# Regressor Based Robust Control for Collaborative Manipulators Handling a Rigid Object 

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#### Abstract

The kinematics and dynamics of two planar rigid manipulators handling a rigid object are studied. The system dynamics is formulated by combining the dynamics of manipulators and the object. A regressor based sliding mode control scheme is developed for the control of object to reach the desired position and orientation. The main advantages of the proposed robust control scheme are that, it can handle quickly varying parameters and alleviate the problem of choosing the upper bounds of the uncertainties in the robust approaches; it also does not need persistency of excitation, and guarantees the exponential convergence of transient behavior. The main problem with this type of control is chattering, which is eliminated with the help of adding boundary layer thickness in the control law. Simulations are performed which confirm the effectiveness of the proposed control approach.


Keywords: Collaborative manipulators, Parameter uncertainty, Robust control, Exponential stability, Regressor

## 1. INTRODUCTION

A single robot alone is not able to grasp and move a long object in a safe and efficient way. Owing to the single arm structure, present day robots are called "handicapped operators" for performing complex tasks. Most tasks in assembly/disassembly, handling large or heavy objects are done efficiently with two robot arms. Collaborative manipulators have many advantages compared to single arm manipulators such as increased load carrying capacity, greater dexterity and manipulability, reduced need for extra auxiliary equipments, efficient use of available workspace and increased productivity by operating each robots in parallel to achieve different tasks at the same time. The first master/slave teleoperated manipulator was used in the nuclear industry (Goertz (1952)) and thereafter the need for two robots for various applications has been realized.

Many control methods for the coordinated motions of manipulator have been developed. In 1980's researchers dealt with master/slave approach, see for example, Luh and Zheng (1989). However, master/slave approach failed due to the kinematic and dynamic uncertainties in uncalibrated slave robot joint measurements. The Hybrid position/force approach was used by various researchers including Hayati (1986) and Dauchez et al. (1989) to achieve coordination between the manipulators when handling an object. Other recent approaches for the coordinated control may be found in Caccavale et al. (2008) and Moosavian and Papadopoulos (2010). However, all these approaches need the accurate information of the dynamic parameters.

[^0]In order to alleviate these problems, both adaptive and robust control algorithms were employed.
In order to adapt to the uncertainties of the model, since late 1980's, adaptive and robust approaches have been introduced. The early works may be found in, for example, Hu and Goldenberg (1989), Walker et al. (1989) and Zribi et al. (2000). It is worth mentioning that until now the adaptive and robust approaches are still the dominant control strategies to deal with the uncertainties in the the coordinated motions of manipulators. Relevant literature includes Gueaieb et al. (2003), Uzmay et al. (2004), Azadi et al. (2006), Gueaieb et al. (2007) and Yagiz et al. (2010) and references therein. However, for adaptive approach, it is evident from Zribi et al. (2000) that, nine parameters are generally required to estimate for each robot and it increases computation burden. For robust approach, the determination of upper bounds for the uncertainties are generally very conservative, which may lead to large control magnitudes.

In order to handle quickly varying parameters and alleviate the problem to choose the upper bounds of the uncertainties, a regressor based control algorithm was proposed by Su et al. (1993) for a single manipulator, where the upper bound can easily be determined. In addition to this advantage, it also does not need persistency of excitation, guarantees the exponential convergence of transient behavior and it is robust against uncertainties in the model. This approach was then extended to the tracking control of uncertain nonholonomic robotic systems (Oya et al. (2003)). In the present paper, we attempt to extend this approach to the control of uncertain collaborative manipulators handling a long and rigid object.

This paper presents a study on the collaborative manipulation of two planar manipulators each with three rigid links used to move an object in the desired position and orientation. The remaining content of the paper is organized as follows. In Section 2, kinematics and dynamics of manipulators and the object are introduced. A regressor based sliding control is developed and exponential stability proof is presented in section 3. In order to validate the proposed controller, simulations are carried out and the results are discussed in section 4 . Finally, section 5 presents the conclusion.

