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Mobile Agent Tracking with Single Frequency Continuous Wave Radar: A Linear Robust Filtering Based Approach ⋆

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Abstract: We present a novel filtering approach to determine the position and velocity information of mobile agents which essentially employs a recently developed linear robust estimation idea as the core approach. Prominent problems engulfing implementations of this nature also include data association and missing information and we directly addressed these and provide a generalized solution to the overall problem. A coherent argument for the realistic usage of robust linear filtering over data collected by a linear phase array is aimed at presenting a generic approach for multiple agent tracking. The underlying system uses Doppler radar measurements and accurately estimate the position and velocity of the mobile agent progressively while addressing the issues of data association and missing information.

Keywords: Doppler radar, linear array, missing information, Estimation and filtering, Multi-agent systems.

1. INTRODUCTION

Accurate tracking and localization of a mobile agent is useful in many defence and commercial applications ranging from urban warfare to diaster and rescue operations, i.e Lin and Ling (2006a,b); Bishop and Pathirana (2007). As the cost of many Radio Frequency(RF) gear has come down significantly in the recent past, many commercial systems particularly aimed at indoor close range applications can be benefited with the use of such hardware. Through-the-Wall Radar Imaging(TWRI) has recently been considered for number of applications, see Bodenstein et al. (1994).

Continuous Wave (CW) radar systems have attracted widespread attention due to its relative simplicity in design and implementation. Single frequency CW radar can measure Doppler frequency shifts with greater accuracy. However, this method is less commonly used for range measurements as more sophisticated systems derived from CW radar are preferred for such applications. These systems are generally expensive, not user friendly and require complex and dedicated hardware in the implementation. Also, CW radar characteristically mitigate clutter and hence can effectively be used in detecting moving agents. Frequency Modulated CW (FMCW) radar and pulsed Doppler radar both have evolved from the CW radar technology. Despite their capability to detect range, CW radar is prone to clutter which needs to be suppressed. CW single frequency radar consisting of two antenna elements can be used to find the Angle of Arrival (AoA),i.e Lin and Ling (2006a), using the phase difference of reflected waves arriving at the receiving antennas. In the case of Doppler, the frequency separations corresponding to the moving agents should be sufficiently large for this system to track multiple agents accurately. In Lin and Ling (2006b), only the location information of the agents could be obtained. For such systems, the location information of agents are acquired and the time derivatives of displacement is used to deduce the velocities of agents. This results in a time lag in velocity estimation particularly in more dynamic settings, and more prone to errors as the position estimation errors are directly translated into velocity deductions. In our Doppler radar approach, two receiving elements are kept half a wave length apart and the other two receiving elements are also placed with the same distance apart. These two sets are placed apart from each other, in a line, facing the mobile agents. We refer to the two elements as a *sensor*. The bearing of an agent with respect to a particular set of elements can be found by measuring the phase difference of the reflected waves within a single sensor. Similarly, the agents bearing with respect to the other two set of receiving elements can be found in the same manner. Using these parameters, the agents location can be tracked. The agents velocity can be derived by the Doppler shifts subtended by the agent in the direction of sensors. In many cases, multiple mobile agents give rise to different Doppler shifts and creates the problem of *data association*. The different frequency components and the phase differences for AoA measurements need to be assigned to the appropriate agent and this needs to be done across the two elements. This essentially poses the so called *data association* problem.

Another important scenario to consider is the similar radial velocities from two or more agents being modulated at the same sensor causing indistinguishable Doppler frequencies. We consider this problem as a *missing information* problem. In

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such instances, the same Doppler frequencies are modulated by the respective agents at that sensor elements causing the discrimination of distinctive frequencies impossible. Then the location and velocity estimation of the agents cannot be recovered. When the number of mobile agents increases, the probability of such occurrences also increases.

2. TRACKING MULTIPLE AGENTS

Tracking multiple agents using a single linear array of sensors are considered. As the number of sensors in the linear array for effective tracking directly affect the physical dimensions of the antenna, obtaining the minimal sensor configuration plays a crucial role in enhancing the versatility of the antenna. Firstly, we investigate the number of sensors in a linear array that is required to successfully track a given number of mobile agents. The two main aspects that need to be considered are:

(1) Identical Doppler frequencies modulated at a sensors due to two distinct mobile agents.
(2) The presence of *ghost* nodes, See Bishop and Pathirana (2007).

A sensor receiving two signals at the same frequency but in different phases, is unable to distinguish the two signals. Instead, it is seen as a signal at the same frequency in a phase which is different to the phases of the two incoming signals. Hence, resolving the AoA of the signal is impossible. Consider a set S of mobile agents and a linear phase array with a set of K distinct elements. Let $|\cdot|$ denote the cardinality of a set.

