Decentralized Control of Multi-robot Systems for Rectangular Aggregation ⋆

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Abstract: This paper addresses a problem of motion formation control of a network of self-deployed autonomous robots. We propose decentralized motion coordination algorithms for the robots so that they collectively move in a rectangular lattice pattern from any initial position. There are no predefined leaders in the group and only local information is required for the algorithms. The algorithms are developed using the ideas of consensus, and their effectiveness is illustrated via numerical simulations.

Keywords: Agents and autonomous systems; autonomous unmanned vehicles; robot deployment; decentralized control; mobile robots; robot flocking with a desired pattern

1. INTRODUCTION

The study of decentralized control laws for groups of autonomous agents has emerged as a challenging research area recently (see e.g.: Jadbabaie et al. (2003); Ren and Beard (2008); Olfati-Saber et al. (2007); Bullo et al. (2009)). In this control framework, the motion of each robot is coordinated using local information such as coordinates or velocities of several other robots that are closest neighbors of the robot at a given time. One approach to developing these local motion is by the inspiration from the animal aggregations, such as schools of fish, flocks of birds or swarms of bees, that are believed to use simple, local motion coordination rules at the individual level (e.g.: Flierl et al. (1999); Shaw (1962); Okubo (1986)). To simulate these behaviors, Vicsek et al. (1995) proposed a simple discrete-time model of a system of several autonomous agents and each agent's motion is updated using a local rule based on its own state and the state of its "neighbors." This simple but interesting model was then analyzed by a number of researchers (see e.g.: Jadbabaie et al. (2003); Savkin (2004); Yu and Wang (2008); Ren and Beard (2008)).

This type of local coordination rule leads to the so-called consensus or agreement scheme. By using this approach a group of agents can be coordinated to achieve a specific formation or geometric structure (see e.g.: Ren and Beard (2008); Olfati-Saber et al. (2007); Cheng and Savkin (2009, 2011)). The advantages of this consensus approach as compared to the traditional leader-follower approach, in e.g. Wang (1991), are that a leader-follower scheme requires a predefined leader and also there is no explicit feedback to the leader from the formation. As a result, the leader may walk away and leave its followers behind. Another approach that is widely adopted is the artificial potential function approach, see e.g. Leonard and Fiorelli (2001). As its name suggests, this approach is based on some potential functions and these functions represent and realize the inter-agent interactions and/or the interactions with the environment. An advantage of this approach is that it naturally leads to a distributed control law and is relatively simple. However, in

general, a potential function approach may generate many local minima, leading to only local results and the pattern that the group forms into cannot be guaranteed.

By taking the consensus approach, the objective of this paper is to develop a decentralized or distributive control strategy for a group of robots so that they form into a rectangular formation and collectively move in this formation from any initial position. In our problem, there are no leaders assigned *a priori*, and the robots have to coordinate with each other in the group relying on some global consensus to achieve and maintain a rectangular pattern. Apart from rectangular patterns, our control strategy can be easily modified to achieve other geometric patterns like triangular or diamond patterns. A potential application of our formation control of a group of robots is for sweep coverage (Choset (2001)) in operations like minesweeping (Cassinis et al. (1999)), boarder patrolling (Kumar et al. (2007)), environmental monitoring of disposal sites on the deep ocean floor (Jeremić and Nehorai (1998)), and sea floor surveying for hydrocarbon exploration (Børhaug et al. (2007)).

The rest of the paper is organized as follows. In Section 2, we formulate the problem of decentralized formation control of a network of mobile robots. An algorithm to address the formation control problem is presented in Section 3. Section 4 presents some simulation results to illustrate the proposed algorithm. The detailed proof of the results will be given in the full version of this paper.

