Preprints of the 18th IFAC World Congress
Milano (Italy) August 28 - September 2, 2011

Sequential probability ratio test using first Laplace distribution for oscillatory fault detection of an hydraulic actuator ⋆

Lamine Fadiga * **Franck Cazaurang** * **Loïc Lavigne** *
Philippe Goupil **

* *Université Bordeaux 1, IMS, Automatic control group, 33405 Talence, FRANCE (Tel:+33-5-40002414; e-mail: firstname.lastname@ims-bordeaux.fr).*
** *Airbus Operations S.A.S, Flight control system, 31060 TOULOUSE CEDEX 09, FRANCE (Tel:+33-5-61183803).*

Abstract: This paper proposes a sequential test for fault detection in the case of Laplace distribution of residual. This approach is adapted from Wald's sequential test and the paper establishes specific test thresholds for Laplace distributions. The suggested algorithm is applied to the detection of oscillatory failure cases on an Airbus A380 elevator actuator characterized by Laplacian distribution of residual. Moreover the proposed application shows that computing workload resulting from the proposed sequential test is similar to the classical Wald test.

1. INTRODUCTION

Hydraulic actuators have many applications in aeronautic such as moving aircraft flight control surfaces. They always need to operate flawlessly in order to keep the highest flight safety level and to not reduce the aircraft control capability. For this purpose powerful fault detection algorithms are necessary. They always need to be improved so that they can correctly monitor actuators always more efficient in the frame of the aircraft design global optimization.

The residuals used to monitor the actuators can be obtained by model-based methods as in Isermann and Ballé [1997], Ding [2008] or Garcia et al. [2009] but they do not give enough information concerning failures on their own. That is why model-based diagnostic methods are often coupled with powerful and reliable statistical fault detection algorithms as shown in Khan et al. [2005] or Ding [2008].

Among statistical diagnostic algorithms, sequential probability ratio tests are the most common since they provide a real-time appraise of the functioning of complex systems (Basseville and Nikiforov [1993]). Wald's sequential test is a well-known case of sequential probability ratio test. Studies have already been conducted concerning its performances in monitoring electro-hydraulic servo-positioning systems by Khan et al. [2005]. Coupling a Wald's sequential test with an observer-based residual, Khan et al. [2005] effectively detect faults due to incorrect supply pressures and sensor faults on the plant. The use of Wald's sequential test was justified by saving of a great number of observations over the most efficient test procedures based on fixed numbers of observations.

Most sequential diagnostic tests are based on a Gaussian distribution since it is the form commonly taken by residuals (Shi et al. [2005]). Wald's sequential test is not an exception to that rule. This test allows to detect early low failure amplitude for Gaussian residuals. However, other distributions may fit residuals better than a normal distribution. A good criterion should be used to evaluate the suitability of the distribution used to fit residuals since the performances of the statistical test will depend on that distribution as proposed by Basseville and Nikiforov [1993]. For example Lilliefors test was used in Shi et al. [2005] to prove that the residual is Gaussian without knowing the parameters of the distribution. Kullback-Liebler distance, used in Kárný [1996] is a good measure of the divergence between two distributions since it takes into account the way values are distributed all over the data set. Yet, as underlined in Shi et al. [2005], a too complicated probability distribution may not be useful for a statistical test since the determination of the decision thresholds may not be implementable due to a considerable computational workload.

In this paper, the residual evaluation step for the detection of oscillatory failure cases of an Airbus A380 aircraft elevator hydraulic actuator is rethought. A novel sequential probability ratio test for Laplace distribution is developed, based on Wald's sequential test. A real flight test data set is used to validate the new method. The determination of the test decision thresholds is also explained.

The paper is divided in 5 sections. The second section presents the model of the hydraulic actuator and some generalities about oscillatory failure cases. The third section is dedicated to the general theory of Wald's sequential test and its adaptation to first Laplace distribution. In the fourth section, this result is applied to the detection of oscillatory failure cases in order to illustrate the performances of the test. The last part presents conclusions and perspectives.