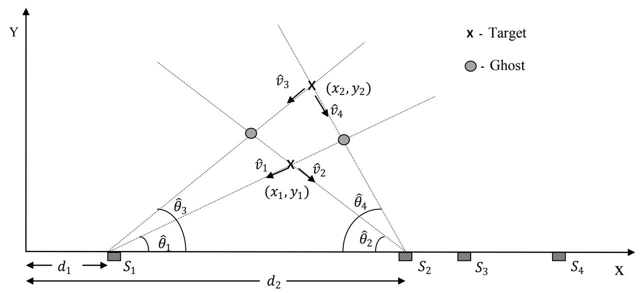

Fig. 1. Angle and Doppler velocity estimation

Proposition 1. Let $U \subseteq S$ modulate the same frequency on $L \subset K$. Then $|U| = 2 \Rightarrow |L| \leq 3$.

Proof 1. This will be provided in a longer version of this paper.

Proposition 2. The following is a necessary and sufficient condition to track S mobiles agents $|L| \geq 3^{|S|}C_2 + |S| + 1$

Proof 2. The proof directly follows from the proposition 1 and the ghost elimination results given in Bishop and Pathirana (2007).

Typically, each *frequency bin* associated with the respective sensor contains S number of frequencies. From the first proposition, a maximum of $3^{|S|}C_2$ number of sensors are need to be discarded in order to guarantee that no remaining sensor contains the same frequency due to two distinct agents. Further $|S| + 1$ sensors are needed to ensure the eradication of ghost nodes. Therefore, the data association problem is solved with $3^{|S|}C_2 + |S| + 1$ number of sensors in the linear array with no ambiguity resulting for any mobile agent trajectory.

For many practical applications, the unprecedented increase in the number of sensors in a linear array when tracking multiple agents causes problems. The enlarged array prevents more modern applications in close range or indoor tracking being realized. Therefore we look at the problem of multiple agent tracking with only two sensor array.

3. MOBILE AND TRACKER DYNAMIC MODEL

In the kinematic modelling of a mobile agent and a receiver sensory (linear) array in cartesian coordinate, the resulting dynamic system is linear. A comprehensive survey of dual body kinematic modelling is presented in Li and Jilkov (2003) and a basic principal approach is given in Savkin et al. (2003) where only the translational kinematics were considered. For certain applications, rotational motion has been considered and the resulting non-linear dynamic models have been used, i.e kinematic parameter estimation in Broida et al. (1990). For the case of radar based tracking, it is suffice to consider only the translational effects and the corresponding linear model (see Li and Jilkov (2003)) as no antenna motion is considered in the plane. In this paper, we consider a stationary receiver array(tracker).

Let the position of the i^{th} mobile agent in each of the traditionally denoted x, y directions with respect to the phase array based coordinate system, be $[x_1^i \ x_2^i]^\top \in \mathbb{R}^2$ with $\top$ denoting transposition. Let the velocity component in each traditionally denoted x, y direction be $[x_3^i \ x_4^i]^\top \in \mathbb{R}^2$ and the acceleration be $[x_5^i, x_6^i]^\top \in \mathbb{R}^2$. Hence, we can define $\mathbf{x^i} = [x_1^i \ x_2^i \ x_3^i \ x_4^i \ x_5^i \ x_6^i]^\top \in \mathbb{R}^6$, and the $\mathbf{x} = [\mathbf{x}^1 \ \cdots \ \mathbf{x}^N]^\top \in \mathbb{R}^{6N}$ such that the state evolves according to,

$$\mathbf{x}(k) = \mathbf{A}\mathbf{x}(k-1) + \mathbf{B}\mathbf{w}(k), \tag{1}$$

where $\mathbf{A}$ and $\mathbf{B}$ are suitably defined transition matrices (see Li and Jilkov (2003)) given by

$$\mathbf{A} = \mathrm{Diag}[\mathbf{\Phi} \cdots \mathbf{\Phi}],$$
$$\mathbf{B} = \mathrm{Diag}[\mathbf{\Psi} \cdots \mathbf{\Psi}],$$

where,

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{I}_2 & k_s\mathbf{I}_2 & \frac{k_s^2}{2}\mathbf{I}_2 \\ \mathbf{O}_2 & \mathbf{I}_2 & k_s\mathbf{I}_2 \\ \mathbf{O}_2 & \mathbf{O}_2 & \mathbf{I}_2 \end{bmatrix}, \mathbf{\Psi} = \begin{bmatrix} \frac{k_s^2}{2}\mathbf{I}_2 \\ k_s\mathbf{I}_2 \\ \mathbf{I}_2 \end{bmatrix}. \tag{2}$$

Notice here, I_2 and O_2 indicate *Identity* and *zero* matrices of 2×2 dimensions respectively.