2. PROBLEM FORMULATION

In our decentralized formation control problem, the objective is to coordinate a group of mobile robots so that they collectively move into a rectangular lattice pattern from any initial deployment. The coordinating algorithm is decentralized that the robots can only access local information. We consider a multi-robot system consisting of n robots labeled 1 through n. Let $V_i(\cdot)$ and $\Theta_i(\cdot)$ be the linear velocity and heading of the robot i, respectively. Let $l(s) = [\cos(s) \quad \sin(s)]^T$ be a unit vector with a given angle $s \in \mathbb{R}$ measured with respect to x-axis and let $p_i = [x_i, y_i]^T$ be the position of the robot i. For

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a given sampling period $T > 0$, the discrete-time kinematic equations of the robots are given by:

$$p_i((k+1)T) = p_i(kT) + V_i(kT)l(\Theta_i(kT))T, \qquad (1)$$

for $i = 1, 2, \ldots, n$, and $k = 0, 1, 2, \ldots$. The velocity V_i satisfies $|V_i(t)| \leq v_{\max}$ for $i = 1, 2, \ldots, n$ and all $t \geq 0$. The initial headings satisfy $\Theta_i(0) \in [0, \pi)$ for all $i = 1, 2, \ldots, n$, and the initial positions of the vehicles are in a bounded set $\mathcal{B} \subset \mathbb{R}^2$ with Lebesgue measure.

Each mobile sensor may have a restricted communication and detection capabilities to reduce the cost of operation, and it has a limited information about other sensors in the group. Therefore, the control law of each mobile senor should be decentralized in the sense that the movement of each sensor relies on the information of its neighbors, e.g., positions, headings, and coordination variables. For instance, the locations of neighboring sensors can be estimated using recursive state estimation methods; see, e.g., Pathirana et al. (2005, 2004). The issue of estimation based on limited measurements was studied in details in Savkin (2006); Matveev and Savkin (2009); Savkin and Cheng (2007).

Robot i, $i = 1, 2, \ldots, n$, has the ability to communicate with another robot in a disk of radius r defined by

$$D_{i,r}(kT) := \{p \in \mathbb{R}^2 : \|p - p_i(kT)\| \leq r\},$$

where $\|\cdot\|$ denotes the Euclidean norm. Let $\mathcal{N}_i(kT)$ be the set of all robots $j, j \neq i$ that at time $t = kT$ belong to the disk $D_{i,r}(kT)$ and $|\mathcal{N}_i(kT)|$ be the number of elements in $\mathcal{N}_i(kT)$. We describe the robot i has $|\mathcal{N}_i(kT)|$ number of *neighbors* at time kT. Let $\mathcal{P}$ be the collection of simple undirected graphs defined on n vertices, representing robots $1, 2, \ldots, n$. For any time $kT \geq 0$, the relationship between neighbors are described by a simple undirected graph $G(kT) \in \mathcal{P}$ with vertex set $\{1, 2, \ldots, n\}$ where i corresponds to the vehicle i. The vertices i and j of the graph, where $i \neq j$, are connected by an edge if and only if the vehicles i and j are neighbors at time kT. To study our problem, we impose the following assumption on $G(kT)$.

Assumption 2.1. The graph $G(kT) \in \mathcal{P}$ is connected for all $k \geq 0$.

For given integers $\gamma, \bar{K} \in \{1, 2, \ldots, n\}$ and scalars $\bar{\psi}, \bar{\phi} \in [0, \pi)$, and $s_1, s_2 > 0$, we define a number points $h_{i,j}(kT)$ relative to robot γ at time kT as shown in Figure 1. The set of locations $\{h_{i,j}(kT)\}$ are defined as follows:

$$\begin{aligned} h_{i,j}(kT) = p_\gamma(kT) &+ s_2(i-1)l(\bar{\phi}) \\ &+ s_1(j-1)l(\bar{\phi} - \pi/2) \end{aligned} \qquad (2)$$

for $i = 1, 2, \ldots, \lfloor n/\bar{K} \rfloor$ and $j = 1, 2, \ldots, \bar{K}$; and for $i = \lceil n/\bar{K} \rceil$ and $j = 1, 2, \ldots, n - \lfloor n/\bar{K} \rfloor \bar{K}$. The $h_{i,j}(kT)$ positions are relative to the position of robot γ. In fact, the integer $\gamma \in \{1, 2, \ldots, n\}$ is not specified at the initial deployment and any robot can eventually take up the $h_{1,1}(kT)$ position. Similarly, the integer $\bar{K}$ and scalar $\bar{\phi}$ are also unspecified at the initial deployment. Thus, the dimensions and the orientation of the rectangular lattice are unknown *a priori*. Moreover, the heading $\bar{\psi}$ of the rectangular lattice is also not specified at the initial deployment. However, the desired speed that the group of robots should move is known to all the robots. We let v_0 be such a desired speed that the group of robots moves.