⋆ This work was supported in part by IMS Laboratory in the frame of L. FADIGA Master thesis

2. MODELLING OF THE HYDRAULIC ACTUATOR AND OSCILLATORY FAILURE CASES

2.1 Actuator model

The non linear model of the hydraulic actuator is based on its physical behaviour. The corresponding equation gives the actuator rod speed as a function of the hydraulic pressure delivered to the actuator and the forces applied on the control surface and reacted by the actuator. The two main contributors are aerodynamics forces and the servo control load in damping mode of the passive actuator in the case of two actuators. The actuator rod speed (for a hydraulic servo control) can be expressed by the following deterministic state-space model (cf. Goupil [2010]):

$$\begin{cases} \dot{x}(t) = V_0(t)\left(\dfrac{\Delta P(t) - \frac{F_{aero}+F_{damping}}{S}}{\Delta P_{ref}}\right)^{\frac{1}{2}} \\ y(t) = x(t) \end{cases} \tag{1}$$

where y represents the actuator rod position, V_0 is the actuator rod speed computed by the Flight Control Computer (FCC), corresponding to the maximum speed of one actuator alone with no load. S is the actuator piston surface area, ΔP is the hydraulic supply pressure really delivered to the actuator and ΔP_{ref} is a reference differential pressure linked to the maximum rod speed performances. F_{aero} represents the aerodynamic forces applied on the control surface. $F_{damping}$ represents the servo control load of the adjacent actuator in damping mode and is defined by:

$$F_{damping} = K_a \dot{x}^2(t), \tag{2}$$

where K_a is the actuator damping coefficient.

With $V_0(t) = K_{ci}K\,(u(t) - x(t))$, equation 1 can also be written as the following:

$$\dot{x}(t) = K_{ci}K\,(u(t)-x(t))\left(\dfrac{\Delta P(t) - \frac{F_{aero}}{S}}{\Delta P_{ref} + \frac{K_a(K_{ci}K(u(t)-x(t)))^2}{S}}\right)^{\frac{1}{2}} \tag{3}$$

where K is the servo control gain and u is the real actuator position command. Saturations of various types (actuators limit positions, maximum orders) are taken into account in the different varying gains (K_{ci}).

This mathematical model of an hydraulic actuator is the one currently used for model-based fault detection on Airbus A380 aircraft (figure 1).

2.2 Oscillatory Failure Cases

Oscillatory Failure Cases (OFC) result in an unwanted control surface oscillation, leading to strong interactions with loads and aero-elasticity when located within actuator bandwidth, see Goupil [2010] and Lavigne et al. [2007] for further information. Consequently, they must be detected in time. Figure 2 shows how the hydraulic actuator is controlled by the FCC and the different places where OFC may appear in the control loop. The residual of the actuator is computed by substracting the measured value provided by the control surface sensor from the estimated control surface position as shown in figure 1.

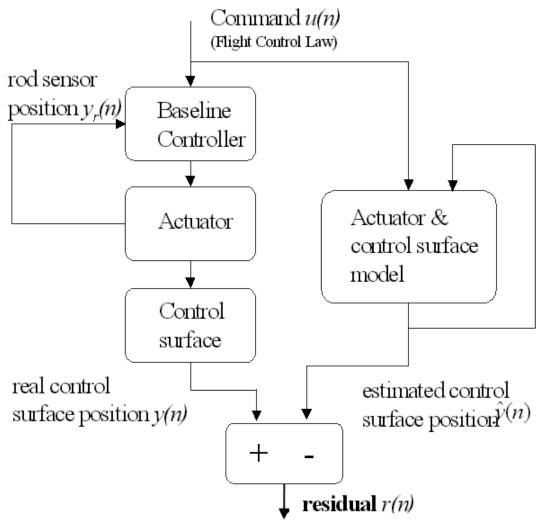

Fig. 1. Principle of model-based diagnostic of the hydraulic actuator.

