

## Artificial Neural Networks, Adaptive and Classical Control for FTC of Linear Parameters Varying Systems

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**Abstract:** Three different schemes for Fault Tolerant Control (FTC) based on Adaptive Control in combination with Artificial Neural Networks (ANN), Robust Control and Linear Parameter Varying (LPV) systems are compared. These schemes include a Model Reference Adaptive Controller (MRAC), a MRAC with an ANN and a MRAC with an  $H_\infty$  Loop Shaping Controller for 4 operating points of an LPV system (MRAC-4OP-LPV, MRAC-NN4OP-LPV and MRAC- $H_\infty$ 4OP-LPV, respectively). In order to compare the performance of these schemes, a coupled-tank system was used as testbed in which two different types of faults (abrupt and gradual) applied in sensor and actuators in different operating points were simulated. The simulation results showed that the use of ANN in combination with an adaptive controller for LPV-based system improves the FTC scheme, delivering a robust FTC system against abrupt and gradual sensor faults. For actuator faults, the only schemes that were fault tolerant were the MRAC- $H_\infty$ 4OP-LPV and the MRAC-4OP-LPV (i.e. the MRAC- $H_\infty$ 4OP-LPV was fault tolerant for actuator faults varying from 0 to 0.5 of magnitude).

**Keywords:** LPV systems, MRAC, Artificial Neural Networks,  $H_\infty$  control.

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### 1. INTRODUCTION

Modern systems and their challenging operating conditions in certain processes increase the possibility of failures causing damages in equipment and/or operators. In these environments the use of automation control (i.e. adaptive and robust control) and intelligent systems is fundamental to minimize the impact of faults. Fault Tolerant Control (FTC) methods have been proposed to ensure the continuous operations even faults appear and to prevent more serious effects.

In the last years, FTC schemes based on Linear Parameter Varying systems have been developed. In Bosche et al. (2009) a FTC structure for vehicle dynamics is developed employing an LPV model with actuator failures. The methodology described in Bosche et al. (2009) is supported on the resolution of Linear Matrix Inequalities (LMIs) using the DC-stability concept and a Parameter-Dependent Lyapunov Matrix (PDLM). In Montes de Oca et al. (2009) an Admissible Model Matching (AMM) FTC method based on LPV fault representation was presented; the faults were considered as scheduling variables in the LPV fault representation allowing an on-line controller adaptation. For instance, in Rodrigues et al. (2007) a FTC methodology for polytopic LPV systems was presented. The most important contribution of Rodrigues et al. (2007) was the development of a Static Output Feedback (SOF) that maintains the system performance using an adequate controller reconfiguration when faults appear.

Although, most of the FTC methods that have been developed are based on classical and modern control theory; classical Artificial Intelligence (AI) approaches such as Artificial Neural Networks (ANN), Fuzzy Logic (FL), ANN-FL and Genetic Algorithms (GA) offer an advantage over those traditional methods used by the control community.

ANN have been applied in FTC because they are helpful to identify, detect and accommodate system faults. ANN are used as fault detectors by estimating changes in process models dynamics (Polycarpou and Helmicki, 1995), as process controllers (Wang and Wang, 1999), and performing both functions: fault detection and control (Perhinschi et al., 2007).

On the other hand, advanced techniques from Robust Control such as  $H_\infty$ , have also been applied to FTC with encouraging results. Dong et al. (2009) proposed an active FTC scheme for a class of linear time-delay systems, using a  $H_\infty$  controller in generalized internal mode architecture in combination with an adaptive observer-based fault estimator. In Xiadong et al. (2008) a dynamic output feedback FTC approach that uses an  $H_\infty$  index for actuator continuous gain faults was proposed. And in Liang and Duan (2004) an  $H_\infty$  FTC approach was used against sensor failures for uncertain descriptor system.

To improve the capabilities of FTC systems, in this paper three different passive FTC approaches based on Adaptive Control, ANN, Robust Control and LPV systems were developed. The first approach is a Model Reference Adaptive Controller for 4 operating points of an LPV system (MRAC-4OP-LPV), the second approach is a combination of an MRAC with an ANN controller for 4 operating points of an

LPV system (MRAC-NN4OP-LPV) and the third scheme is a MRAC in combination with an  $H_\infty$  Loop Shaping controller for 4 operating points of an LPV system (MRAC- $H_\infty$ 4OP-LPV). The MRAC was chosen as a based controller because guarantees asymptotic output tracking, it has a direct physical interpretation and it is easy for implementation.