Also $\mathbf{w}(k) \in \mathbb{R}^{2N}$ is an uncertainty parameter that encompasses the target's maneuvers and k_s is the sampling time. Our subsequent estimation algorithm is derived quite generally and permits a large class of linear dynamic models to be employed. Figure 1 depicts the modulation of doppler velocities and bearing measurements in $\mathbb{R}^2$.

Remark 1. Without loss of generality, the coordinate system is chosen with sensor 1 at the origin and sensor 2 is positioned at distance $d > 0$ away on the positive x_1-axis.

Here we consider a point target (or N number of point features) that obeys a linear dynamic model such as those described in Li and Jilkov (2003). Any arbitrary number of point targets can be included in this model and object rigidity is not required since each point is tracked independently. However, the data association problem (also known as the feature point association problem) Bar-Shalom and Li (1993) exists in practice for tracking multiple point targets.

4. LINEAR ROBUST FILTERING WITH NONLINEAR DOPPLER RADAR

Now we outline the measurement model and the subsequent measurement conversion technique along with the robust linear filter which we derive as the state estimator. Assume the associated frequency bin of each of the two sensors contains f_1^d and f_2^d modulated by the i^{th} mobile agent. Let the corresponding Doppler velocities and the AoA signals modulated by the agent on the two sensors be $\hat{v}_1, \hat{v}_2$ and $\hat{\theta}_1, \hat{\theta}_2$ respectively. We can state the measurement model for the i^{th} agent as follows.

Let $\hat{\mathbf{y}}$ denote the true values of the measured doppler velocities and the angles at the two sensors with ν_i and ε_i for $i = [1, 2]$ denoting the corresponding measurement noise.

$$\hat{\mathbf{y}}(k) = \begin{bmatrix} \hat{v}_1(k) \\ \hat{v}_2(k) \\ \hat{\theta}_1 \\ \hat{\theta}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{-x_3^i x_1^i - x_2^i x_4^i}{\sqrt{\left(x_1^i\right)^2 + \left(x_2^i\right)^2}} + \nu_1 \\ \dfrac{x_3^i\left(d - x_1^i\right) - x_2^i x_4^i}{\sqrt{\left(d - x_1^i\right)^2 + \left(x_2^i\right)^2}} + \nu_2 \\ \arcsin\left(\dfrac{x_2^i}{\sqrt{(x_1^i)^2 + (x_2^i)^2}}\right) + \varepsilon_1 \\ \arcsin\left(\dfrac{x_2^i}{\sqrt{(d - x_1^i)^2 + (x_2^i)^2}}\right) + \varepsilon_2 \end{bmatrix}$$

Now we can write the noisy locations of (x_1^i, x_2^i) and the directional velocities (x_3^i, x_4^i) in the following converted measurement form $(\mathbf{m})$:

$$\mathbf{m} \triangleq [\,\hat{x}_1\ \hat{x}_2\ \hat{x}_3\ \hat{x}_4\,]^\top = \begin{bmatrix} -\dfrac{d \sin(\theta_2 + \varepsilon_2)\cos(\theta_1 + \varepsilon_1)}{\sin(\theta_1 - \theta_2 + \varepsilon_1 - \varepsilon_2)} \\ \dfrac{d \sin(\theta_1 + \varepsilon_1)\sin(\theta_2 + \varepsilon_2)}{\sin(\theta_1 - \theta_2 + \varepsilon_1 - \varepsilon_2)} \\ \dfrac{-(v_1 + \nu_1)\sin(\theta_2 + \varepsilon_2) + (v_2 + \nu_2)\sin(\theta_1 + \varepsilon_1)}{\sin(\theta_1 - \theta_2 + \varepsilon_1 - \varepsilon_2)} \\ \dfrac{-(v_1 + \nu_1)\cos(\theta_2 + \varepsilon_2) + (v_2 + \nu_2)\cos(\theta_1 + \varepsilon_1)}{\sin(\theta_1 - \theta_2 + \varepsilon_1 - \varepsilon_2)} \end{bmatrix} \quad (3)$$

Here, the $\hat{x}_1 = x_1(\text{noisy}) - d$. Assume the angle measurement error is bounded. i.e $\|\varepsilon_i\| \leq |\alpha|$ for $i = 1, 2$ and $\|\nu_i\| \leq \gamma\|v_i\|$ for $i = 1, 2$.