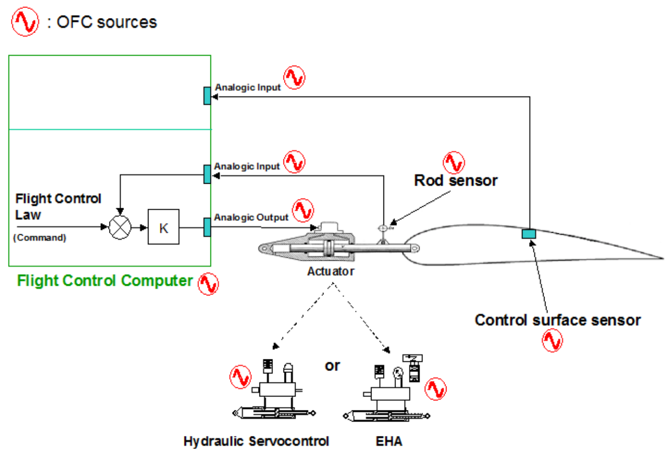

Fig. 2. OFC sources localization in the servo-loop control of a control surface.

OFC is said to be liquid when it results in a sinusoidal signal added to the nominal servo-loop signal or solid when the sinusoidal signal replaces the nominal signal. Early and robust detection of OFC is very important because it has an impact on the structural design of the aircraft. Goupil [2010] demonstrated that the model-based approach implemented in the A380 permits stringent requirements to be met with low computational cost. This solution is currently used on in-service Airbus A380 to ensure OFC detection, providing a complete coverage of such events. However, for upcoming and future aircraft, it could be required to detect OFC with less important amplitude in less time while keeping a good robustness. The following sequential probability ratio test could help to achieve this goal when used as a residual evaluation technique instead of the current one (Goupil [2010]).

3. SEQUENTIAL TESTS FOR FAULT DETECTION

3.1 Wald's sequential test

This diagnostic algorithm is generally used when the residual of the plant follows a Gaussian distribution. Non-sequential diagnostic tests are dependent on the number of observations in the sample which is being tested. In practice, a moving window is used on the residual. The width of that window is carefully determined in order to maximize the performances of the algorithm. For Wald's sequential test, the width of the sample is not specified. A decision is made as soon as there are enough observations in the sample so that error probabilities are inferior to set values. These values are the non detection and false alarms probabilities, respectively P_{ND} and P_F. Further information can be found in Basseville and Nikiforov [1993].

Let H_0 be the hypothesis of normal functioning of the actuator and H_1 be the hypothesis of a faulty functioning. The probability density functions associated with H_0 and H_1 have been determined and are noted $p(x_k|H_0)$ and $p(x_k|H_1)$ where x_k is the observation at the instant k. Let A be the decision threshold corresponding to the selection of the hypothesis H_0 and B the threshold for H_1. The likelihood ratio for Wald's sequential test is given by :

$$\lambda_k = \frac{p(x_1, x_2, ..., x_k|H_1)}{p(x_1, x_2, ..., x_k|H_0)} \tag{4}$$

The threshold is determined by considering the extreme case when the likelihood ratio is equal to B :

$$p(x_1, x_2, ..., x_k|H_1) = B \cdot p(x_1, x_2, ..., x_k|H_0)$$

$$\int_B^\infty p(x_1, x_2,\ ..., x_k|H_1) dx_1...dx_k = B \cdot \int_B^\infty p(x_1, x_2, ..., x_k|H_0) dx_1...dx_k$$

$$\int_B^\infty p(x_1, x_2, ..., x_k|H_1) dx_1...dx_k = 1 - P_{ND}$$
$$\int_B^\infty p(x_1, x_2, ..., x_k|H_0) dx_1...dx_k = P_F$$

Hence :

$$B = \frac{1 - P_{ND}}{P_F}. \tag{5}$$

A can be found by following the same reasoning :

$$A = \frac{P_{ND}}{1 - P_F}. \tag{6}$$

The resulting diagnostic rule is :
- H_0 is chosen when $\lambda_k \leq A$,
- H_1 is chosen when $\lambda_k \geq B$,
- No decision can be made when $A \leq \lambda_k \leq B$. The previous decision is kept.