This paper is organized as follows: section 2 describes the main building blocks; in section 3 the proposed schemes are shown; section 4 presents the results, in section 5, a comparison with similar approaches is presented, and finally, section 6 addresses the conclusions.

## 2. BACKGROUND

The background theory implemented to implement the MRAC-4OP-LPV, MRAC-NN4OP-LPV and the MRAC- $H_\infty$ 4OP-LPV approaches will be described.

### 2.1 LPV Systems

The Linear Parameter Varying (LPV) systems depend on a set of variant parameters over time. These systems can be represented in state space. The principal characteristic is the matrix representation function of one or more variable parameters over time. The continuous time representation of an LPV system is:

$$\begin{aligned} \dot{x}(t) &= A(\varphi(t))x + B(\varphi(t))u \\ y(t) &= C(\varphi(t))x + D(\varphi(t))u \end{aligned} \quad (1)$$

where  $x(t) \in \mathbb{R}^n$  represents the state space vector,  $y(t) \in \mathbb{R}^m$  is the measurement or output vector,  $u \in \mathbb{R}^p$  is the input vector,  $\varphi(t)$  represents the parameters variation over time  $t$  and  $A(\cdot)$ ,  $B(\cdot)$ ,  $C(\cdot)$  and  $D(\cdot)$  are the continuous function of  $\varphi(t)$ .

An LPV system can be obtained through different methodologies; if the physical representation of the nonlinear system is obtained, the Jacobian Linearization method, the State Transformation Method and the Substitution Function method can be used. The main objective of these methodologies is to occult the nonlinearity of the system in any variable in order to get the LPV system. If the experimental data model is obtained, the LPV system can be created using the Least Square Estimation for different operating points of the system (Apkarian et al., 1995; Bamieh and Giarré, 2001; Marcos and Balas, 2004).

### 2.2 MRAC controller

The Model Reference Adaptive Control (MRAC), implements a close loop controller where the adaptation mechanism adjusts the controller parameters to match the process output with the reference model output. The reference model is specified as the ideal model behaviour that the system is expected to follow (see Figure 1). This type of controller behaves as a close loop controller because the actuating error signal is fed to the controller in order to reduce the error to achieve the desired output value (Whitaker et al., 1958). The controller error is calculated as follows:

$$e(t) = y(t) - y_m(t) \quad (2)$$

where  $y(t)$  is the process output and  $y_m(t)$  is the reference model output. To reduce the error, a cost function was used in the form of

$$J(\theta) = \frac{1}{2} e^2(\theta) \quad (3)$$

where  $\theta$  is the adaptive parameter inside the controller.

The function  $J(\theta)$  can be minimized if the parameters  $\theta$  change in the negative direction of the gradient  $J$ , this is called the gradient descent method and it is represented by:

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma \frac{\partial e}{\partial \theta} e \quad (4)$$

where  $\gamma$  is the speed of learning. The implemented MRAC is a second order system and has two adaptation parameters: adaptive feedforward gain ( $\theta_1$ ) and adaptive feedback gain ( $\theta_2$ ). These parameters will be updated to follow the reference model. The final equations of the two adaptation parameters are:

$$\frac{\partial e}{\partial \theta_1} = \left( \frac{a_{1r}s + a_{0r}}{s^2 + a_{1r}s + a_{0r}} \right) u_c \rightarrow \frac{d\theta_1}{dt} = -\gamma \frac{\partial e}{\partial \theta_1} e = -\gamma \left( \frac{a_{1r}s + a_{0r}}{s^2 + a_{1r}s + a_{0r}} \right) u_c e \quad (5)$$

$$\frac{\partial e}{\partial \theta_2} = - \left( \frac{a_{1r}s + a_{0r}}{s^2 + a_{1r}s + a_{0r}} \right) y \rightarrow \frac{d\theta_2}{dt} = -\gamma \frac{\partial e}{\partial \theta_2} e = \gamma \left( \frac{a_{1r}s + a_{0r}}{s^2 + a_{1r}s + a_{0r}} \right) y e \quad (6)$$

where  $a_{1r}$  and  $a_{0r}$  are the coefficients of the second order model.