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} = \begin{bmatrix} -d\dfrac{\omega_1\omega_2 \sin(\theta_2)\cos(\theta_1)}{\omega_3 \sin(\theta_1 - \theta_2)} \\ d\dfrac{\omega_1\omega_2 \sin(\theta_1)\sin(\theta_2)}{\omega_3 \sin(\theta_1 - \theta_2)} \\ \dfrac{-(v_1 + \nu_1)\omega_2 \sin(\theta_1) + (v_2 + \nu_2)\omega_1 \sin(\theta_1)}{\omega_3 \sin(\theta_1 - \theta_2)} \\ \dfrac{-(v_1 + \nu_1)\omega_2 \cos(\theta_1) + (v_2 + \nu_2)\omega_1 \cos(\theta_1)}{\omega_3 \sin(\theta_1 - \theta_2)} \end{bmatrix} \quad (4)$$

with the following condition

$$\sqrt{1 - \sin^2 \tau\alpha} \leq \omega_i \leq \frac{1}{\sqrt{1 - \sin^2 \tau\alpha}}$$

satisfied where

$$\tau = \left\{ \begin{matrix} 1 & i = 1, 2 \\ 2 & i = 3 \end{matrix} \right\}. \quad (5)$$

Remark 2. We have used the same variable i.e ω_i, to represent both sin and cos terms as the error variation is identical in either case(This will be illustrated in an extended version of this paper).

$[\hat{x}_1\ \hat{x}_2\ \hat{x}_3\ \hat{x}_4]'$ provides a well-defined system of measurement equations with associated noise tolerances for the measurements. Now assume that the target motion is described by the system in equation(1) where the matrix $\mathbf{A}$ is non-singular. Let $0 < p_0 \leq 1$, $\alpha \in [0, 2\pi]$ and $0 \leq \gamma < 1$ be given constants and suppose that the system initial condition $\mathbf{x}(0)$, noise $\mathbf{w}(k)$ and the actual measurement noise ν_i and ε_i $\forall i \in \{1, 2\}$ satisfy the following assumption.

Assumption 1. The following inequalities simultaneously hold with probability p_0 :

$$|\varepsilon_i| \leq |\alpha|, \ \ |\nu_i| \leq \gamma|v_i| \ \forall i \in \{1, 2\},$$

$$(\mathbf{x}(0) - \mathbf{x}_0)^\top \mathbf{N}(\mathbf{x}(0) - \mathbf{x}_0) + \sum_{0}^{T-1} \mathbf{w}(k)'\mathbf{Q}(k)\mathbf{w}(k) \leq \tau. \quad (6)$$

Here $\mathbf{x}_0$ is a given initial state estimate vector, $\mathbf{N} = \mathbf{N}^\top$ and $\mathbf{Q} = \mathbf{Q}^\top$ are given positive definite weighting matrices, $\tau > 0$ is a given constant associated with the system and $T > 0$ is a given terminal time.

Our solution to the state estimation problem involves the following Riccati difference equation,

$$\begin{aligned} \mathbf{F}(k+1) &= \hat{\mathbf{B}}\left[\hat{\mathbf{B}}^\top \mathbf{S}(k)\hat{\mathbf{B}} + \mathbf{Q}\right]^{-1} \hat{\mathbf{B}}^\top \mathbf{S}(k)\hat{\mathbf{A}}, \\ \mathbf{S}(k+1) &= \hat{\mathbf{A}}^\top \mathbf{S}(k)\left[\hat{\mathbf{A}} - \mathbf{F}(k+1)\right] + \mathbf{C}^\top U(k+1)\mathbf{C} \\ &\quad -\mathbf{K}^\top \mathbf{K}, \quad \mathbf{S}(0) = \mathbf{N}. \end{aligned} \quad (7)$$

where $\hat{\mathbf{A}} \triangleq \mathbf{A}^{-1}$ and $\hat{\mathbf{B}} \triangleq \mathbf{A}^{-1}\mathbf{B}$. We also define

$$\mathbf{C} \triangleq \begin{bmatrix} \beta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 \end{bmatrix}, \quad \mathbf{K} \triangleq \begin{bmatrix} \gamma_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_2 & 0 & 0 \end{bmatrix} \quad (8)$$

$$\begin{aligned} \text{where } \beta_1 &= \frac{\cos 2\alpha\left(1 + \cos^4\alpha\right)}{2\cos^2\alpha}, \\ \beta_2 &= \frac{(1+\gamma)}{2\cos 2\alpha \cos\alpha} + \frac{(1-\gamma)\cos\alpha\cos 2\alpha}{2} \\ \gamma_1 &= \frac{\cos 2\alpha\left(1 - \cos^4\alpha\right)}{2\cos^2\alpha}, \\ \gamma_2 &= \frac{(1+\gamma)}{2\cos 2\alpha \cos\alpha} - \frac{(1-\gamma)\cos\alpha\cos 2\alpha}{2}. \end{aligned}$$