The next part will show that this approach can be extended to first Laplace distribution.

3.2 Wald's sequential test for first Laplace distribution residuals

The probability density function of the first Laplace distribution for a sample x is given by:

$$f(x) = \frac{1}{2 \cdot b} \exp\left(-\frac{|x - \mu|}{b}\right) \tag{7}$$

where μ is the mean of the data set and b is a scale parameter related to its variance σ^2 by:

$$\sigma^2 = 2 \cdot b^2. \tag{8}$$

The parameter b can be estimated using maximum likelihood method:

$$b = \frac{1}{N} \sum_{i=1}^{N} |x_i - \mu|. \tag{9}$$

where N is the number of observations.

A novel Wald's sequential test is calculated using the first Laplace distribution. Let $x = x_1, ..., x_N$ be a data set following the first Laplace distribution whose mean is μ_0 and variance related scale parameter b_0. x is supposed to be the residual in normal functioning (H_0) of an hydraulic actuator. The corresponding probability density function is:

$$p\left(x_i|H_0\right) = \frac{1}{2 \cdot b_0} \exp\left(-\frac{|x_i - \mu_0|}{b_0}\right). \tag{10}$$

In case of failure we assume that the residual can be fitted by the following first Laplace distribution defined by μ_1 and b_1 parameters:

$$p\left(x_i|H_1\right) = \frac{1}{2 \cdot b_1} \exp\left(-\frac{|x_i - \mu_1|}{b_1}\right). \tag{11}$$

Proposition 1. Assuming that the residual can be fitted by the above first Laplace distributions, then the variance based test decision relation is given by:

$$\ln A - k \ln\left(\frac{b_0}{b_1}\right) \leq \sum_{i=1}^{k} \left(-\frac{|x_i - \mu_1|}{b_1} + \frac{|x_i - \mu_0|}{b_0}\right) \leq \ln B - k \ln\left(\frac{b_0}{b_1}\right). \tag{12}$$

Proof.

The likelihood ratio for sample k is given by:

$$\lambda_k = \frac{p(x_1, x_2, ..., x_k|H_1)}{p(x_1, x_2, ..., x_k|H_0)} \tag{13}$$

Supposing that the $x_i, i = 1 \ldots k$ are independent of each other, λ_k can be rewritten as the following:

$$\lambda_k = \frac{p(x_1|H_1)p(x_2|H_1)...p(x_k|H_1)}{p(x_1|H_0)p(x_2|H_0)...p(x_k|H_0)} \quad (14)$$

$$\lambda_k = \prod_{i=1}^{k} \frac{\frac{1}{2 \cdot b_1} \exp\left(-\frac{|x_i-\mu_1|}{b_1}\right)}{\frac{1}{2 \cdot b_0} \exp\left(-\frac{|x_i-\mu_0|}{b_0}\right)} \quad (15)$$

$$\lambda_k = \left(\frac{b_0}{b_1}\right)^k \prod_{i=1}^{k} \exp\left(-\frac{|x_i-\mu_1|}{b_1} + \frac{|x_i-\mu_0|}{b_0}\right) \quad (16)$$

$$\lambda_k = \left(\frac{b_0}{b_1}\right)^k \exp\left[\sum_{i=1}^{k} \left(-\frac{|x_i-\mu_1|}{b_1} + \frac{|x_i-\mu_0|}{b_0}\right)\right] \quad (17)$$

then thresholds are evaluated by taking the natural logarithm of the above expression in the inequality $\ln A \leq \ln \lambda_k \leq \ln B$.