## 2. MODELING OF THE SYSTEM

### 2.1 Kinematics of the Manipulators

The kinematic relations of each manipulator can be written with respect to transformation matrices of each links. Fig. 1 shows two planar manipulators with corresponding end-effectors grasping an object. The co-ordinate frame $X_{1} Y_{1}$ and $X_{2} Y_{2}$ shown in Fig. 1 are fixed frames and xy-frame is a moving coordinate frame which is attached to the beam. $X_{e 1} Y_{e 1}$ and $X_{e 2} Y_{e 2}$ are the end-effector frames attached at each end of the end-effectors. The end-effector positions and orientations $e_{1}=\left\{x_{1}, y_{1}, \theta\right\}^{T}$ and $e_{2}=\left\{x_{2}, y_{2}, \theta\right\}^{T}$, are represented with respect to a reference frame $X_{1} Y_{1}$, respectively. In general, velocity of each end-effector of the manipulator is related to joint velocity of the manipulator through Jacobian matrix.


Fig. 1. Two planar rigid manipulators grasping an object
For the two manipulators, the end-effector velocities $\dot{e}_{1}$ and $\dot{e}_{2}$ and manipulator joint velocities $\dot{q}_{1}$ and $\dot{q}_{2}$ are related through Jacobian matrices $J_{1}$ and $J_{2}$ as follows,

$$
\begin{equation*}
\{\dot{e}\}=[J]\{\dot{q}\} \tag{1}
\end{equation*}
$$

Differentiating (1) gives,

$$
\begin{equation*}
\{\ddot{e}\}=[\dot{J}]\{\dot{q}\}+[J]\{\ddot{q}\} \tag{2}
\end{equation*}
$$

where,

$$
\{\dot{e}\}=\left\{\begin{array}{l}
\dot{e}_{1} \\
\dot{e}_{2}
\end{array}\right\} ; \quad J=\left[\begin{array}{cc}
J_{1} & 0 \\
0 & J_{2}
\end{array}\right]
$$

### 2.2 Dynamics of the Manipulators

In general, Lagrange dynamic equation of manipulator in joint space can be expressed as,

$$
\begin{equation*}
M_{i}\left(q_{i}\right) \ddot{q}_{i}+C_{i}\left(q_{i}, \dot{q}_{i}\right) \dot{q}_{i}+G_{i}\left(q_{i}\right)=\tau_{i}+J_{i}^{T} f_{i} \tag{3}
\end{equation*}
$$

$M_{i}\left(q_{i}\right)(i=1,2)$ represents symmetric positive definite inertia matrix, $C_{i}\left(q_{i}, \dot{q}_{i}\right) \dot{q}_{i}$ is the vector due to coriolis and centrifugal components, $G_{i}\left(q_{i}\right)$ represents the vector of gravitational components, $\tau_{i}$ is the vector of input torque applied at each joint of the manipulator, $f_{i}$ is the interaction forces/moment between the manipulator and object, $J_{i}$ is Jacobian matrix of manipulator and $q_{i}$ is the vector of joint angles.
Assembling the equations for the two manipulators in joint space we have,

$$
\begin{equation*}
M_{r} \ddot{q}+C_{r} \dot{q}+G_{r}=\tau+J^{T} f \tag{4}
\end{equation*}
$$

where,

$$
\begin{gathered}
M_{r}=\left[\begin{array}{cc}
M_{1} & 0 \\
0 & M_{2}
\end{array}\right] ; C_{r}=\left[\begin{array}{cc}
C_{1} & 0 \\
0 & C_{2}
\end{array}\right] ; \quad G_{r}=\left\{\begin{array}{l}
G_{1} \\
G_{2}
\end{array}\right\} \\
\tau=\left\{\begin{array}{l}
\tau_{1} \\
\tau_{2}
\end{array}\right\} ; J=\left[\begin{array}{cc}
J_{1} & 0 \\
0 & J_{2}
\end{array}\right] ; f=\left\{\begin{array}{l}
f_{1} \\
f_{2}
\end{array}\right\} ; q=\left\{\begin{array}{l}
q_{1} \\
q_{2}
\end{array}\right\}
\end{gathered}
$$