Definition 2.1. Given n autonomous robots, a set of decentralized control algorithms is said to be a rectangular formation control for the robots if for almost all initial robot po-
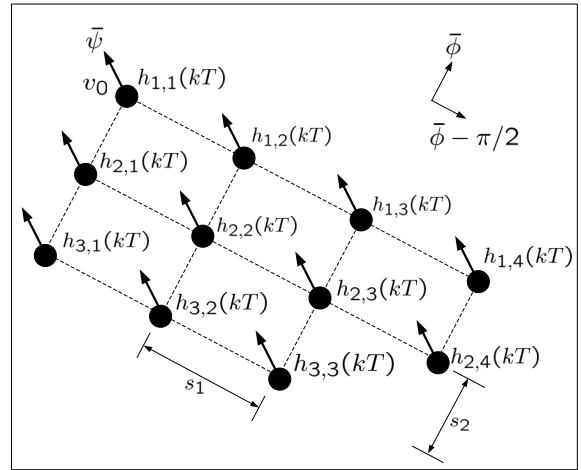

Fig. 1. A group of robots moving in the rectangular formation in the direction $\bar{\phi}$ with speed v_0 ($n = 11$, $\bar{K} = 4$).

sitions, there exist a robot $\gamma \in \{1, 2, \ldots, n\}$, integer $\bar{K} \in \{1, 2, \ldots, n\}$, and scalars $\bar{\phi}, \bar{\psi} \in [0, \pi)$; and for each $h_{i,j}(kT)$ location, there exists a unique index $z_{i,j} \in \{1, 2, \ldots, n\}$ such that the following condition holds:

$$\lim_{k \to \infty} \|p_{z_{i,j}}(kT) - h_{i,j}(kT)\| = 0. \qquad (3)$$

In Definition 2.1, almost all means for all initial conditions except for a set of initial conditions that has Lebesgue measure (area) zero.

3. ALGORITHM

In this section, a control algorithm will be presented for the coordination of a group of moving robots to achieve a rectangular formation. In brief, the algorithm consists of two stages. During the first stage, the robots coordinate and align themselves into a line formation. Once the robots are aligned and each has been assigned an identity (ID) based on this alignment, they start forming $\lceil n/\bar{K} \rceil$ number of parallel lines. The distance between these lines is s_2. Depending on $\bar{K}$ and n, the last row of robots may not form a complete line as shown in Fig. 1.

3.1 First Stage

In the following, a set of decentralized control laws will be proposed for the robots to achieve a line formation during the first stage. Since the control laws for the robots are distributed or decentralized, they rely on the local information of each robot. Information such as locations and some coordination variables of a robot's neighbors is available to the robot. The coordination variables are mainly for coordinating the motion of a robot with other robots in the group.

First, for robot i, we introduce the coordination variable $K_i(kT)$ that takes a value in the discrete set $\{1, 2, \ldots, n\}$. The initial value of this variable satisfies $K_i(0) \in \{1, 2, \ldots, n\}$. This coordination variable will characterize the dimensions of the rectangular lattice that the robots will form. In particular, the number of robots along one side of a rectangular pattern. At any time $k = 1, 2, 3, \ldots$, robot i, $i = 1, 2, \ldots, n$, updates $K_i(kT)$ using the following "nearest neighbor rule": the average $\mathcal{A}_i(t)$ of $K_i(kT)$ is defined as

$$\mathcal{A}_i(kT) :=$$

$$\frac{1}{1 + |\mathcal{N}_i(kT)|} \left(K_i(kT) + \sum_{j \in \mathcal{N}_i(kT)} K_j(kT) \right). \quad (4)$$

It is clear that $\mathcal{A}_i(kT) \in [1, n]$. Now we define $K_i((k+1)T)$ of robot i as

$$K_i((k+1)T) := \lfloor \mathcal{A}_i(kT) \rfloor. \quad (5)$$

Next we introduce the coordination variable $\psi_i(kT)$ for robot i. This variable will be used to define the heading of the rectangular formation. The coordination variable $\psi_i(kT)$ has initial value $\psi_i(0) \in [0, \pi)$ and $\psi_i(0) = \Theta_i(0)$. The variable $\psi_i(\cdot)$ is updated as follows:

$$\psi_i((k+1)T) :=$$

$$\frac{1}{1 + |\mathcal{N}_i(kT)|} \left(\psi_i(kT) + \sum_{j \in \mathcal{N}_i(kT)} \psi_j(kT) \right). \quad (6)$$

