Easy Path Planning and Robust Control for Automatic Parallel Parking

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Abstract: This paper deals with path planning and the associated control for a car-like vehicle in parallel parking problem. Our path planning method is purely based on a geometric approach such as minimal turning radii, which can be determined by the geometry of a vehicle and its maximum steering angle. The main strategy for parallel parking comes from retrieving a vehicle from the parking slot. This procedure is reversible and applicable for the parallel parking maneuvering. The theoretical minimum length of parallel parking slot for parking in one trial is given. The proposed planning method is independent of the initial position of the vehicle if the vehicle position is in parallel to the parking space. For the control aspect, a model-free approach is proposed to compensate for neglected dynamics at chassis level due for example to road slopes. Convincing simulation results are presented.

Keywords: Path planning, trajectory planning, vehicle dynamics, automotive control, feedback control, PID control, robustness

1. INTRODUCTION

Driving assistance systems such as automatic parallel parking assist, have been extensively studied in recent years. This kind of system is designed to improve comfort and security of driving in constrained environments where a lot of attentions and experiences are needed to maneuver the vehicle. Since the task of parallel parking requires many repeated maneuvers in a very limited space, its nature is different from the normal driving in a huge environment. Many methods to tackle the parking problem have been presented, and they can be divided into two categories : one based on stabilization of the vehicle to a target point and the other, based on planning a feasible path that connects initial and goal configurations.

The methods based on stabilization can be divided again into two groups. The first group is the one based on Lyapunov function: e.g. in Lee et al. [1999] the authors propose a method using Lyapunov functions to stabilize the vehicle firstly to a desired line corresponding to the parking place, and secondly to a desired point corresponding to the middle of the parking space. This method is simple and easy to apply but the behavior of the vehicle depends very strongly on the Lyapunov function gains. Then, it is necessary to adjust the gains in accordance with the detected dimension of parking space but this is not an easy task. The second group is the one based on learning human skills using Fuzzy logic (e.g. Zhao and Collins Jr [2005]) or neural network (e.g. Jenkins and Yuhas [1993]) in order to acquire the empirical technique of human experts. These methods do not need any path planning and their associated tracking control but they are limited to the experience of human experts and not easily extended to more general cases.

One of the most conventional methods based on path planning is the one proposed by Laumond et al. [1994] that carries out the path planning in two phases:

- Step 1: Plan a geometric collision-free path without taking non-holonomic constraints of the vehicle into account.

- Step 2: Perform subdivisions on the path until all endpoints can be linked to their neighbors by an admissible collision-free path using a local planner.

This method can be applied for parallel parking problem in general cases by using Reed&Shepp's shortest path (Reeds and Shepp [1990]) as a local planner in Step 2.

The interesting work we have considered is the one using a geometric approach (Lo et al. [2003]) which is based on retrieving a vehicle from parallel parking bay. In real life, when the parking bay is large enough, human driver usually steers the front wheel to maximum angle for retrieving. When the vehicle is retrieved, the driver steers the front wheel to maximum angle again but in reverse direction until the car is parallel to the road. Since this procedure is reversible thus it can be applied in parallel parking problem. This steering scenario forms a simple path composed by two identical minimum circles connected by a tangent point. In order to use this path for parallel parking, it is necessary to know the correct position where to begin the parking maneuvers in accordance with the initial position of the car. For this usage, the authors used a looked-up table pre-calculated off line and they said that the necessary minimum length of the parking bay is the radius of the circle forming the path. The usage of look-up table can not cover all cases and we need more precise calculation for the minimum length of the parking bay. Our approach will be based on this geometric method but without look-up table and more precise formula for the minimum length will be given.

Significant and practical key issues in motion planning are safety and optimality. In this paper, we will focus on the maneuvering path planning which will meet these key issues quite well. In section 2, we will recall some geometric properties related on the kinematics of the vehicle and define turning radii of the vehicle. Then a path planning for the simplest parallel parking scenario in one trial will be shown in section 3. Section 4 is devoted to the generation of reference time trajectories. Then we will propose a robust grey-box control strategy in section 5 to follow the planned path. Finally, our conclusion will be presented in section 6.