□

The previous test is used in the next section to detect liquid OFC in an hydraulic actuator of an aircraft elevator.

4. APPLICATION

The test developed in the previous section is now applied to the residual of an elevator actuator from an experimental data set provided by an Airbus A380 FCC during dynamic flight operations as shown in figure 3 representing the position of the elevator.

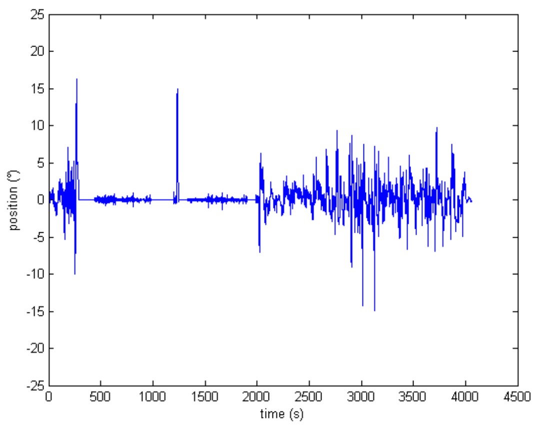

Fig. 3. Position of the elevator.

Figures 4 and 5 present respectively the residuals in normal conditions and a zoom in the residual with a liquid OFC with an amplitude of 0.5° and a frequency of 3Hz injected at time 500s.

The first step in applying the sequential test is determining the distribution followed by the residual in normal conditions. Kullback-Leibler distance will enable the choice between normal distribution and Laplace distribution.

Given two probability distributions P_1 and P_2, Kullback-Leibler distance also called relative entropy measures the

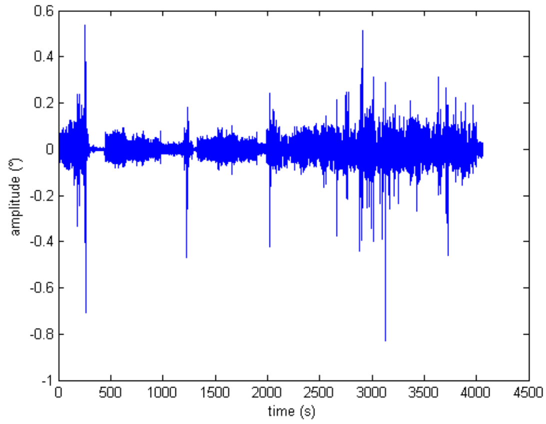

Fig. 4. Residual of the actuator without failure.

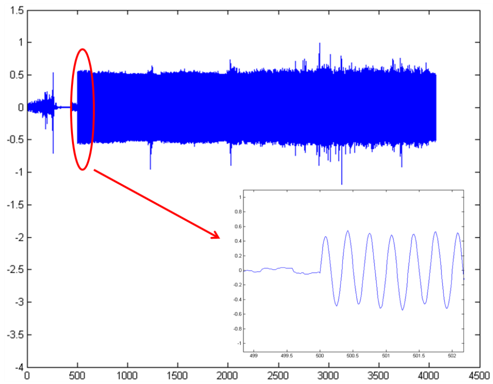

Fig. 5. Residual of the actuator with 0.5° of amplitude and 3Hz of frequency OFC inserted at time 500s.

dissimilarity between those distributions. Kullback-Leibler distance $D_{KL}(P_1, P_2)$ of P_2 in relation to P_1 is given by:

$$D_{KL}(P_1, P_2) = \sum_i P_1(i) \log \frac{P_1(i)}{P_2(i)}. \quad (18)$$

The less $D_{KL}(P_1, P_2)$ is, the more P_2 is close to P_1.

In practice, after determining the parameters for the Gaussian and Laplacian distributions, their Kullback-Leibler distances in relation to the residual of the hydraulic actuator are calculated. The distances for Gaussian distribution and Laplace distribution are given in table 1. The residual is nearly three times closer to Laplace distribution than to the Gaussian one as shown in figure 6.