In addition, we introduce the variable $\xi_i(kT) \in \{1, 2, 3 \ldots\}$ with $\xi_i(0) = 1$. This variable characterizes the row in the formation that robot i belongs to at time kT. During the first stage, $\xi_i(\cdot) \equiv 1$. At time kT, we let $\mathcal{S}_i(kT) = \{j \in \mathcal{N}_j(kT) : \xi_j(kT) = \xi_i(kT)\}$. The set $\mathcal{S}_i(kT)$ contains the neighbors of robot i that belong to the same row as robot i. Similar to $\psi(kT)$, another coordination variable $\phi_i(kT)$ for $i = 1, 2, \ldots, n$ is introduced and it will define the orientation of the rectangular formation. The coordination variable $\phi_i(kT)$ has initial value $\phi_i(0) \in [0, \pi)$, and we define:

$$\mathcal{H}_i(kT) :=$$

$$\frac{1}{1 + |\mathcal{S}_i(kT)|} \left(\phi_i(kT) + \sum_{j \in \mathcal{S}_i(kT)} \phi_j(kT) \right), \quad (7)$$

for $i = 1, 2, \ldots, n$, where $|\mathcal{S}_i(kT)|$ denotes the number of elements in the set $\mathcal{S}_i(kT)$. The coordination variable $\phi_i(kT)$ is then updated by

$$\phi_i((k+1)T) = \mathcal{H}_i(kT). \quad (8)$$

In contrast to $K_i((k+1)T)$, $\psi_i((k+1)T)$ and $\phi_i((k+1)T)$ will take any value in the interval $[0, \pi)$ rather than from a discrete set. Using $\phi_i(kT)$, we define a variable that is the projection of $p_j(kT)$ (i.e., the position of robot j) in the direction $\phi_i(kT)$ as $c_{i,j}(kT) = l^T(\phi_i(kT))p_j(kT)$, for $j \in \mathcal{S}_i(kT) \cup \{i\}$.

Similar to (7), we define the average of $c_{i,j}(\cdot)$, $j \in \mathcal{S}_i(kT) \cup \{i\}$ for robot i as follows:

$$\mathcal{M}_i(kT) :=$$

$$\frac{1}{1 + |\mathcal{S}_i(kT)|} \left(c_{i,i}(kT) + \sum_{j \in \mathcal{S}_i(kT)} c_{i,j}(kT) \right), \quad (9)$$

for $k = 1, 2, \ldots$. For each robot i, we introduce the coordination variable $\mathcal{F}_i(kT) = c_{i,i}(kT)$. Using $\phi_i(kT)$ and $\mathcal{F}_i(kT)$, a line $\mathcal{L}_i(kT)$ that robot i belongs to is defined as:

$$\mathcal{L}_i(kT) = \{p \in \mathbb{R}^2 : l^T(\phi_i(kT))p = \mathcal{F}_i(kT)\} \quad (10)$$

for $i = 1, 2, \ldots, n$. This line is instrumental in determining the control for robot i. To develop the control action along $\mathcal{L}_i(kT)$, we let $q_j^i(kT)$ be the projection of the position of robot $j \in \mathcal{S}_i(kT) \cup \{i\}$ on the line $\mathcal{L}_i(kT)$ at time kT, and it is given by $q_j^i(kT) = l^T(\phi_i(kT) - \pi/2)p_j(kT)$. Using this, we define robots $\alpha, \beta \in \mathcal{S}_i(kT)$, provided that if they exist, such that