2. GEOMETRIC PROPERTIES

The car-like robot is subject to the limit of steering angle: $-\beta_{max} < \beta < \beta_{max}$. When the velocity v is constant and the steering angle β is fixed, the vehicle movement is almost a circle. The turning radius of that circle depends on the steering angle β. The steering angle therefore influences the minimum radius of the circle. This feature of the kinematics of the vehicle helps to plan a simple path with circular motions.

Consisting in parking the vehicle with a minimum number of maneuvers, the shortest path can be done with circles of minimum radius (Reeds and Shepp [1990]). In this context, the motion planning for the automatic parallel parking necessitates the study on turning radii which can be determined by geometric parameters of the vehicle and the steering angle β.

First, we define the parameters describing the dimension of the vehicle (see Fig. 1): front and rear overhang p, wheelbase e, overall length of vehicle l, overall width of vehicle w and steering angle β.

The turning radius is the radius of the circle created by a vehicle when it turns with a fixed steering angle. It is defined by a virtual wheel located in the middle of the front axle, using the steering angle β and the wheelbase e. By trigonometry:

$$R = \frac{e}{sin\beta}.$$

Two other turning radii can be defined: the inner radius R_i which is the smallest radius formed by the inside rear wheel and the outer radius R_e which is the largest radius formed by the outside front corner of the vehicle.

The inner radius R_i can be calculated from the radius R by applying the Pythagorean theorem:

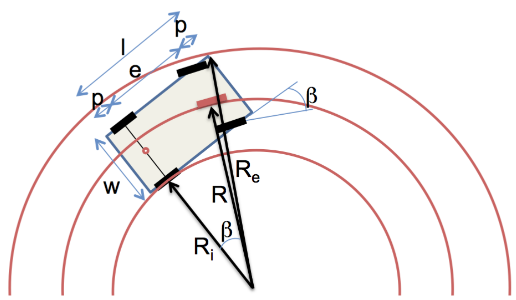

Fig. 1. Geometry of the vehicle and turning radii

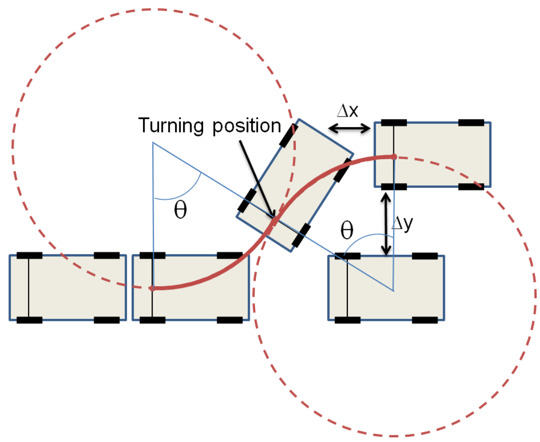

Fig. 2. Strategy for parallel parking in one trial

$$R_i = \sqrt{R^2 - e^2} - \frac{w}{2} = \sqrt{\frac{e^2}{sin^2\beta} - e^2} - \frac{w}{2} = \frac{e}{tan\beta} - \frac{w}{2}.$$

Similarly, the external radius R_e can be calculated from R by using the Pythagorean theorem:

$$R_e = \sqrt{(R_i + w)^2 + (e+p)^2} = \sqrt{(\sqrt{R^2 - e^2} + \frac{w}{2})^2 + (e+p)^2}$$

The more we increase the steering angle β, the more turning radii become obviously small. In the following sections, when the steering angle β is maximal, the minimum radii will be denoted by $(R_{min}, R_{i_{min}}, R_{e_{min}})$.

3. PATH PLANNING IN ONE TRIAL

For the automatic parallel parking system, normally ultra-sonic sensors or infra-red sensors are used to measure the parking space. These sensors can not measure the exact depth of the space if there is no wall and generally we park the vehicle in a line with neighboring vehicles. Therefore we assume in the following sections that the depth of the space is equal to the width of the vehicle.

3.1 Main strategy

We will tackle the problem of parallel parking by retrieving vehicle from the parking space as proposed in Lo et al. [2003]. That is, we can reason in the opposite way: if we are able to retrieve the car without reversing of the longitudinal velocity, we can park the car without velocity sign change too. This is what we call in the sequel a maneuver "in one trial". We assume that the parking space is large enough. To exit from the parking slot, one first reverses the car to the end before touching the vehicle behind. Then one steers to the left at maximum to make an appropriate orientation to go out and counter-steers at maximum until the vehicle is parallel to the road. This procedure is reversible and therefore applicable to the problem of parallel parking. With this maneuver, the corresponding path followed by the midpoint of the rear axle is the one composed by two circular arcs of minimum radius connected by a tangential point as shown in Fig. 2.