### 2.3 Kinematics of the object

Consider a beam of length $L$ and mass $m$ which is rigidly grasped by the two manipulators. The mass center position and orientation (pose) with respect to reference frame $X_{1} Y_{1}$ is represented as $c_{0}=\left\{x_{0}, y_{0}, \theta\right\}^{T}$. All kinematic relations are written with respect to $X_{1} Y_{1}$ frame.
The left end pose of the beam is given by,

$$
\begin{equation*}
\left\{e_{1}\right\}=\left\{c_{o}\right\}-\left\{\frac{L}{2} \cos \theta \quad \frac{L}{2} \sin \theta \quad 0\right\}^{T} \tag{5}
\end{equation*}
$$

and the right end pose of the beam is given by,

$$
\begin{equation*}
\left\{e_{2}\right\}=\left\{c_{o}\right\}+\left\{\frac{L}{2} \cos \theta \quad \frac{L}{2} \sin \theta \quad 0\right\}^{T} \tag{6}
\end{equation*}
$$

Differentiating (5) and (6) yields as,

$$
\{\dot{e}\}=\left[\begin{array}{ccc}
1 & 0 & \frac{L}{2} \sin \theta \\
0 & 1 & -\frac{L}{2} \cos \theta \\
0 & 0 & 1 \\
1 & 0 & -\frac{L}{2} \sin \theta \\
0 & 1 & \frac{L}{2} \cos \theta \\
0 & 0 & 1
\end{array}\right]\left\{\begin{array}{c}
\dot{x}_{0} \\
\dot{y}_{0} \\
\dot{\theta}
\end{array}\right\}
$$

$$
\begin{equation*}
\{\dot{e}\}=[R]\{\dot{X}\} \tag{7}
\end{equation*}
$$

Differentiating (7) gives the acceleration as,

$$
\begin{equation*}
\{\ddot{e}\}=[\dot{R}]\{\dot{X}\}+[R]\{\ddot{X}\} \tag{8}
\end{equation*}
$$

where $R$ is the transformation matrix which relates the end-effector velocity and object velocity.

### 2.4 Dynamics of the object

In general, the Euler dynamic model of the rigid beam can be written as,

$$
\begin{equation*}
M_{b} \ddot{X}+C_{b} \dot{X}+G_{b}=F_{b}(-f) \tag{9}
\end{equation*}
$$

$$
\begin{gathered}
M_{b}=\left[\begin{array}{ccc}
m & 0 & 0 \\
0 & m & 0 \\
0 & 0 & \frac{m L^{2}}{12}
\end{array}\right] ; \ddot{X}=\left\{\begin{array}{c}
\ddot{x}_{0} \\
\ddot{y}_{0} \\
\ddot{\theta}
\end{array}\right\} ; G_{b}=\left\{\begin{array}{c}
0 \\
m g \\
0
\end{array}\right\} \\
F_{b}=\left[\begin{array}{cccccc}
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 \\
\frac{L}{2} \sin \theta & -\frac{L}{2} \cos \theta & 1-\frac{L}{2} \sin \theta & \frac{L}{2} \cos \theta & 1
\end{array}\right]
\end{gathered}
$$

where $M_{b}$ is the inertia matrix of the beam. Assuming that the beam does not experience any centrifugal and coriolis forces (Zribi et al. (2000)) results in $C_{b}=\{0,0,0\}^{T}$. The gravitational force is represented by $G_{b}$ and $F_{b}$ denotes the grasp matrix. The negative sign $(-f)$ indicates the reaction forces/moments at the two ends of the beam due to the applied forces/moments by the manipulators.

### 2.5 Combined Dynamics

In order to obtain the combined system dynamics, the dynamic equations of manipulators and beam must be combined. However, the dynamic equation of beam is represented in cartesian space and should be converted into joint space. The resulting joint space dynamics of beam can be combined together with the manipulators dynamics (4) which is already given in joint space. Here in this section, initially the beam dynamics is transformed into joint space and finally combined dynamic equations will be developed.