$$q_\alpha^i(kT) < q_i^i(kT) < q_\beta^i(kT). \quad (11)$$

During the first stage, if both robots α and β exist, then we define

$$\mathcal{Q}_i(kT) = (q_\alpha^i(kT) + q_\beta^i(kT))/2. \quad (12)$$

If α exists but not β, then we define

$$\mathcal{Q}_i(kT) = (q_\alpha^i(kT) + q_i^i(kT) + s_1)/2. \quad (13)$$

On the other hand, if β exists but not α, then we define

$$\mathcal{Q}_i(kT) = (q_\beta^i(kT) + q_i^i(kT) - s_1)/2. \quad (14)$$

Before introducing our decentralized control laws, we define

$$\begin{aligned} \bar{v}_i(kT) &= (\mathcal{Q}_i(kT) - q_i^i(kT))/T \\ \hat{v}_i(kT) &= (\mathcal{M}_i(kT) - \mathcal{F}_i(kT))/T \end{aligned} \quad (15)$$

for $i = 1, 2, \ldots, n$ and $k = 1, 2, \ldots$. Using (15), we introduce the following variables:

$$v_i(kT) = \sqrt{\bar{v}_i(kT)^2 + \hat{v}_i(kT)^2};$$

$$\theta_i(kT) = \begin{cases} \phi_i(kT) + \xi_i(kT) - \pi/2, & \text{if } \hat{v}_i(kT) \geq 0 \\ \phi_i(kT) - \xi_i(kT) - \pi/2, & \text{if } \hat{v}_i(kT) < 0, \end{cases}$$

where $\xi_i(kT) := \cos^{-1}(\bar{v}_i(kT)/v_i(kT))$. Now we are in position to introduce a set of decentralized control laws that is described by:

$$V_i(kT)\Theta_i(kT) = v_i(kT)\theta_i(kT) + v_0 l(\psi_i(kT)). \quad (16)$$

3.2 Second Stage

The first stage of the algorithm presented previously will drive the group of robots into a line formation. However, our objective is to drive the robots into a rectangular lattice pattern as shown in Fig 1. Therefore, the purpose of the second stage of the algorithm is to meet the objective and it will be presented in this subsection.

As mentioned in Section 2, the dimension and the orientation $\bar{\phi}$ of the rectangular lattice are not predefined. In fact, these parameters will only be known by the robots when a global consensus is reached via sharing of local information. The rules (6), (8) and (9) guarantee that all robots will be aligned and moving in a line formation. In other words, the coordination variables have reached their respective consensus values. In addition, using the rule (4), the variable $K_i(\cdot)$ of each robot will also reach a consensus value $\bar{K}$ that belongs to $\{1, 2, \ldots, n\}$, i.e, $K_i(\cdot) = \bar{K}$ for $i = 1, 2, \ldots, n$. The value $\bar{K}$ will characterize the number of robots in each row of the final formation.

Once the robots are aligned and $K_i(kT)$ has reached a consensus value $\bar{K}$, the leftmost robot initiates a counting sequence and the robots then count from the leftmost robot to the rightmost robot. By doing so, each robot has a unique number or ID defined by $N_i \in \{1, 2, \ldots, n\}$ that characterizes its location counted from the leftmost robot. By letting the leftmost robot be robot γ and the rightmost robot be δ, we have $N_\gamma = 1$ and $N_\delta = n$ and the robots between them from left to right have numbers from 2 to $n - 1$. So long as the rightmost robot has $N_\delta = n$ (meaning that all the robots have been assigned IDs), the algorithm moves to the second stage. In other words, before moving to second stage, there exists a set $\{z_1, z_2, \ldots, z_n\}$, that is a permutation of the set $\{1, 2, \ldots, n\}$, such that robot $z_1 = \gamma$ is at the leftmost position of the line, robot z_2 is at the right hand side of robot z_1. Therefore, the right most position is taken by robot $z_n = \delta$.

During the second stage, the robots that have the ID number N_i greater than $\bar{K}$ will move down to form a number of

lines that are parallel to the first line of robots. The distance between these lines is s_2. At the beginning of second stage, the position of robot $z_{\bar{K}}$ is passed to robot z_n via robots $z_{\bar{K}}, z_{\bar{K}+1}, \ldots, z_{n-1}$. Robot z_n uses this information and starts to move to the position that is below robot $z_{\bar{K}}$ with distance s_2 between them. At the same time, robots $z_{\bar{K}+1}, z_{\bar{K}+2}, \ldots, z_{n-1}$ set their variable $\xi_i(\cdot)$ to 2, since they start forming a second layer of robot array.