We note that the lengths of these two arcs are identical when the vehicle is positioned in parallel after the exit. That is, the central angles of these arcs are identical. The more the central angle increases, the more the lateral displacement $\triangle y$ and longitudinal displacement $\triangle x$ at the end of maneuver will increase (see Fig. 3). In other words, for different angles, the final positions will be different and this position can cover a large set of lateral displacement $\triangle y$ from its minimum value. To perform the automatic parallel parking, the exit procedure will be reversed. In this case, the final position of the exit will be the starting position of the parking maneuver and it is necessary to know the longitudinal displacement $\triangle x$ and the associated counter-steering position in accordance with the lateral displacement $\triangle y$. If the vehicle's lateral displacement and the length of the parking slot are bigger than their minimum value, the vehicle can reverse the exit procedure to be parked correctly.

3.2 Minimum length of the parking space

In this section, the minimum length of the parking space to park a car in one trial will be precisely calculated. At the exit, the outer front corner will determine if the vehicle collides (see Fig. 4). The outer radius $R_{e_{min}}$ should be therefore taken into account. The minimum length L_{min} of the parking space can be calculated by using the Pythagorean theorem applied to the triangle C_1BA:

$$L_{min} = p + \sqrt{R_{e_{min}}^2 - R_{i_{min}}^2}.$$

Therefore if the length of the space is theoretically longer than L_{min}, we can succeed parallel parking without collision. The minimum lengths calculated for several commercial models are summarized in Table 1.

3.3 Minimum lateral displacement

Now, we need to calculate the minimum lateral displacement $\triangle y_{min}$, dead zone in other words where the vehicle can not be found after the exit. This dead zone is related to the minimum central angle θ_{min} of the arc. When the vehicle leaves the parking space if the two arcs to be followed are too short, the vehicle collides with the vehicle ahead. It is therefore necessary to calculate the minimum angle θ_{min} to avoid the collision and to check out for a dead zone on the lateral displacement $\triangle y$.