	Gaussian	Laplace
Kullback-Leibler distances	0.9965	0.3632

Table 1. Kullback-Leibler distances of normal and Laplace distributions in relation to the residual.

Figure 7 presents the distribution of the residual with a 0.5° of amplitude failure after a long enough period. The main peaks are now located on the amplitude of the failure.

Results given by a variance-based Gaussian Wald's sequential test and the new Laplacian sequential probability ratio test for the detection of liquid OFC are now compared.

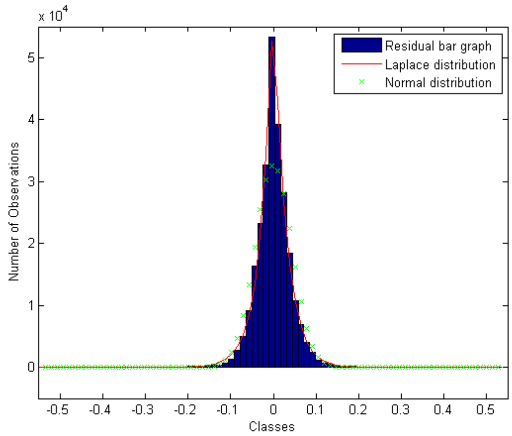

Fig. 6. Residual bar graph and associated Laplace and Normal distributions.

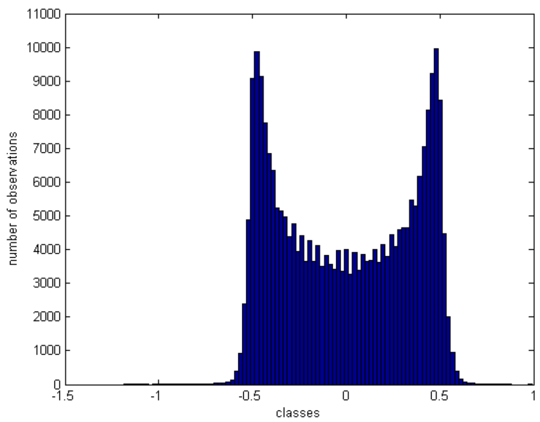

Fig. 7. Residual bar graph with a 0.5° of amplitude OFC.

1^{st} **case**: Gaussian

For a Gaussian distribution $\aleph(\mu, \sigma^2)$ with mean μ and variance σ^2, Wald's inequality on the variance is:

$$2 \cdot \frac{\ln(A) + \frac{k}{2} \ln\left(\frac{\sigma_1^2}{\sigma_0^2}\right)}{\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}} \leq \sum_{i=1}^{k} (x_i - \mu)^2 \leq 2 \cdot \frac{\ln(B) + \frac{k}{2} \ln\left(\frac{\sigma_1^2}{\sigma_0^2}\right)}{\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}} \tag{19}$$

where x_i is the i-th observation, σ_0^2 is the variance limiting the area without failure and σ_1^2 is the variance due to the failure. These values are experimentally set to $\sigma_0^2 = (3.6\sigma)^2$ and $\sigma_1^2 = (3.7\sigma)^2$ in order to minimize detection times.

2^{nd} **case**: Laplace

Relation (12) is taken with the following values for the determination of the thresholds: $b_0 = 7b$ and $b_1 = 8b$ where b is the scale parameter obtained when the residual was fitted by first Laplace distribution. μ_1 enables to chose the minimum amplitude of the failure the user wants to detect. Indeed, when the OFC appears for a long enough period the shape of the residual probability density is given in figure 8. Now two peaks which can be considered as two Laplace distributions located at plus and minus the

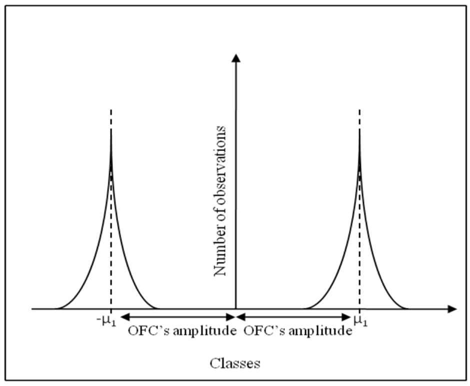

Fig. 8. Shape of the faulty residual probability density after a long period.