To drive robot z_n below robot $z_{\bar{K}}$ with distance s_2, the robot uses the following rules:

$$\mathcal{Q}_{z_n}(kT) =$$
$$\begin{cases} (q_{z_n}(kT) + q_{z_{n-1}}(kT))/2, & \text{if } q_{z_{n-1}}(kT) > q_{z_{\bar{K}}(kT)}, \\ (q_{z_n}(kT) + q_{z_{\bar{K}}}(kT))/2, & \text{otherwise.} \end{cases}$$
$$\mathcal{M}_{z_n}(kT) = (\mathcal{F}_{z_n}(kT) + c_{z_n,\bar{K}}(kT) - s_2)/2, \tag{17}$$

where $c_{z_n,\bar{K}}(\cdot) = l^T(\phi_{z_n}(\cdot))p_{\bar{K}}(\cdot)$. By using (17), the position of robot z_n will satisfy

$$\lim_{k \to \infty} (p_{z_{\bar{K}}}(kT) - p_{z_n}(kT)) = s_2 l(\bar{\phi}). \tag{18}$$

At the same time, robots $z_{\bar{K}+1}, z_{\bar{K}+2}, \ldots, z_{n-1}$ will follow robot z_n to go below robots $z_1, z_2, \ldots, z_{\bar{K}}$ since $\xi_{z_{\bar{K}+1}}(\cdot)$, $\xi_{z_{\bar{K}+2}}(\cdot), \ldots, \xi_{z_n}(\cdot)$ are all set to 2. In addition, there will be the distance of s_2 separating between these two rows of robots. Using algorithms that are similar to the ones for stage 1, the robots $z_{\bar{K}+1}, z_{\bar{K}+2}, \ldots, z_{n-1}$ will align themselves forming a second layer of robots that is parallel to the first one.

If $n = 2\bar{K}$, the number of robots in the second row will equal to $\bar{K}$ and a rectangular formation is achieved. On the other hand, if $n > 2\bar{K}$, the number of robots in the second row will excess $\bar{K}$ and a rectangular pattern cannot be achieved, since there will be $n - 2\bar{K}$ excess of robots that will move beyond robot z_1. One way to achieve the configuration as shown in Fig. 1 is that the $n - 2\bar{K}$ excess robots move down in the direction of $-l(\bar{\phi})$ below robot z_1, instead of moving beyond robot z_1. Once these robots have moved down, they again can start forming the third layer of robots in the opposite direction to the second row.

Similarly, if $n = 3\bar{K}$, three rows of robots will be formed and each row has $\bar{K}$ number of robots in it. However, if $n > 3\bar{K}$, the $n - 3\bar{K}$ excess robots will move down in the direction of $-l(\bar{\phi})$ below robot z_n. By repeating this process, there will be $\lfloor n/\bar{K} \rfloor$ number of rows formed with $\bar{K}$ number of robots in each row. For the last row, i.e., row $\lceil n/\bar{K} \rceil$, there will be $n - \lfloor n/\bar{K} \rfloor \bar{K}$ number of robots in it. In order to achieve this, we need to modify $\bar{v}_i(\cdot)$ and $\hat{v}_i(\cdot)$ for robots that have ID's greater than $\bar{K}$ (i.e. robots that are not in the first row).

After reaching the second stage, we will define two imaginary lines $\mathcal{W}_1(kT)$ and $\mathcal{W}_2(kT)$ that represent the sides of a rectangle along the direction $l(\bar{\phi})$. These two parallel lines can be defined as:

$$\mathcal{W}_1(kT) := \{p \in \mathbb{R}^2 | (p - p_{z_1}(kT))^T l(\bar{\phi} - \pi/2) = 0\}$$
$$\mathcal{W}_2(kT) := \{p \in \mathbb{R}^2 | (p - p_{z_{\bar{K}}}(kT))^T l(\bar{\phi} - \pi/2) = 0\}. \tag{19}$$

The distance between these lines is $(\bar{K} - 1)s_1$.