We now consider a set of state equations of the form

$$\begin{aligned} \boldsymbol{\eta}(k+1) &= \left[\hat{\mathbf{A}} - \mathbf{F}(k+1)\right]^\top \boldsymbol{\eta}(k) \\ &\quad + \mathbf{C}^\top V(k+1)\mathbf{m}(k+1), \ \ \boldsymbol{\eta}(0) = \mathbf{N}\mathbf{x}_0 \\ g(k+1) &= g(k) + \mathbf{m}(k+1)^\top W(k+1)\mathbf{m}(k+1) \\ &\quad -\boldsymbol{\eta}(k)^\top \hat{\mathbf{B}}\left[\hat{\mathbf{B}}^\top \mathbf{S}(k)\hat{\mathbf{B}} + \mathbf{Q}(k)\right]^{-1} \hat{\mathbf{B}}^\top \boldsymbol{\eta}(k), \\ g(0) &= \mathbf{x}_0^\top \mathbf{N}\mathbf{x}_0. \end{aligned}$$

Notice that the matrices U, V, W are appropriately defined to account for the missing information case considered subsequently. For the full information case, the matrices are evaluated to be identity matrices.

The above state equations (9) and Riccati equations (7) can simply be thought of as a robust implementation of the standard linear Kalman Filter for uncertainties obeying Assumption 1, e.g. see Anderson and Moore (1979); Savkin and Petersen (1998). Now we are in a position to present the main result of this section.

Theorem 1. Let $0 < p_0 \leq 1$ be given, and suppose that Assumption 1 holds. Then the state $\mathbf{x}(T)$ of the system (1) with probability $p \geq p_0$ belongs to the ellipsoid

$$E_T \triangleq \left\{ \begin{array}{l} \mathbf{x}_T \in \mathbf{R}^n : \\ \|(\mathbf{S}(T)^{\frac{1}{2}}\mathbf{x}_T - \mathbf{S}(T)^{-\frac{1}{2}}\boldsymbol{\eta}(T))\|^2 \\ \leq \rho + \tau \end{array} \right\} \quad (9)$$

$$\text{where } \rho \triangleq \boldsymbol{\eta}(T)^\top S(T)^{-1}\boldsymbol{\eta}(T) - g(T)$$

and $\boldsymbol{\eta}(T)$ and $g(T)$ are defined by the equations (9). Also, we require $\rho + \tau \geq 0$.

Proof 3. This will be presented in a longer version of the paper

A point value state estimate can be obtained from the bounded ellipsoidal set's center and is given by $\widehat{\mathbf{x}} = \mathbf{S}(k)^{-1}\boldsymbol{\eta}(k)$. We have thus proved that our algorithm's estimation error is bounded in a probabilistic sense when the relevant uncertainties obey Assumption 1. The sum quadratic constraint given in Assumption 1 accommodates a large class of non-linear and dynamic process noise characteristics. As the Gaussian noise is bounded within the first standard deviation with a probability $p_0 \approx 0.68$ and within two standard deviations with probability $p_0 \approx 0.95$ etc, we lose no generality by considering uncertainties satisfying Assumption 1. That is, Gaussian distributed measurement, process and initial condition errors form special case of Assumption 1 which defines a larger class of uncertainties. We solve the problem in the linear domain and our algorithm permits very large initial errors. No similar proofs exist for the extended Kalman filter (EKF) or the majority of other approaches that employ some form of a Taylor-series based approximation. Indeed, the fact that we can prove bounded tracking performance with arbitrarily large initial condition errors is a novel contribution.

5. LINEAR ARRAY WITH TWO SENSORS

Reduction of the number of sensors in the linear array to obtain more practical physical dimensions for many applications introduces the problem of data association. We look at this problem under two scenarios as below :

- Complete information : There is no overlapping of frequencies in either frequency bin as in figure 2 (for four mobile agents).
- Incomplete information : At least one frequency bin contains less than S frequencies. If any of the two or more frequency(in figure 2) components coincided to an indistinguishable level, then we are unable to recover the respective velocity components.