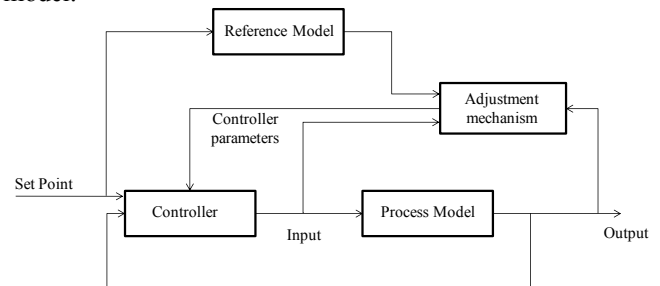


Fig. 1. Model Reference Adaptive Controller (MRAC) general scheme.

### 2.3 Artificial Neural Networks (ANN)

ANN are mathematical models that try to mimic the biological neural system. An artificial neuron have multiple input signals  $x_1, x_2, \dots, x_n$  entering the neuron using connection links with specific weights  $w_1, w_2, \dots, w_n$  or  $\sum_{i=1}^n w_n x_i$  named the net input, and also have a firing threshold  $b$ , an activation function  $f$  and an output of the neuron that is represented by  $y = f(\sum_{i=1}^n w_i x_i - b)$ . The firing threshold  $b$  or bias can be represented as another weight by placing an extra input node  $x_0$  that takes a value of 1 and it has a  $w_0 = -b$  (Nguyen et al., 2002).

An ANN can have a feedback or a feed-forward structure. The ANN need to be trained from examples, in a process called supervised learning. Once a successfully training is done, the ANN is ready if and only if the networks reproduce the desired outputs from the given inputs. The most common methodology is the backpropagation algorithm, where the weights of the ANN are determined by using iteration until the output of the network is the same as the desired output. In addition, unsupervised learning uses a mechanism for changing values of the weights according to the input values, this mechanism is named self-organization.

### 2.4 $H_\infty$ Loop Shaping Controller

The  $H_\infty$  control theory is a robust technique developed in Zames (1981) to achieve robust performance and stabilization in an implemented system. This control theory uses the  $H_\infty$

norm which is the frequency response magnitude to maximum singular value of the interested transfer function (i.e. peak gain or worst case disturbances). The standard configuration problem for an  $H_\infty$  controller is shown in Figure 2 (Skogestad and Postlethwaite, 2005):

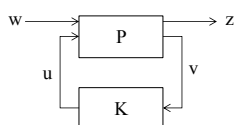


Fig. 2. General  $H_\infty$  controller configuration (Zames, 1981). where  $K$  is the  $H_\infty$  controller,  $P$  is a generalized plant,  $u$  are the control variables,  $w$  the exogenous signals,  $z$  the error signals which have to be minimized to achieve the control objectives and  $v$  the measured variables. In terms of state space, the above is rewritten as (Skogestad and Postlethwaite, 2005):

$$\begin{bmatrix} z(s) \\ v(s) \end{bmatrix} = P(s) \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} \quad (7)$$

$$u(s) = K(s)v(s) \quad (8)$$

$$P(s) = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \quad (9)$$

In which the linear fractional transformation of  $w$  to  $z$  is given by:

$$z = F_1(P, K)w \rightarrow F_1(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \quad (10)$$

where the  $H_\infty$  control implicate the minimization of  $H_\infty$  norm of  $F_1(P, K)$ .

$$\|F_1(P, K)\|_\infty = \sup \sigma(F_1(P, K)(j\omega)) \quad (11)$$

### 3. PROPOSED SCHEMES

A two tank second order coupled-tank system was used for LPV-based approaches validation. This coupled-tank system is composed by two cylindrical tanks (Figure 3): an upper and a lower tank. A pump is used to transport water from reservoir to tank 1, the outlet flow of tank 1 flows to tank 2, and finally the outlet flow of tanks 2 end in the reservoir (Abdullah and Zribi, 2009; Apkarian, 1999). The water levels of the tanks are measured using differential pressure sensors.