Using these lines, we first consider the case when $\xi_i(kT)$ is even (i.e., the even row). If both robots α and β exist as defined by (11), and $p_\alpha(kT) \notin \mathcal{W}_1(kT)$ or $p_\beta(kT) \notin \mathcal{W}_2(kT)$, then we use (12) for $\mathcal{Q}_i(kT)$. If β exists but not α, or if β exists and α is on $\mathcal{W}_1(kT)$, then $\mathcal{Q}_i(kT)$ is defined by (14). If α exists but

not β, then $\mathcal{Q}_i(kT)$ is defined by (13). If robot i hits $\mathcal{W}_1(kT)$ and there are no other robots in $\mathcal{S}_i(kT)$ that are on $\mathcal{W}_1(kT)$, robot i is then placed at $\mathcal{W}_1(kT)$ and $\mathcal{Q}_i(kT) = q^i_{\eta,1}(kT)$, where $q^i_{\eta,1}(kT) = l^T(\phi_i(kT) - \pi/2)\eta_{i,1}(kT)$ and $\eta_{i,1}(kT) := \mathcal{L}_i(kT) \cap \mathcal{W}_1$. When robot i moves along the line $\mathcal{W}_2(kT)$ from row $\xi_i(kT) - 1$ and robots α and β do not exist, then robot i is placed at $\mathcal{W}_2(kT)$ and define $\mathcal{Q}_i(kT) = q^i_{\eta,2}(kT)$, where $q^i_{\eta,2}(kT) = l^T(\phi_i(kT) - \pi/2)\eta_{i,2}(kT)$ and $\eta_{i,2}(kT) := \mathcal{L}_i(kT) \cap \mathcal{W}_2$. For the case with odd $\xi_i(kT)$ and $\xi_i(kT) \neq 1$, we can define $\mathcal{Q}_i(kT)$ in a similar manner as for the case with even $\xi_i(kT)$.

If robot i is the robot placed at $\mathcal{W}_1(kT)$ and $\xi_i(kT)$ is odd, or at $\mathcal{W}_2(kT)$ and $\xi_i(kT)$ is even, then we define $\bar{S}_i(kT)$, γ_i and $\bar{\mathcal{M}}_i(kT)$ such that

$$\bar{S}_i(kT) := \{j \in \mathcal{N}_i(kT) \mid \xi_j(kT) = \xi_i(kT) - 1\};$$
$$\bar{\mathcal{M}}_i(kT) := (c_{i,\gamma_i}(kT) + c_{i,i}(kT) - s_2)/2,$$

where $\gamma_i := \arg\min_{j \in \bar{S}_i(kT)} |l^T(\phi_i(kT))(p_j(kT) - p_i(kT))|$. The value $\bar{\mathcal{M}}_i(kT)$ is for maintaining the distance between 2 rows of robots at s_2. Using $\mathcal{M}_i(kT)$, $\bar{\mathcal{M}}_i(kT)$ and $\mathcal{F}_i(kT)$, we introduce

$$\hat{\mathcal{M}}_i(kT) = \begin{cases} \mathcal{F}_i(kT) - s_2, & \text{if } \mathcal{Q}_i(kT) < q^i_{\eta,1}(kT) \\ & \text{or } \mathcal{Q}_i(kT) > q^i_{\eta,2}(kT) \\ \bar{\mathcal{M}}_i(kT), & \text{if } p_i(kT) \in \mathcal{W}_1 \text{ or } \mathcal{W}_2; \\ \mathcal{M}_i(kT), & \text{otherwise,} \end{cases} \tag{20}$$

and also

$$\hat{\mathcal{Q}}_i(kT) = \begin{cases} q^i_i(kT), & \text{if } \mathcal{Q}_i(kT) < q^i_{\eta,1}(kT) \\ & \text{or } \mathcal{Q}_i(kT) > q^i_{\eta,2}(kT) \\ \mathcal{Q}_i(kT), & \text{otherwise.} \end{cases} \tag{21}$$

To update the row number, we have

$$\xi_i((k+1)T) =$$
$$\begin{cases} \xi_i(kT), & \text{if } q^i_{\eta,1}(kT) < \mathcal{Q}_i(kT) < q^i_{\eta,2}(kT); \\ \xi_i(kT) + 1, & \text{otherwise.} \end{cases} \tag{22}$$

Using $\hat{\mathcal{M}}_i(kT)$ and $\hat{\mathcal{Q}}_i(kT)$, we modify $\bar{v}_i(kT)$ and $\hat{v}_i(kT)$ in (15) as follows:

$$\bar{v}_i(kT) = (\hat{\mathcal{Q}}_i(kT) - q^i_i(kT))/T$$
$$\hat{v}_i(kT) = (\hat{\mathcal{M}}_i(kT) - \mathcal{F}_i(kT))/T. \tag{23}$$

Again the control law is described by (16).