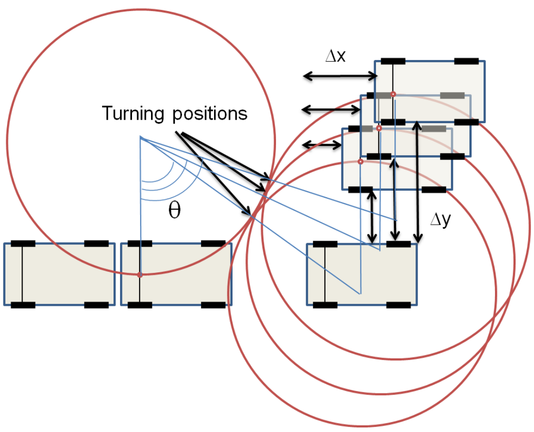

Fig. 3. Exit trajectories with several different turning positions

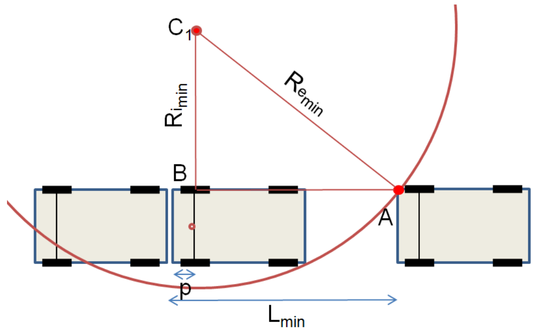

Fig. 4. Calculation of minimum length of the parking space

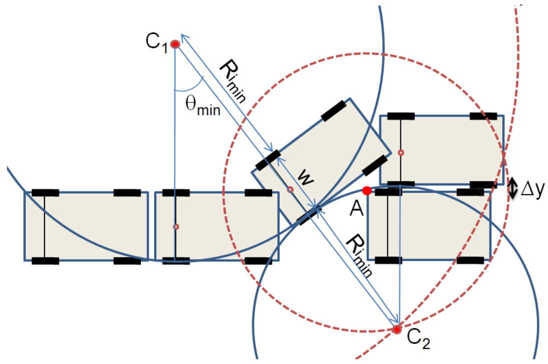

Fig. 5. Calculation of $\triangle y_{min}$ and θ_{min}

Here we suppose that the length of the parking space is equal to L_{min}. In this case, the right rear wheel has a high probability of collision, and therefore the radius $R_{i_{min}}$ associated with the right rear wheel should be taken into account. The minimum angle is reached when the right rear wheel passes through the point A (the left rear corner of the front vehicle, see Fig. 5). That is, the minimum angle is the angle that allows the minimum inner radius $R_{i_{min}}$ to pass through A without collision. This angle can be calculated geometrically.

At the exit, the right rear wheel creates a circle of radius $R_{i_{min}} + w$ with center C_1 then a circle of radius $R_{i_{min}}$ with center C_2 (see Fig. 5). If we imagine another circle of radius $R_{i_{min}}$ with center A, C_2 is the intersection between the circle of radius $2R_{i_{min}} + w$ with center C_1 and the circle of radius $R_{i_{min}}$ with center A. In this case, we can calculate the intersection points of these two circles and we take the point C_2 (see Bourke [1997] for details). Then we can easily compute θ_{min} and $\triangle y_{min}$.

According to calculation, the minimum central angle θ_{min} can put the vehicle in parallel to the parking slot with an almost zero lateral shift $\triangle y \approx 0$ (as we can see in Fig. 5). $\triangle y_{min}$ calculated for several commercial models are almost zero (see Table 1). [1] This means that most of commercial cars have no constraint on their initial lateral displacement when they park.

[1] These computations have been done by using geometry of commercial vehicles found in e.g. Etai [1997].

Table 1. Summary of geometric calculations for several commercial vehicles

Model	MINI	AUDI A6	MEGANE	PICASSO	BOXER
$\triangle y_{min}$ (m)	**0.021**	**0.0034**	**0.0035**	**0.0052**	**0.0034**
L_{min} (m)	**5.60**	**7.07**	**6.52**	**6.23**	**6.92**

3.4 Calculation of the longitudinal starting position and the turning position in accordance with the initial lateral position

In order to execute the parallel parking in one trial, it was necessary to know the longitudinal displacement $\triangle x$ and the associated turning position according to the initial lateral displacement $\triangle y$. In other words, when the vehicle arrives and is positioned in parallel in the vicinity of the parking space, the vehicle will measure the size of the space and its initial lateral position y_i. Then, if we can directly compute the longitudinal starting position x_s where the vehicle starts to turn into and the turning position P_t in accordance with y_i, the vehicle can first move longitudinally up to x_s and start steering to the right, while moving back up to P_t. Then it can counter-steer to the left to enter into the parking slot. In this section we develop analytical calculations for these relationships.

Once the vehicle size and the maximum steering angle β_{max} are known, the minimum radius $(R_{min}, R_{i_{min}}, R_{e_{min}})$ can be calculated. Since in our methodology, the vehicle is supposed to follow two arcs tangentially connected, the rotation center $C_1(x_{c_1}, y_{c_1})$ is always perpendicular to the final position of the parking maneuver with a distance of $R_{i_{min}} + w/2$ and we define this distance by radius $\widetilde{R}_{min}$ (see Fig. 6). Once the geometry of the parking space is available, the center C_1 can then be easily determined. Similarly the center C_2 is always perpendicular to the start position of parking maneuver (x_s, y_s) with a distance of $\widetilde{R}_{min}$. Since the vehicle is supposed to move longitudinally from (x_i, y_i) to (x_s, y_s), y_{c_2} of the center C_2 is still far from y_i with the same distance $\widetilde{R}_{min}$ whatever the initial longitudinal position x_i is. Therefore:

$$y_{c_2} = y_i - \widetilde{R}_{min}.$$

Now we can calculate the point $P_t(x_t, y_t)$ where the vehicle counter-steers. First y_t can be calculated by the symmetry between the two centers C_1 and C_2:

$$y_t = (y_{c_1} + y_{c_2})/2.$$

Then x_t can be calculated by applying the Pythagorean theorem:

$$x_t = x_{c_1} + \sqrt{\widetilde{R}^2_{min} - (y_t - y_{c_1})^2}.$$

Finally, we can calculate the longitudinal displacement x_s by the symmetry:

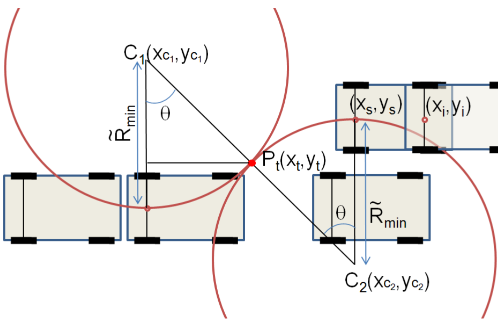

Fig. 6. Calculation of x_s and P_t

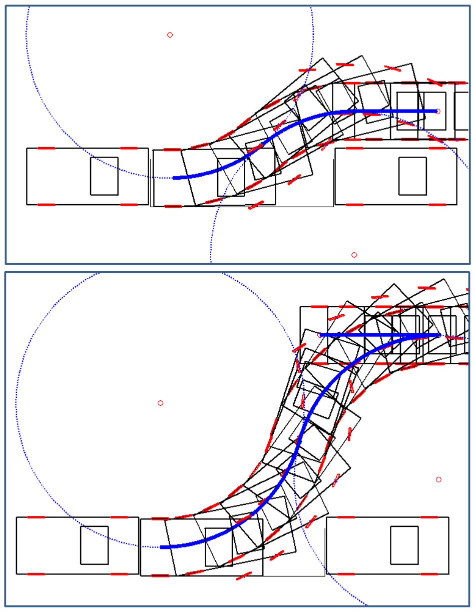

Fig. 7. Simulations on Matlab, vehicle length = 3.62 m, space length = 5.44 m

$$x_s = x_{c_2} = 2x_t - x_{c_1}.$$

Thus, we have shown how to calculate the longitudinal starting position x_s and the turning position P_t in accordance with the initial lateral position y_i. Now, we have all information needed to plan a path. Since we know the two rotation centers (C_1 and C_2) and the radius $\widetilde{R}$, we can connect (x_s, y_s), $P_t(x_t, y_t)$ and the final position by the associated arcs of circle. By connecting (x_s, y_s) with the initial position (x_i, y_i), a path from the initial position to the final position can be finally planned.

The complete strategy for path planning was simulated on our simulation environment using Matlab. As we can see in Fig. 7, the planned paths are composed of 2 identical circular arcs of minimal radius and a straight line segment. It shows also that our method is completely independent of the initial position of the vehicle. The only constraint is that the vehicle should be in parallel to the parking space whatever its initial position is. But this is what we usually do to perform parallel parking. As a consequence, we can obtain the second one of Fig. 7 that a human driver will never realize.

Remark 1. Our algorithm can be extended to a maneuver "in n trials" if the free space is too small to park the car in one trial (Choi [2010]). These results cannot be published for the moment, due to a confidentiality clause.

4. GENERATION OF REFERENCE TRAJECTORIES

Now we can plan a geometrical path which consists of 4 segments: 2 straight segments and 2 circular arcs of minimal radius, and this can be expressed in cartesian coordinates $(x, y(x))$ (see for example Fig. 8). The last straight segment is added to position the vehicle in the middle of the parking slot at the end of the parking maneuver. Our goal is to make the vehicle well follow the generated path. In order to do this, we need to build reference time trajectories $x_{ref}(t)$, $y_{ref}(t)$, functions of the longitudinal velocity $v_{ref}(t)$ and the steering angle $\beta_{ref}(t)$. This can be done through for example a kinematic model of the car (see Choi [2010] and Lo et al. [2003]).

In our parking process, the reference velocity will have a trapezoidal shape at each maneuver. The length of each segments of the geometric path can be easily calculated, then if we know the desired constant acceleration a_{des} to attain the maximal constant velocity v_{max}, the reference velocity trajectory can be easily generated.

First, the time needed to reach v_{max} is given by $t_1 = \frac{v_{max}}{a_{des}}$. Then the distance over t_1 is $d_1 = \frac{a_{des} t_1^2}{2}$. The time during which the vehicle stays at a constant speed v_{max} can be calculated by $t_2 = \frac{d - 2d_1}{v_{max}}$, where d is the distance of a segment of the geometric path.