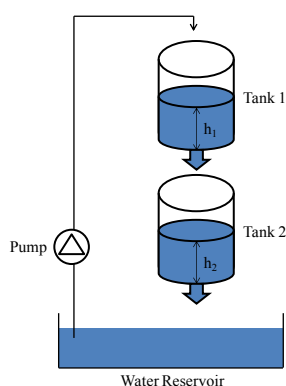


Fig. 3. Coupled-tank system designed by Apkarian (1999). Pan et al. (2005) proposes the following model for this process. Table 1 describes the variables.

$$\dot{h}_1(t) = -a_1/A_1 \sqrt{2g} \sqrt{h_1(t)} + k_p/A_1 u(t) \quad (12)$$

$$\dot{h}_2(t) = -a_1/A_2 \sqrt{2g} \sqrt{h_1(t)} - a_2/A_2 \sqrt{2g} \sqrt{h_2(t)} \quad (13)$$

$$y(t) = h_2(t) \quad (14)$$

In Table 1, the variables definition involves in the above system are explained.

Table 1. Variables definition of the coupled tank system

Variable	Definition	Value
$h_1, h_2$	water level of tanks 1 and 2	-
$A_1, A_2$	cross-section area of tanks 1 and 2	15.5179 cm <sup>2</sup>
$a_1, a_2$	cross-section area of the outflow orifice of tanks 1 and 2	0.1781 cm <sup>2</sup>
$U$	pump voltage	-
$k_p$	pump gain	3.3 cm <sup>3</sup> /V s
$G$	gravitational constant	981 cm/s <sup>2</sup>
$\alpha_4$	approximation constant	2.981 x 10 <sup>-7</sup>
$\alpha_3$	approximation constant	-3.659 x 10 <sup>-5</sup>
$\alpha_2$	approximation constant	1.73 x 10 <sup>-3</sup>
$\alpha_1$	approximation constant	-4.036 x 10 <sup>-2</sup>
$\alpha_0$	approximation constant	0.583

An LPV model of the system is computed by a polynomial fitting technique that approximates  $\sqrt{h_i}$  for  $0 \leq h_i \leq 30$  cm with  $\varphi_i h_i$ , where (Forsythe and Malcolm, 1977):

$$\varphi_i = \alpha_4 h_i^4 + \alpha_3 h_i^3 + \alpha_2 h_i^2 + \alpha_1 h_i + \alpha_0 \quad (15)$$

The parameters  $\varphi_1$  and  $\varphi_2$  are bounded with the following values:

$$0.1 = \underline{\varphi}_1 \leq \varphi_1 \leq \overline{\varphi}_1 = 0.6 \quad 0.1 = \underline{\varphi}_2 \leq \varphi_2 \leq \overline{\varphi}_2 = 0.6 \quad (16)$$

The LPV form ends in:

$$\begin{aligned} \dot{x} &= A(\varphi)x + Bu \\ y &= Cx \end{aligned} \quad (17)$$

where:

$$x = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (18)$$

$$A(\varphi) = \begin{bmatrix} -0.5085\varphi_1 & 0 \\ 0.5085\varphi_1 & -0.5085\varphi_2 \end{bmatrix} \quad (19)$$

$$B = \begin{bmatrix} 0.2127 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (20)$$

#### 3.1 MRAC 4 Operating Points LPV Controller (MRAC-4OP LPV)

To start testing the LPV model of the two tank system, an MRAC scheme including the four extreme operating points was developed (see Figure 4).

Table 2. Model Reference  
 Adaptive Controller of the 4 Operating Points.