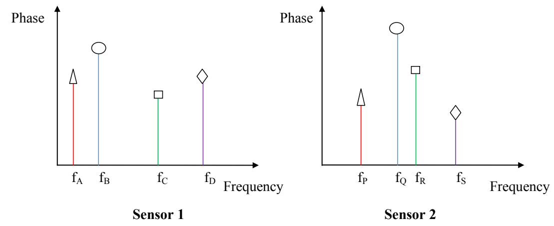

Fig. 2. Instantaneous frequency and the phase distribution at the two sensors

5.1 Data association with complete information

Here we assume that each sensor receives all the (N) measurements distinctively. That is the frequencies are easier to distinguish at each sensor node. Let the measurement at receiver 1 and 2 be $\mathbf{L}_1 = \{(\hat{v}_1^1, \hat{\theta}_1^1), \cdots, (\hat{v}_1^N, \hat{\theta}_1^N)\}$, $\mathbf{L}_2 = \{(\hat{v}_2^1, \hat{\theta}_2^1), \cdots, (\hat{v}_2^N, \hat{\theta}_2^N)\}$. Define the set $S = \{\chi = [\chi_1^1\ \chi_2^1\ \chi_3^1\ \chi_4^1, \cdots, \chi_1^i\ \chi_2^i\ \chi_3^i\ \chi_4^i, \cdots, \chi_1^N\ \chi_2^N\ \chi_3^N\ \chi_4^N]^\top : (\chi_1^i, \chi_3^i) \in \mathbf{L}_1 \text{ and } (\chi_2^i, \chi_4^i) \in \mathbf{L}_2, \{\chi_1^i\ \chi_2^i\ \chi_3^i\ \chi_4^i\} \neq \{\chi_1^j\ \chi_2^j\ \chi_3^j\ \chi_4^j\}\ \forall\, i, j \in [1, \cdots, N]\}$. Notice that the cardinality, $|S| = N!$

Remark 3. The doppler velocities and the corresponding AoA signal measurements are related to the doppler frequency at the receiver. The measurements are modelled so that the first set of measurement is obtained from the first receiver. Only one of the combinations actually corresponds to the actual physical distribution of the mobile agents. The other combinations consists of ghost node combinations.

Let $\chi = [\chi_1^1\, \chi_2^1\, \chi_3^1\, \chi_4^1, \cdots, \chi_1^i\, \chi_2^i\, \chi_3^i\, \chi_4^i, \cdots, \chi_1^N\, \chi_2^N\, \chi_3^N\, \chi_4^N]^\top \in S$ and let $[Y_1^i\ Y_2^i\ Y_3^i\ Y_4^i]^\top = f(\chi_1^i, \chi_2^i, \chi_3^i, \chi_4^i)$, where f denotes the measurement conversion function implemented in equation 3. Therefore $\chi \leftrightarrow Y = [Y_1^1\, Y_2^1\, Y_3^1\, Y_4^1 \cdots Y_1^i\, Y_2^i\, Y_3^i\, Y_4^i \cdots Y_1^N\, Y_2^N\, Y_3^N\, Y_4^N]^\top$ is a one-to-one mapping. Let

$$E^i = \begin{bmatrix} Y_1^i(k) \\ Y_2^i(k) \end{bmatrix} - \begin{bmatrix} Y_1^i(k-1) + k_s Y_3^i \\ Y_2^i(k-1) + k_s Y_4^i \end{bmatrix}. \quad (10)$$

$$\text{Then } \quad \chi : \min_{\chi \in S} \sum_{i=1}^{N} \|E^i\|, \quad (11)$$

should corresponds to the combination of the actual mobile agents positions and hence exclude all the ghost nodes. That is the two consecutive state estimates(converted measurements) are closest for the case of actual mobile agents. i.e Only real agents behave according to the estimated dynamics and not the ghost nodes.

It is also a crucial practical measure to maintain the same order of measurements in the filtering implementations. This can be explained graphically as given in figure 2. When the doppler frequencies are sufficiently apart and the order is maintained, the ordering(of Y) for the filter is maintained with no intricacies arise. If the frequency components overlap, then the scenario described as *missing information* exists as the Doppler velocities and the corresponding angles are not available. Indeed the problem becomes intricate when the frequency components cross each other due to the respective mobile agent motion denying the maintenance of the order of the states corresponding to the agents in the filtering process.

Assume χ^o corresponds to the optimal combination given by equation 11 and the corresponding converted measurement vector be $Y^o = [Y_1^1\ Y_2^1\ Y_3^1\ Y_4^1 \cdots Y_1^i\ Y_2^i\ Y_3^i\ Y_4^i \cdots Y_1^N\ Y_2^N\ Y_3^N\ Y_4^N]^\top$. Let $T = \{Z = [Z_1^1\ Z_2^1\ Z_3^1\ Z_4^1 \cdots Z_1^i\ Z_2^i\ Z_3^i\ Z_4^i \cdots Z_1^N\ Z_2^N\ Z_3^N\ Z_4^N]^\top : \{Z_1^j\ Z_2^j\ Z_3^j\ Z_4^j\} \subset Y^o\ \forall j \in [1, \cdots, N],\ \{Z_1^j\ Z_2^j\ Z_3^j\ Z_4^j\} \neq \{Z_1^k\ Z_2^k\ Z_3^k\ Z_4^k\}\ \forall j \neq k\}$ Then, $z : \min_{z \in T} \|z - \hat{x}(k-1)\|$, will provide the relevant order of the measurement vector to be used in the robust linear filter. Here $\hat{x}(k-1)$ indicates the previous state estimate of the robust filter.