Using (1), (7) can be rewritten as,

$$
\begin{equation*}
\dot{X}=R^{\dagger} J \dot{q} \tag{10}
\end{equation*}
$$

Differentiating (10) gives,

$$
\begin{equation*}
\ddot{X}=\dot{R}^{\dagger} J \dot{q}+R^{\dagger}(\dot{J} \dot{q}+J \ddot{q}) \tag{11}
\end{equation*}
$$

Substituting (11) into (9) gives us the beam dynamics in the joint space,

$$
\begin{equation*}
M_{b} R^{\dagger} J \ddot{q}+M_{b}\left(\dot{R}^{\dagger} J+R^{\dagger} \dot{J}\right) \dot{q}+G_{b}=F_{b}(-f) \tag{12}
\end{equation*}
$$

Incorporating (12) into (4) gives the combined dynamics (manipulators and beam dynamics) in joint space,

$$
\begin{equation*}
M_{j s} \ddot{q}+C_{j s} \dot{q}+G_{j s}=\tau_{j s} \tag{13}
\end{equation*}
$$

where,

$$
\begin{aligned}
& M_{j s}=\left(M_{r}+J^{T} F_{b}^{\dagger} M_{b} R^{\dagger} J\right) \\
& C_{j s}=C_{r}+J^{T} F_{b}^{\dagger} M_{b}\left(\dot{R^{\dagger}} J+R^{\dagger} \dot{J}\right) \\
& G_{j s}=G_{r}+J^{T} F_{b}^{\dagger} G_{b}
\end{aligned}
$$

where $R^{\dagger}$ and $F_{b}^{\dagger}$ represents the pseudo inverse matrices.
The combined dynamics (13) has following properties which are essential for designing control algorithm and stability analysis and these properties can be proved.
Property 1: $M_{j s}$ is a symmetric positive definite matrix.

Property 2 : The matrix $M_{j s}$ and $C_{j s}$ in (13) satisfies

$$
\begin{equation*}
X_{1}^{T}\left(\dot{M}_{j s}-2 C_{j s}\right) X_{1}=0, \quad \forall X_{1} \neq 0 \tag{14}
\end{equation*}
$$

where $X_{1}$ is any arbitrary vector. That is $\left(\dot{M}_{j s}-2 C_{j s}\right)$ is a skew-symmetric matrix.

Property 3: There exists a vector $\alpha_{j s} \in \mathbf{R}^{v}$ which solely depends on manipulator and beam dynamic parameters (lengths, masses and moments of inertia etc.) such that

$$
\begin{equation*}
M_{j s} \ddot{q}+C_{j s} \dot{q}+G_{j s}=Y_{j s}(\ddot{q}, \dot{q}, q) \alpha_{j s} \tag{15}
\end{equation*}
$$

where $Y_{j s} \in \mathbf{R}^{u X v}$ is called regressor (Slotine and Li (1991)) of the combined dynamic system represented in joint space.

## 3. CONTROLLER DESIGN

### 3.1 Sliding Mode Control

The control law is formulated in such a way that the object will move from the given initial pose to final pose and simultaneously the two manipulators are also moved in a prescribed way. In order to handle bounded uncertainties of the parameters, a robust control algorithm is developed. Defining the tracking error as,

$$
\begin{equation*}
e_{r r}=q-q_{d} \tag{16}
\end{equation*}
$$

the auxiliary trajectory can also be defined as,

$$
\begin{equation*}
\dot{q}_{r}=\dot{q}_{d}-\lambda_{j s} e_{r r} \tag{17}
\end{equation*}
$$

where $\lambda_{j s}$ is a positive definite matrix whose eigenvalues are strictly in the right half of the complex plane.
The sliding surface can be chosen as,

$$
\begin{equation*}
S_{j s}=\dot{q}-\dot{q}_{r}=\dot{e}_{r r}+\lambda_{j s} e_{r r} \tag{18}
\end{equation*}
$$

The sliding mode controller can be given as,

$$
\begin{equation*}
\tau_{j s}=Y_{j s} \psi_{j s}-K_{D} S_{j s} \tag{19}
\end{equation*}
$$

where $K_{D}$ is a positive definite gain matrix, $Y_{j s}\left(\ddot{q}_{r}, \dot{q}_{r}, \dot{q}, q\right)$ is regressor matrix, $\alpha_{j s}$ is the parameter vector and $\psi_{j s}=$ $\left[\psi_{1} \ldots \psi_{m}\right]^{T}$ are the switching functions which are given by,

$$
\begin{equation*}
\psi_{j s}=-\beta_{j s} \frac{Y_{j s}^{T} S_{j s}}{\left\|Y_{j s}^{T} S_{j s}\right\|} \tag{20}
\end{equation*}
$$

where $\beta_{j s} \geq\left\|\alpha_{j s}\right\|$ and $\beta_{j s}$ is upper bound of $\alpha_{j s}$ which can easily be determined and shows the main advantage over the other robust approaches in the literature.

