \le 20 \\ 2 & 10 \le t \end{cases}$$

Note that $B(x)f(t)$ has the structure:

$$B(x)f(t) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ B_{31}(x) & B_{32}(x) \\ B_{41}(x) & B_{42}(x) \end{bmatrix} f(t) \qquad (33)$$

Hence, the term $Ef(t)$ of Eq. (1) can be re-written, for this case, as $Ef_{new1}(t)$

where $E = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$, $f_{new}(t) = \begin{bmatrix} f_{new1}(t) \\ f_{new2}(t) \end{bmatrix} = B(x)f(t)$,

This shows that although $B(x)$ is time-varying, the fault distribution can still be represented via a constant distribution matrix E operating on a transformed (but bounded) fault $f_{new}(t)$. $f_{new}(t)$ is bounded since both $B(x)$ and $f(t)$ are bounded and $\hat{f}(t) = B^{++}(x)$. By using the Matlab LMI toolbox a solution to (15) and (23) can be obtained (shown to 4 decimal places) as:

$$P = \begin{bmatrix} 64.6210 & 0.1039 & 4.5897 & 1.0857 \\ 0.1039 & 65.5793 & 1.0596 & 4.3703 \\ 4.5897 & 1.0596 & 0.4053 & 0.1678 \\ 1.0857 & 4.3703 & 0.1678 & 0.3676 \end{bmatrix}$$

$$L = \begin{bmatrix} 4.5897 & 1.0596 & 0.4053 & 0.1678 \\ 1.0857 & 4.3703 & 0.1678 & 0.3676 \end{bmatrix},$$

$$G = \begin{bmatrix} 0.0623 & 0.0471 \\ 0.0471 & 0.0504 \end{bmatrix}$$

Fig.3 shows the two faults $f_1(t)$ & $f_2(t)$ and their estimates $f_{e1}(t)$ & $f_{e2}(t)$ soon converge to their true values after an oscillatory transient period. Fig.4. gives the TLM response of two cases (a) with FTC action applied and (b) without FTC action. Without the FTC fault compensation function, the plant state variables are strongly affected by the occurrence of faults. The angle and angular velocity for Joint-1 oscillate around the reference points, whilst for Joint-2, the angle and angular velocity follow their reference levels with steady-state following errors. It is clear that after compensation the angle state responses become almost independent of the fault effects.

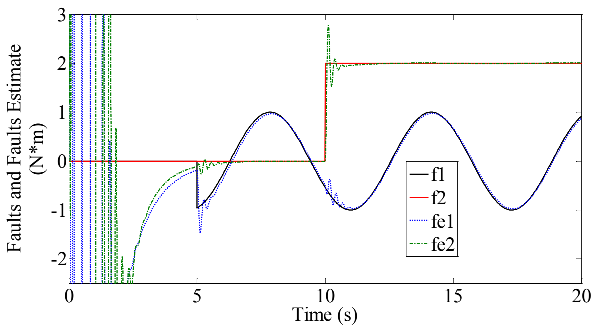

Fig.3. $f_1(t), f_2(t)$ & fault estimates f_{e1}(t); f_{e2}(t))

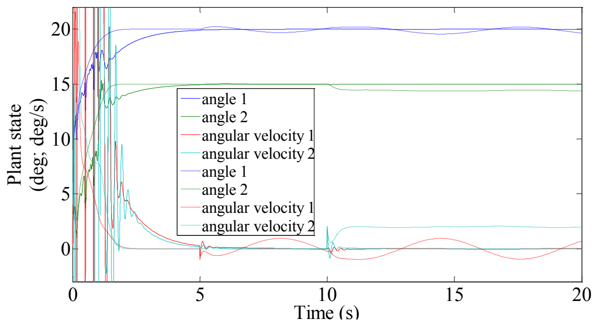

Fig.4. Fault TLM responses (initial conditions: 10; 3; 0; 0) with fault compensation (solid) and no compensation (dotted).

From the above results, good FTC performance is achieved as the fault is accurately estimated and compensated on-line. Some oscillations appear in the fault estimate plot, as shown in the upper part of Fig.3. These are due to the high derivatives of the fault signals at the events t = 5s and t = 10s. The significance of this can be seen in Eq. (11) which shows that the derivatives of a fault signal (and it's estimate) enter into the estimation error analysis, although the actual fault signal is not required in the fault estimator. Even if this requirement is a potential limitation of this method the results show that the estimation error signals have very low magnitudes. From this study it is understood that the proportional term $\dot{e}$ (the estimation equivalent of proportional control) in $\dot{\hat{f}} = L(\dot{e} + e)$ as introduced by (Zhang, Jiang and Cocquempot, 2008) provides good estimator design freedom for minimising this effect.

5. CONCLUSIONS

This paper proposes a strategy of active (direct) FTC for systems that have no unique equilibria, making use of fault estimation and compensation via MRC design. When all the required assumptions are satisfied the aim of the combined on-line fault estimation and compensation is described in terms of a pre-designed reference model. The reference model is used to derive the fault estimator parameters as well as the controller structure. The controller stability guarantee is provided by the stability of the reference model, whilst the estimator stability arises from the solution of an appropriate LMI-based Lyapunov condition. The fact that the estimator parameters are based on a reference model, rather than on the plant itself is an important improvement over existing methods. Furthermore, the approach is important for systems that have no unique equilibria i.e. that cannot be uniquely linearised.

The paper describes an example of FTC for a non-linear TLM dynamical system with independently acting joint faults. The fault estimation errors are very small and good control compensation performance is demonstrated. Further research on this approach will inevitably involve a deeper understanding of robustness issues that may be applied to further enhance the performance of the method.

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