Operation Point	MRAC Equations for Each Operating Point		
	Reference Model = Process Model	Adaptive feed forward update rule ( $\theta_1$ )	Adaptive feedback update rule ( $\theta_2$ )
$\varphi_1=0.1$ $\varphi_2=0.1$	$\frac{0.01082}{s^2+0.1017s+0.002586}$	$-\gamma \left( \frac{0.1017s+0.002586}{s^2+0.1017s+0.002586} \right) e$	$\gamma \left( \frac{0.1017s+0.002586}{s^2+0.1017s+0.002586} \right) e$
$\varphi_1=0.1$ $\varphi_2=0.6$	$\frac{0.01082}{s^2+0.3559s+0.01551}$	$-\gamma \left( \frac{0.3559s+0.01551}{s^2+0.3559s+0.01551} \right) e$	$\gamma \left( \frac{0.3559s+0.01551}{s^2+0.3559s+0.01551} \right) e$
$\varphi_1=0.6$ $\varphi_2=0.1$	$\frac{0.06489}{s^2+0.3559s+0.01551}$	$-\gamma \left( \frac{0.3559s+0.01551}{s^2+0.3559s+0.01551} \right) e$	$\gamma \left( \frac{0.3559s+0.01551}{s^2+0.3559s+0.01551} \right) e$
$\varphi_1=0.6$ $\varphi_2=0.6$	$\frac{0.06489}{s^2+0.6102s+0.09309}$	$-\gamma \left( \frac{0.6102s+0.09309}{s^2+0.6102s+0.09309} \right) e$	$\gamma \left( \frac{0.6102s+0.09309}{s^2+0.6102s+0.09309} \right) e$

### 3.2 MRAC Neural Network 4 Operating Points LPV Controller (MRAC-NN4OP-LPV)

Four ANN controllers were added to the MRAC-4OP-LPV Controller. To create and train the ANN controller, the desired reference levels of the tanks were introduced as well as the measured levels (when the system has no faults) as the inputs and the outputs of the ANN, respectively. The created ANN is a two-layer feed forward ANN with 50 sigmoid hidden neurons and a linear output neuron. To train the network the Levenberg-Maquard backpropagation algorithm was used. This training algorithm is a combination of Gauss-Newton and gradient descent methods which integrates the benefits of the global and local convergence properties from the gradient descent and Gauss-Newton methods, respectively (Ye, 2004). The global implementation of the MRAC-NN4OP-LPV can be observed in Figure 4 as a feedforward controller.

### 3.3 MRAC $H_\infty$ Loop Shaping 4 Operating Points LPV Controller (MRAC- $H_\infty$ 4OP-LPV)

An  $H_\infty$  controller was design for each of the 4 operating points. The  $H_\infty$  control proposed in this work was designed by using the loop shaping method and the following steps were realized: First, the worst faulty condition (30% of deviation from the nominal operation point) were simulated and identified in the form of a Laplace function. Second, these functions are compared against the non-fault process. Third, a loop shaping control synthesis is performed to calculate an optimal  $H_\infty$  controller for the Laplace fault-functions. This controller shapes the sigma plot of the Laplace fault-function and obtains the desired loop shaping with a precision parameter called  $\gamma$  (e.g. if  $\gamma$  should be  $\geq 1$  with  $\gamma = 1$  being a perfect match). Figure 4 also shows the implementation of the MRAC-  $H_\infty$ 4OP-LPV Controller as a feedback controller.

The input of both controllers must be persistently exciting in order to converge to the desired output value.

## 4. RESULTS

Two different types of faults were simulated: abrupt and gradual. Abrupt faults were simulated with a step function.

Gradual faults were implemented with ramp functions. Abrupt faults in actuators represent a pump stuck and in sensors a constant bias. Gradual fault could be a progressive loss of electrical power in pump, and a drift in sensors.

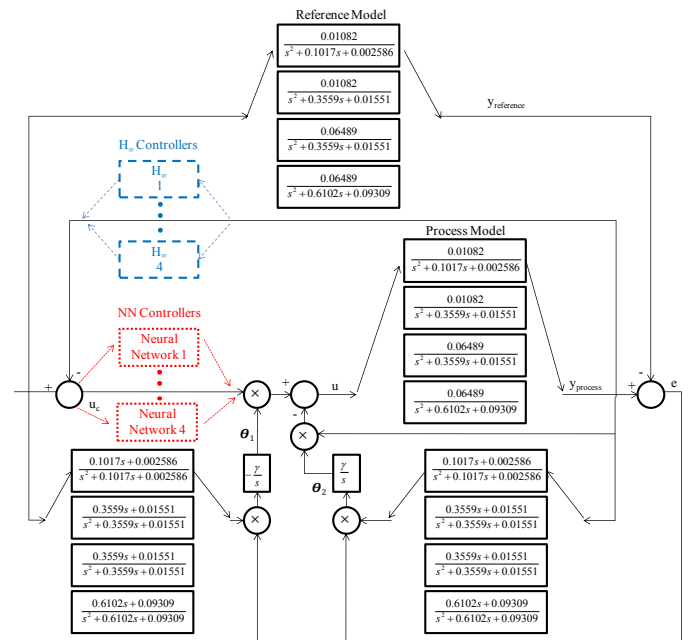


Fig. 4. MRAC+NN+ $H_\infty$  4 Operating Points LPV Controllers.