At each transition between maneuvers, we increase the steering angle linearly to its maximum value, with a time t_3 needed to rotate wheels from $-\beta_{max}$ to β_{max} during which the vehicle remains stationary. During $2t_1 + t_2$, it keeps its maximum steering angle $\pm\beta_{max}$.

Thus, with the same calculations for each segment and the same value t_3, we can obtain a reference trajectory of the velocity of a trapezoidal shape. For example, when $a_{des} = 1m/s^2$, $v_{max} = 1m/s$ and $t_3 = 2s$, the generated reference velocity trajectory for the geometric path of Fig. 8 is illustrated in Fig. 10. As we can see in the last part of this figure, if the segment length is too short, the shape of the velocity becomes an isosceles triangle. The generated reference trajectory steering angle $\beta_{ref}(t)$ is shown in Fig. 9.

With times t_1, t_2 and t_3, the reference acceleration $a_{ref}(t)$ is also obtained as in Fig. 9, which can be viewed as a 'feedforward' term.

Once the reference trajectories of velocity and steering angle are generated, the time functions of $x_{ref}(t)$ and $y_{ref}(t)$ can be calculated numerically, for example by integrating the kinematic model of the vehicle (Lo et al. [2003]) (see Fig. 10).

5. ROBUST GREY-BOX CONTROL

5.1 *Engine/Brake torque generation*

In order to make the vehicle follow the reference trajectories, we can elaborate a desired acceleration with feedback terms as follows:

$$u = a_{ref} - K_D(v - v_{ref}) - K_P(s - s_{ref}), \tag{1}$$

where K_D and K_P are derivative and proportional gains, s_{ref} is the reference traveled distance which is equal to the total area of the reference velocity profile.

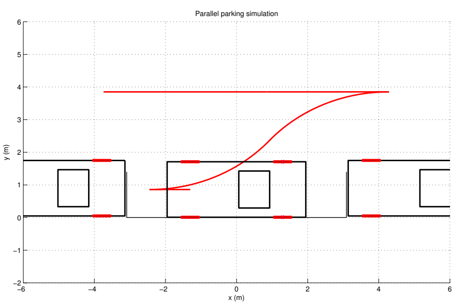

Fig. 8. Geometric path for parallel parking

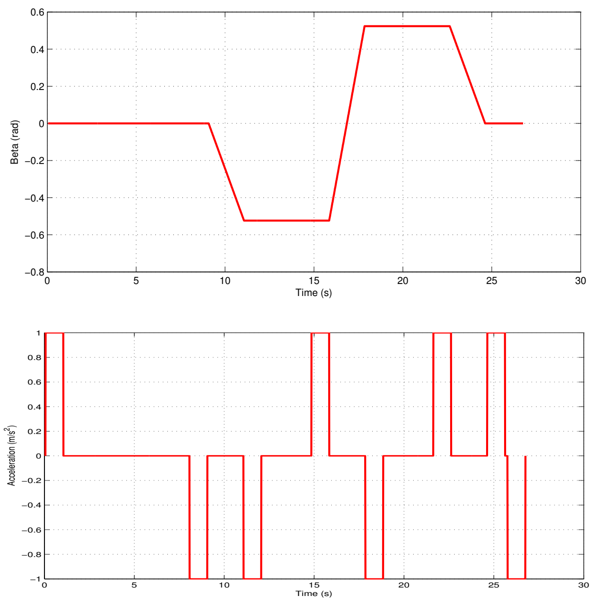

Fig. 9. Reference trajectories of steering angle $\beta_{ref}(t)$ and acceleration $a_{ref}(t)$.

Now it is necessary to compute desired torque τ_d corresponding to the desired acceleration u. Using the formula we already developed in Villagra et al. [2009], we obtain :

$$\tau_d = I \sum_{i=1}^{4} \dot{\omega}_i + rMu. \tag{2}$$

where I is the wheel rotation inertia moment, ω_i is the wheel angular velocity, r is the tire radius, and M is the total weight of vehicle.

Let us point out that the steering control is applied independently of the engine/brake torque when the vehicle is at rest as already mentioned in section 4.