5.2 *Data association with incomplete information*

The reduction in the number of sensor elements inevitably increases the probability of instances where two or more mobile agents modulate the same frequency at any one of the two sensors. This results in inaccurate information. That is, $\mathbf{y}(k)$ is incomplete or not available for certain time t. Let $\mathbf{M}(t) = [\mathcal{M}^1(t)\ \cdots\ \mathcal{M}^4(t)]^\top$ be a given vector for $t = 1, 2, \cdots, T$ such that $\mathcal{M}^i \in \{0, 1\}$, for $i = 1, \cdots, 4$. Then the matrix $\mathrm{M} \triangleq [\mathbf{M}(1) \cdots \mathbf{M}(T)]^\top$, is referred as the *incomplete* matrix. Together with, $\mathcal{M}$, define two sequences of matrices :

$$E(t) = \mathrm{Diag}[\mathcal{M}^1(t)\ \mathcal{M}^2(t)\ \cdots\ \mathcal{M}^4(t)]$$
$$\hat{E}(t) = [\tilde{\mathcal{M}}^1(t)\ \tilde{\mathcal{M}}^2(t)\ \cdots\ \tilde{\mathcal{M}}^4(t)]^\top \qquad (12)$$

where $\mathcal{M}^i(t) + \tilde{\mathcal{M}}^i(t) = 1$. In the Riccati equation (7) and (9), U, V and W is defined to account for the missing information.

$$U = EWE,\ \ V = EW,\ \ W = I - \hat{E}(\hat{E}^\top \hat{E})^{-1}\hat{E}^\top. \quad (13)$$

Remark 4. For the case of no missing information, $\hat{E}$ is the zero vector and E is the identity matrix. This ensures that the U, V and W are evaluated as identity matrices.

6. ILLUSTRATIVE EXAMPLES

In this section, we look at a simulation scenario where the location of four agents were tracked using doppler measurements. The simulation parameters are given in Table 1. Both filters are initialized with Gaussian distributed values about the true initial state with $\sigma = 5\%$ of the true initial values. The EKF is known to be potentially unstable without correct initialization. The true initial state is $[-100\ 250\ 1\ \ -1\ 0.0001\ 0.0001]^\top$. The

Table 1. Simulation Parameters

Input	Value	Comments
$\mathbf{w_1}$	$[-1\ \ 2]' \sin(2\pi t)$	Agent 1 Accel. Input
$\mathbf{w_2}$	$[1\ \ -2]' \sin(2\pi t)$	Agent 2 Accel. Input
$\mathbf{w_3}$	$[-1\ \ 1]' \sin(2\pi t)$	Agent 3 Accel. Input
$\mathbf{w_4}$	$[-1\ \ 1]' \sin(2\pi t)$	Agent 4 Accel. Input
$v_i, i \in 1, 2$	$\sigma_{vi} = 0.005$	Gaussian Meas. Noise 1
$\varepsilon_i, i \in 1, 2$	$\sigma_{vi} = 0.005$	Gaussian Meas. Noise 2
$[\mathbf{N}_R,\ \ \mathbf{Q}_R]$	$[10^-4I_6,\ \ 10^-9I_2] \cdot \mathbf{I}$	Robust Filter Parameters
$[\mathbf{R}_E,\ \ \mathbf{Q}_E]$	$[10^5 I_2,\ \ 10^-10] \cdot \mathbf{I}$	EKF Uncertainty
$T\ \ @\ \ t_s$	$12s\ \ @\ \ 0.1s$	Track Duration, Periodicity

EKF parameters, i.e. $\mathbf{Q}_E$ and $\mathbf{R}_E$ were tuned fairly accurately. The initial covariance of the EKF is also tuned assuming the initial error statistics known to the tracking system. That is, for the EKF parameters we assumed perfect knowledge of all the relevant error statistics and tuned around these true values in order to get the best performance. On the other hand, for the robust filter we simply used the identity matrix for both the initial and process noise weightings. For the robust filter, α is taken as two times the first standard deviations of the Gaussian measurement noise. We plot the accurate tracking performed by the robust linear filter in figure 3 where the actual trajectories of the four agents are accurately estimated. The significant error in the initial condition(position and velocity) is corrected faster as depicted in figure 4 where a clear advantage of using the robust filter is evident in terms of convergence and accuracy against the Extended Kalman Filter(EKF). Even though the EKF velocity estimate converges when tracking maneuverable targets, it is evident that the prominent difference in the convergence rate is primarily due to the underlying linear formulation. The proposed linear robust filter fundamentally uses the measurement conversion technique which is essentially a computation of 2D coordinates of an agent in a closed-form fashion. Therefore, our robust estimator exhibits superior performance. In contrast, the EKF does not contain any computation of 2D coordinates of a target and is based on linearization and Taylor series approximations. It causes the accumulation of errors resulting in divergence for large uncertainties.