### 3.2 Stability analysis:

Differentiating the sliding surface (18) with respect to time gives,

$$
\begin{equation*}
\dot{S}_{j s}=\ddot{q}_{d}-\ddot{q}_{r} \tag{21}
\end{equation*}
$$

Mutiplying both sides of (21) by $M_{j s}$ and utilizing (13), (21) can be rewritten as,

$$
\begin{equation*}
M_{j s} \dot{S}=\tau_{j s}-C_{j s} \dot{q}-G_{j s}-M_{j s} \ddot{q}_{r} \tag{22}
\end{equation*}
$$

Adding and subtracting $C_{j s} \dot{q}_{r}$ in (22) and utilizing (18), (22) can be rewritten as,

$$
\begin{equation*}
M_{j s} \dot{S}_{j s}=\tau_{j s}-Y_{j s}\left(\ddot{q}_{r}, \dot{q}_{r}, \dot{q}, q\right)-C_{j s} S_{j s} \tag{23}
\end{equation*}
$$

Consider a Lyapunov function candidate as,

$$
\begin{equation*}
V\left(t, S_{j s}\right)=\frac{1}{2} S_{j s}^{T} M_{j s} S_{j s} \tag{24}
\end{equation*}
$$

Differentiating (24) with respect to time gives,

$$
\begin{equation*}
\dot{V}\left(t, S_{j s}\right)=S_{j s}^{T} M_{j s} \dot{S}_{j s}+\frac{1}{2} S_{j s}^{T} \dot{M}_{j s} S_{j s} \tag{25}
\end{equation*}
$$

Substituting (23) into (25) and also utilizing property 2 given in (14), above equation yields into,

$$
\begin{equation*}
\dot{V}\left(t, S_{j s}\right)=S_{j s}^{T}\left[\tau-Y_{j s}\left(\ddot{q}_{r}, \dot{q}_{r}, \dot{q}, q\right) \alpha_{j s}\right] \tag{26}
\end{equation*}
$$

where,

$$
M_{j s} \ddot{q}_{r}+C_{j s} \dot{q}_{r}+G_{j s}=Y_{j s}\left(\ddot{q}_{r}, \dot{q}_{r}, \dot{q}, q\right) \alpha_{j s}
$$

Substituting the control law given in (19) and (20) into (26) one can have,

$$
\begin{equation*}
\leq-S_{j s}^{T} K_{D} S_{j s}-\beta_{1}\left\|Y_{j s}^{T} S_{j s}\right\|+\left\|S_{j s}^{T} Y_{j s}\right\|\left\|\alpha_{j s}\right\| \tag{27}
\end{equation*}
$$

Taking transpose of $\left\|S_{j s}^{T} Y_{j s}\right\|$ and also $\beta_{1} \geq\left\|\alpha_{j s}\right\|$ gives us

$$
\begin{equation*}
\dot{V}\left(t, S_{j s}\right) \leq-S_{j s}^{T} K_{D} S_{j s} \tag{28}
\end{equation*}
$$

It is well known that $K_{D}=M_{j s} \kappa_{1}$ where $\kappa_{1}$ can be considered as a least eigenvalue.
Hence, (28) can be rewritten as,

$$
\begin{equation*}
\frac{d V\left(t, S_{j s}\right)}{d t} \leq-S_{j s}^{T} M_{j s} \kappa_{1} S_{j s} \tag{29}
\end{equation*}
$$

Using (24), (29) can be rewritten as,

$$
\begin{equation*}
\frac{d V\left(t, S_{j s}\right)}{d t} \leq-2 \kappa_{1} V\left(t, S_{j s}\right) \tag{30}
\end{equation*}
$$

The solution of the above equation is,

$$
\begin{equation*}
V\left(t, S_{j s}\right) \leq V\left(0, S_{j s}(0)\right) e^{-2 \kappa_{1} t} \tag{31}
\end{equation*}
$$

It is evident from the above equation that the sliding surface will converge exponentially to zero. Thus the sliding surface is related to the tracking error $e_{r r}$ in (18) which also converges exponentially to zero.