5.2 *Grey-box feedback control*

In fact, external forces like road slop, rolling resistance and aerodynamic forces are neglected when elaborating the desired torque τ_d of (2), but the feedback terms in u of (1) are not robust enough to tackle them. Therefore the model-free control technique (Fliess and Join [2008]) will be applied in this section in order to guarantee robustness of the system against neglected external dynamics. The procedure described in Choi et al. [2009] and Villagra et al. [2009] can here be applied in a similar way. Since some specific dynamics are very well known, it is worth to integrate them in our predictive scheme, this is the reason why we call this method "grey-box control".

The expression between the desired torque and the measured acceleration γ_x can then be written as follows:

$$\gamma_x = \frac{1}{rM}(\tau_d - I \sum_{i=1}^{4} \dot{\omega}_i) + F, \tag{3}$$

where F represents all the neglected external dynamics. Now since from (2),

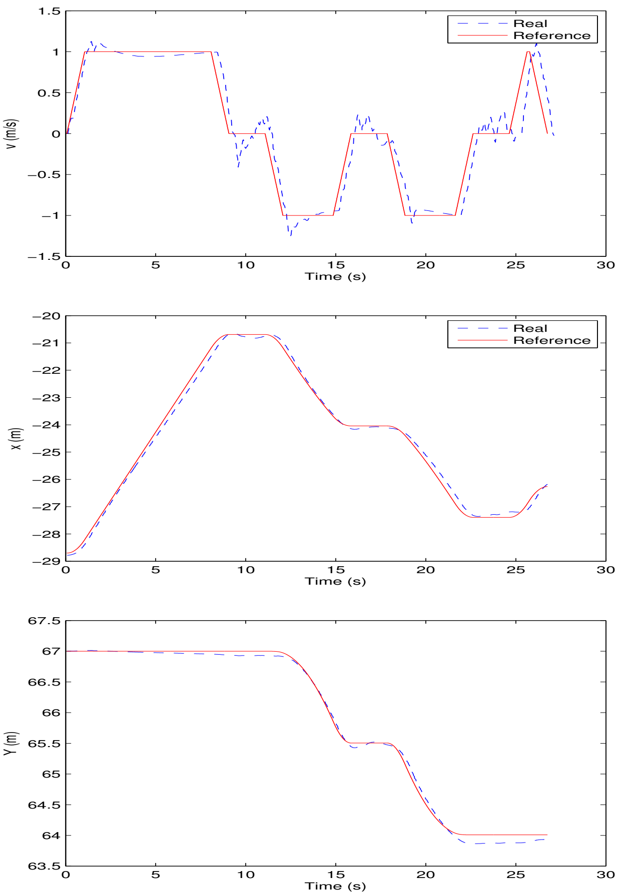

Fig. 10. Result with the model-free strategy

$$u = \frac{1}{rM}(\tau_d - I\sum_{i=1}^{4}\dot{\omega}_i),$$

(3) can be rewritten by $\gamma_x = u + F$. Therefore F can be estimated by $\hat{F}_k = \gamma_{x_k} - u_{F_{k-1}}$. Then we can correct immediately the desired acceleration u by $u_{F_k} = u_k - \hat{F}_k$. Our final desired torque can then be obtained by

$$\tau_{d_k} = I\sum_{i=1}^{4}\dot{\omega}_i + rMu_{F_k}.$$

SiVIC (realistic simulation environment dedicated to prototyping and evaluating ADAS, Gruyer et al. [2006]) was used to test our model-free control strategy. Road slope of 5° was added in the parking environment. Some interesting results are shown in Fig. 10, where the effect of the slope of the parking slot is well compensated by our grey-box control strategy and the reference positions x and y are well tracked.

6. CONCLUSION

We have proposed an easy path planning method based on a geometric approach for the parallel parking problem in a known environment. Once the vehicle is parallel to the parking place whose length is superior to its minimum L_{min}, our method is independent of the initial position of the vehicle and can plan a simple path composed of 2 identical circular arcs of minimum radius and a straight line segment. A robust control strategy for parking using model-free approach to compensate all the unmodeled external dynamics such as road slop was also studied.

We will communicate in our next publication the extension of our algorithm for a parking "in n trials" according to the available length of the parking slot. A theoretical justification on its complexity is given and compared with other planners (e. g. Laumond et al. [1994]) and existing commercial products (such as the Park4u developed by Valeo). Experimental results have also been developed but they cannot be detailed here due to a confidentiality clause. [2]

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[2] However some videos can be found in http://www.youtube.com/user/laraimara showing the automatic parking in n trials of an electric car on a flat ground.