For each of the three proposed MRAC-4OP-LPV, MRAC-NNOP-LPV, MRAC- $H_\infty$ 4OP-LPV controllers both faults were tested obtaining the results shown if Table 3. These results explain the range in which the methodologies are robust (R), fault tolerant (FT), unstable (U) or degraded system (DS). For example, the MRAC-NNOP-LPV is robust for the four operating points.

Figures 5 and 6 show the simulation results. In these figures the output of the reference model and the process model are plotted. In Figure 5 the MRAC-4OP-LPV, the MRAC-NN4OP-LPV and the MRAC- $H_\infty$ 4OP-LPV controllers are compared. The three controllers are working in an operating point ( $\varphi_1=0.1$  and  $\varphi_2=0.1$ ), the left column figures present an abrupt-sensor fault of magnitude 7 (which means a 70% deviation from nominal value) introduced at time 5,000 s. It can be observed that the MRAC-NN4OP-LPV is the only robust scheme against the fault. On the other hand, in the right column of Figure 5 an abrupt-actuator fault of magnitude 0.25 (2.5 % deviation from nominal value) was introduced at time 5,000 s. It is shown that the MRAC-NN4OP-LPV has a steady oscillation (degraded system) at all times of the simulation.

In Figure 6 the same controllers are tested working in the operating point  $\varphi_1=0.6$  and  $\varphi_2=0.1$ . Left column shows gradual-sensor fault of magnitude 3 (30 % deviation from nominal value) at time 5,000 s. It can be seen that the MRAC-NN4OP-LPV is the only robust scheme. Right column of Figure 6 presents gradual-actuator fault of magnitude 0.25 introduced at time 5,000 s. For this case, the MRAC-4OP-LPV has the smaller deviation of the normal

operating value at the beginning of the fault, but a steady oscillation is observed.

Table 3. Results of the three MRAC schemes in the four Operation Points of the LPV system