3.3 Main Result

Theorem 3.1. Consider n robots and their dynamics are described by the equation (1). Suppose that Assumption 2.1 holds. Then the decentralized control algorithms (15), (16) and (23) are a rectangular formation control for the robots.

Proof: The proof of Theorem 3.1 will be given in the full version of this paper.

4. SIMULATION RESULTS

In this section, we present simulation results to illustrate the proposed algorithm. In the first simulation, the robots form into a rectangular formation and move in the direction of $\bar{\psi}$, as shown in Fig. 2. It is clear that all the robots got aligned first in the first stage. As soon as they reached the second stage, the
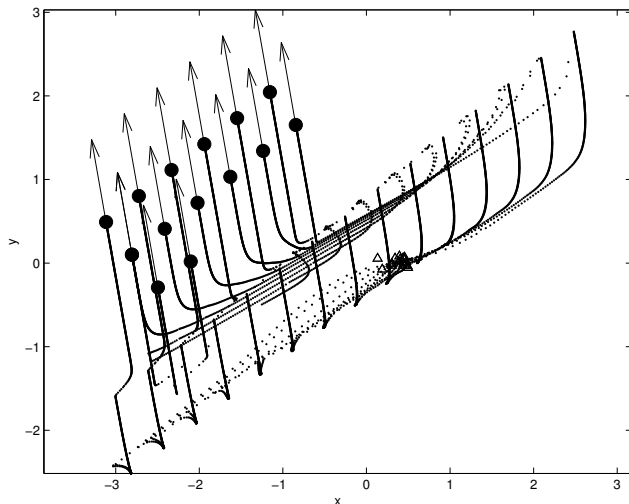
stacle avoidance, and physical constraints of the robots, are currently under investigation.

Fig. 2. Rectangular formation ($n = 14$, $\bar{K} = 6$): robot positions ($\triangle$-initial, $\bullet$-final).
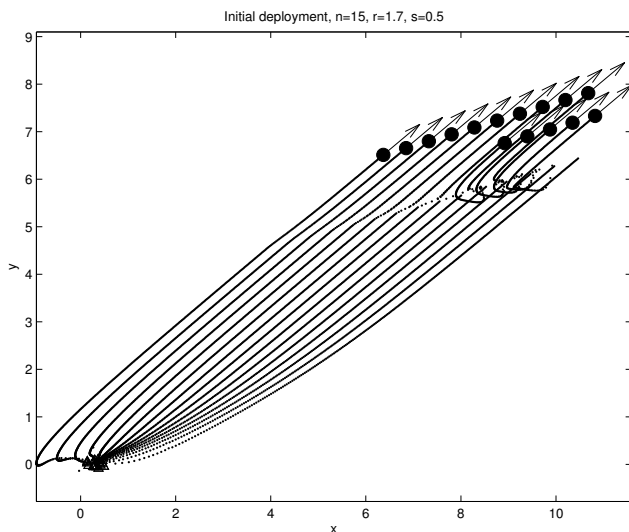

Fig. 3. Rectangular formation ($n = 15$, $\bar{K} = 10$): robot positions ($\triangle$-initial, $\bullet$-final).

robots formed into a rectangular lattice. Hence, in this case, we have 14 robots and the consensus value of $\bar{K}$ was 6, there were 2 robots in the third row. In the second simulation, as shown in Fig. 3, the robots formed into 2 rows. Again the robots got aligned in the first stage and then they formed 2 rows of robots in the second stage. In this cases, 5 robots were in the second row as there were 15 robots and the consensus value of $\bar{K}$ was 10.

5. CONCLUSIONS

In this paper, a set of decentralized formation control algorithms was developed to coordinate a group of mobile robots so that they collectively move into a rectangular lattice pattern from any initial deployment. The algorithm was developed using the consensus approach and it requires only local information. Numerical simulations were performed to illustrate the proposed algorithms. To address the limitations of the current results, issues such as, collision avoidance between robots, ob-

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