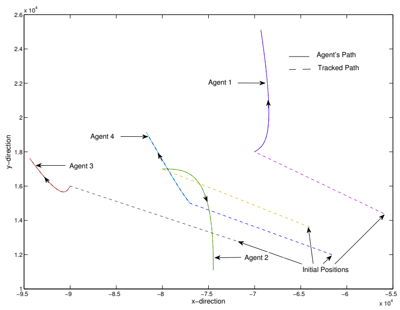

Fig. 3. Actual and estimated trajectories of four mobile agents

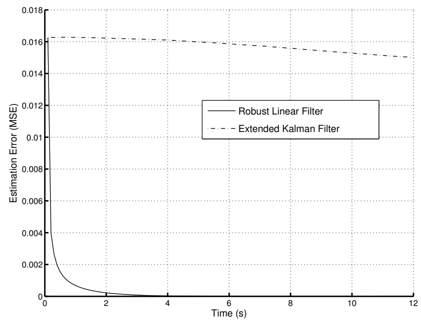

Fig. 4. The estimation error of the agents using EKF and LRF

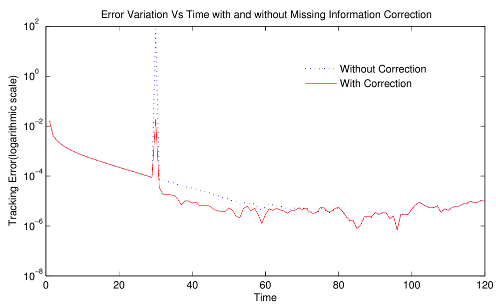

Fig. 5. Linear Robust Filtering with missing information

Figure 5 shows the performance of the modified form of the linear robust filter for the case of missing information. In a doppler radar application with multiple agents, missing certain measurements in the time series can be a common occurrence. Here we use the modified version of the linear filter in comparison to the proceeding case of full information(using the previous measurement for the missing instance) development and demonstrate the superior adaptation of the modified version directly aimed at mitigating the effect due to missing information.

6.1 Real Experimental Data

We setup a Doppler information acquisition system consisting of two sensors (receivers) and obtain data from a person moving in a well defined path in an indoor setting. The receiving sensors (one sensor composed of two antennas kept 6cm apart) are kept 0.5m apart on the x axis as shown in the figure 7. The signal generator is set to transmit 2.4GHz continuous wave RF at 12dBm. The person moved in a marked path on the ground and the reflected RF waves form the moving person is captured by the antennas and was feed into AD8347 direct conversion quadrature demodulator. Then the consequent output was feed in to the NI USB-6009 for analog to digital conversion. Next the digitized data are collected by the computer and FFT is performed to find the frequencies and corresponding phase differences to measure the angle of arrival(AoA) of the object at consecutive time step.

Experimental setup depicted in figure 6 is used to estimate position of a person via Doppler based measurements and the actual physical measurements based path is shown in figure 7. The estimation error with and without the filter is shown in figure 8.

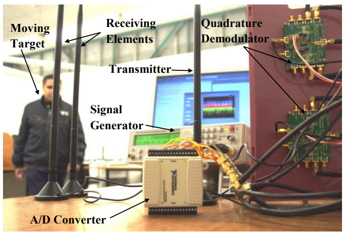

Fig. 6. The experimental setup for tracking a person

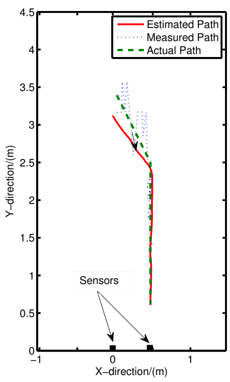

Fig. 7. The estimated and actual path of the person

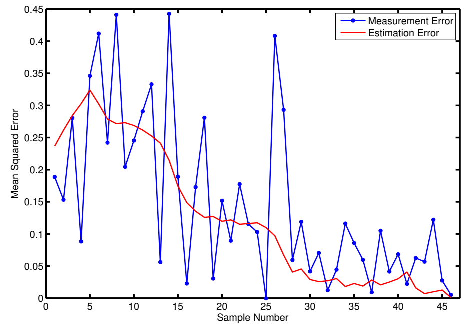

Fig. 8. The measurement and the estimation error of the person

7. CONCLUSION

In this paper we derived a linear state estimator with provable performance limits for doppler radar based tracking of multiple agents. We use a novel measurement conversion approach that does not use Taylor-series approximations and allows us to derive a new algorithm exploiting the strength of fundamentally strong approach of linear filtering. A significant contribution of this technique is the mathematically rigorous proof of the boundedness of the filtering error. No such results are known for the extended Kalman filter. We considered the case of a minimal structure for the receiver antenna with merely two sensors as well as addressing the practical issue of missing information.

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