Approach	Sensor Faults		Actuator Faults	
	Abrupt	Gradual	Abrupt	Gradual
<b>Process Model 1</b>				
MRAC-4OP-LPV	$f < +/- 4.4 \rightarrow R$	$f < +/- 4.4 \rightarrow R$	$0 < f < 0.25$	$+/- 0 < f < +/- 0.25$
	$+/- 4.4 < f < +/- 7.8$	$+/- 4.4 < f < +/- 7.8$	$\rightarrow FT$	$\rightarrow FT$
MRAC-NN4OP-LPV	$R$	$R$	$f > 0.25 \rightarrow U$	$f > +/- 0.25 \rightarrow U$
MRAC- $H_{\infty}$ 4OP-LPV	$f < 5.1 \rightarrow R$	$f < +/- 5.1 \rightarrow R$	$0 < f < 0.5$	$+/- 0 < f < +/- 0.5$
	$5.1 < f < 16 \rightarrow FT$	$+/- 5.1 < f < +/- 16$	$\rightarrow FT$	$\rightarrow FT$
MRAC-4OP-LPV	$f > 16 \rightarrow U$	$f > +/- 16 \rightarrow U$	$f > 0.5 \rightarrow U$	$f > +/- 0.5 \rightarrow U$
<b>Process Model 2</b>				
MRAC-4OP-LPV	$f < 4.9 \rightarrow R$	$f < +/- 4.9 \rightarrow R$	$0 < f < 0.25$	$+/- 0 < f < +/- 0.25$
	$4.9 < f < 8.2 \rightarrow FT$	$+/- 4.9 < f < +/- 8.2 \rightarrow FT$	$\rightarrow FT$	$\rightarrow FT$
MRAC-NN4OP-LPV	$R$	$R$	$f > 0.25 \rightarrow U$	$f > +/- 0.25 \rightarrow U$
MRAC- $H_{\infty}$ 4OP-LPV	$f < 6.2 \rightarrow R$	$f < +/- 6.2 \rightarrow R$	$0 < f < 0.5$	$+/- 0 < f < +/- 0.5$
	$6.2 < f < 15.7 \rightarrow FT$	$+/- 6.2 < f < +/- 15.7$	$\rightarrow FT$	$\rightarrow FT$
MRAC-4OP-LPV	$f > 15.7 \rightarrow U$	$f > +/- 15.7 \rightarrow U$	$f > 0.5 \rightarrow U$	$f > +/- 0.5 \rightarrow U$
<b>Process Model 3</b>				
MRAC-4OP-LPV	$f < 0.5 \rightarrow R$	$f < +/- 0.5 \rightarrow R$	$0 < f < 0.25$	$+/- 0 < f < +/- 0.25$
	$0.5 < f < 0.7$	$+/- 0.5 < f < +/- 0.7$	$\rightarrow FT$	$\rightarrow FT$
MRAC-NN4OP-LPV	$R$	$R$	$f > 0.25 \rightarrow U$	$f > +/- 0.25 \rightarrow U$
MRAC- $H_{\infty}$ 4OP-LPV	$f < 2.7 \rightarrow R$	$f < +/- 2.7 \rightarrow R$	$0 < f < 0.5$	$+/- 0 < f < +/- 0.5$
	$2.7 < f < 4.9 \rightarrow FT$	$+/- 2.7 < f < +/- 4.9$	$\rightarrow FT$	$\rightarrow FT$
MRAC-4OP-LPV	$f > 4.9 \rightarrow U$	$f > +/- 4.9 \rightarrow U$	$f > 0.5 \rightarrow U$	$f > +/- 0.5 \rightarrow U$
<b>Process Model 4</b>				
MRAC-4OP-LPV	$f < 0.5 \rightarrow R$	$f < +/- 0.5 \rightarrow R$	$0 < f < 0.25$	$+/- 0 < f < +/- 0.25$
	$0.5 < f < 0.7$	$+/- 0.5 < f < +/- 0.7$	$\rightarrow FT$	$\rightarrow FT$
MRAC-NN4OP-LPV	$R$	$R$	$f > 0.25 \rightarrow U$	$f > +/- 0.25 \rightarrow U$
MRAC- $H_{\infty}$ 4OP-LPV	$f < 2.2 \rightarrow R$	$f < +/- 2.2 \rightarrow R$	$0 < f < 0.5$	$+/- 0 < f < +/- 0.5$
	$2.2 < f < 4.8$	$+/- 2.2 < f < +/- 4.8$	$\rightarrow FT$	$\rightarrow FT$
MRAC-4OP-LPV	$f > 4.8 \rightarrow U$	$f > +/- 4.8 \rightarrow U$	$f > 0.5 \rightarrow U$	$f > +/- 0.5 \rightarrow U$

FT = Fault Tolerant, R = Robust, U = Unstable, DS=Degraded System

### 5. COMPARISON WITH SIMILAR APPROACHES

Some other MRAC approaches in the last years have been developed, in the context of FTC systems, but there are several differences from the methods proposed in this work. Cho et al. (1990) proposed a method for FTC systems using a pole assignment controller and a MRAC controller to guarantee the system performance in the presence of a fault, this scheme is different from ours because it did not use any AI method such as ANN. In the case of robust  $H_{\infty}$  controller, although there are some publications where the  $H_{\infty}$  technique has been combined with other schemes (Lian et al., 2002), to the best of our knowledge there are no reports concerning the combination of MRAC with  $H_{\infty}$  for LPV systems. For the MRAC controller for LPV systems or LPV controls just a few studies had been developed (see Mayasato, 2007;

Abdullah and Zribi, 2009), although none of them included any AI techniques.