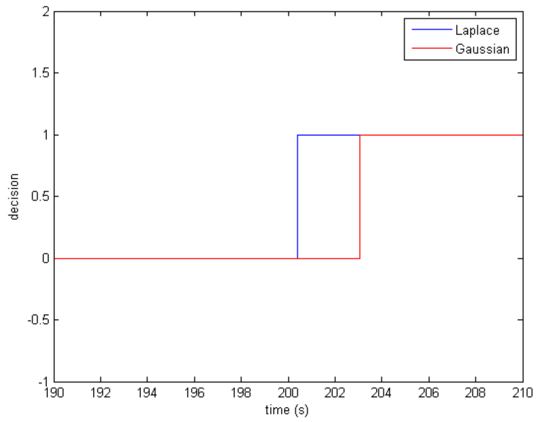

Fig. 9. Tests decision with 0.5° of amplitude and 3Hz of frequency OFC inserted at time 200s.

amplitude of the OFC overshadow the initial distribution centered around 0. from a practical point of view, one can observe that if μ_1 is set at the position of the peak on the right, failures with an amplitude superior or equal to μ_1 are detected more quickly. For this application μ_1 is set to 0.5°.

The following tables show simulation results for 0.5° of amplitude and variable frequency liquid OFC injected at $200s$ where the command signal is dynamic on one hand and faults with 1° of amplitude and variable frequencies on the other hand, with $\mu = 0.5°$.

Error signal freq. (Hz)	0.1	1	2	3	4	5
Gaussian	3.23	4.23	4.11	4.09	4.08	4.06
Laplace	0.86	0.28	0.42	0.44	0.45	0.47
Error signal freq. (Hz)	6	7	8	9	10	
Gaussian	4.08	4.11	4.11	4.09	4.05	
Laplace	0.40	0.48	0.55	0.48	0.45	

Table 2. Detection times (s) for a liquid failure with 0.5° of amplitude and variable frequency with $\mu_1 = 0.5°$

The new sequential probability ratio test with first Laplace distribution gives better results than Gaussian Wald's sequential test. Detection times are divided by 4 at the minimum for a 1° of amplitude OFC and by 9 for a 0.5° of amplitude OFC. The use of Laplace distribution enables

Error signal freq. (Hz)	0.1	1	2	3	4	5
Gaussian	1.62	0.81	0.86	0.8	0.84	0.86
Laplace	0.59	0.20	0.17	0.16	0.19	0.17
Error signal freq. (Hz)	6	7	8	9	10	
Gaussian	0.87	0.84	0.86	0.83	0.84	
Laplace	0.19	0.19	0.22	0.17	0.19	

Table 3. Detection times (s) for a liquid failure with 1° of amplitude and variable frequency with $\mu_1 = 0.5°$

to greatly improve the test. Moreover the test does not depend on the frequency of the OFC since detection times are nearly the same all over the frequency range apart from $0.1Hz$.

5. CONCLUSION

A novel sequential test for first Laplace distribution based on Wald's sequential test was developed in this paper. The test was applied to an Airbus A380 aircraft elevator hydraulic actuator. Results showed that variable amplitude and frequency oscillatory liquid faults were correctly detected with detection times always better than those of Wald's Gaussian sequential test. The fact that the minimum detectable amplitude can be added to the test parameters greatly improved its performances for amplitude superior or equal to that minimum. Moreover from an industrial point of view the test is easy to tune as the required detection level is a clear input of the algorithm. A test campaign not presented in this paper will enable to verify the performances of the algorithm with residuals in various flight conditions and for solid OFC.

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