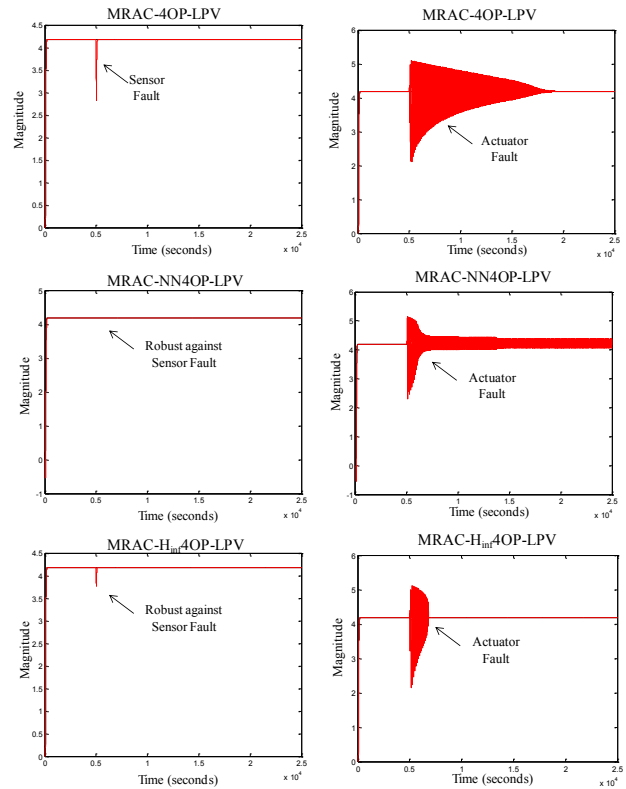


Fig. 5. Comparison between the 3 MRAC schemes with an abrupt-sensor fault of 70% and an abrupt-actuator fault of 2.5%.

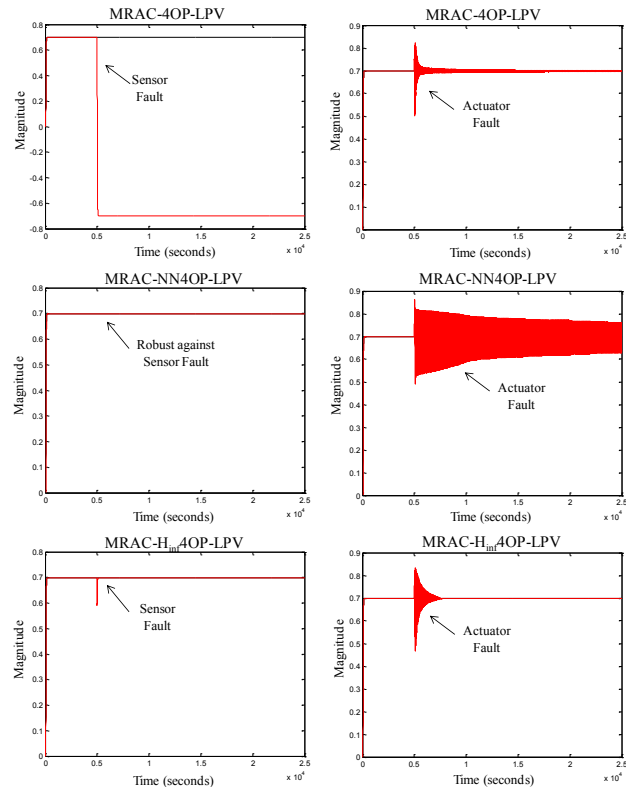


Fig. 6. Comparison between the 3 MRAC schemes with a gradual-sensor fault of 30% and a gradual-actuator fault of 2.5%.

## 6. CONCLUSIONS

For sensor faults, the MRAC-NN4OP-LPV methodology showed the best results because it was fault tolerant against the faults no matter the operation point and the magnitude of the fault. This method resulted the best scheme because is a combination of two type of controllers, one is a Model Reference Adaptive Controller (MRAC) and the other one is an ANN-based controller designed to follow the ideal trajectory (non-faulty trajectory). Both controllers were designed to work in 4 operating points of an LPV system giving them the possibility of control each of these operating points. For the actuator fault, the only schemes that were fault tolerant were the MRAC-H<sub>∞</sub>4OP-LPV and the MRAC-4OP-LPV. The MRAC-H<sub>∞</sub>4OP-LPV and the MRAC-4OP-LPV were able to compensate completely the abrupt and gradual actuator fault approximately after 5,000 and 15,000 s, respectively.

As future work of this article, experiments using the nonlinear model of the system instead of the LPV plant were realized. In these experiments chattering in the plant output was observed. This chattering was reduced using the design of the MRAC controller based on Lyapunov theory because it guarantees the system stability. In addition, experiments using a real physical coupled tank system are being carried out.

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