## 4. SIMULATION

The two planar manipulators each with three links move the rigid beam from initial position and orientation of center of mass (Ahmad and Zribi (1993)) ( $0.51 \mathrm{~m} ; 0.36 \mathrm{~m}$; $90^{\circ}$ ) to final position and orientation ( $0.55 \mathrm{~m} ; 0.36 \mathrm{~m} ; 90^{\circ}$ ) is considered for the simulation. The motion of each joint angles of first manipulator from $\left(0^{\circ} ;-45^{\circ} ;-45^{\circ}\right)$ to ($10.35^{\circ} ;-21.5^{\circ} ;-58.2^{\circ}$ ) and correspondingly the second manipulator from the initial joint angles ( $0^{\circ} ; 45^{\circ} ; 45^{\circ}$ ) to final joint angles $\left(10.35^{\circ} ; 21.5^{\circ} ; 58.2^{\circ}\right)$ are considered. The parameters of identical manipulators and beam are given in tables 1 and 2. Also, the control parameters are given in table 3.
It is observed from the Fig. 2 that, the beam approaches towards its final position along X directions within 0.3 secs and along Y directions as shown in Fig. 3. Due to highly


Fig. 2. Motion of Beam along X direction


Fig. 3. Motion of Beam along Y direction


Fig. 4. Orientation of Beam


Fig. 5. Joint-1 of Manipulator-1


Fig. 6. Joint-2 of Manipulator-1


Fig. 7. Joint-3 of Manipulator-1

Table 1. Parameters of the manipulators

| Link | Length (m) | Mass $(\mathrm{kg})$ | Moment of inertia $\left(\mathrm{kg} \cdot \mathrm{m}^{2}\right)$ |
| :---: | :---: | :---: | :---: |
| 1 | 0.3 | 1.0 | 0.30 |
| 2 | 0.3 | 1.0 | 0.30 |
| 3 | 0.05 | 0.4 | 0.15 |

Table 2. Parameters of the beam

| Parameter | Value |
| :---: | :---: |
| Mass $(m)$ | 1.0 kg |
| Length $(L)$ | 0.1 m |
| Moment of Inertia $(I)$ | $0.2 \mathrm{~kg} \cdot \mathrm{~m}^{2}$ |

Table 3. Control parameters

| Parameter | Value |
| :---: | :---: |
| $K_{D}$ | $\operatorname{diag}(20)$ |
| $\lambda_{j s}$ | $\operatorname{diag}(50)$ |
| $\boldsymbol{\beta}_{j s}$ | 3 |



Fig. 8. Joint-1 of Manipulator-2


Fig. 9. Joint-2 of Manipulator-2
nonlinear presence of the parameters, a small deviation to the final value occurs initially. It can be also seen from Fig. 4 that, orientation of the beam reaches its desired value after 0.2 secs. These results are comparatively better than the results presented in Zribi et al. (2000) and also parameter estimation is avoided. Due to strict space limitations, the regressor matrix for the combined dynamic system is not provided and this can be available upon request. Fig. 5-10 show that, the manipulators also achieved their desired joint angular motions.
Due to the sliding condition given in (20), the control law (19) is discontinuous across the sliding surface and this causes the chattering phenomenon. Chattering is the undesirable phenomenon of oscillations which has finite frequency and amplitude. The chattering leads to high control activity and correspondingly low control accuracy, high wear of moving mechanical parts and also high heat losses in electrical power circuits (Utkin and Lee (2006)). It may excite unmodeled high frequency dynamics which are not considered during initial modeling of the systems. This phenomenon is observed in all the sliding surfaces. For example, the sliding variable 6 shown in Fig. 11 has chattering effect. This phenomenon is also observed in the input control torques which are shown in the Figs. 12


Fig. 10. Joint-3 of Manipulator-2
and 13. We have not shown sliding behavior for all the sliding variables and control torques. However, this kind of behavior also exists in the simulation results.
In order to overcome the chattering, the discontinuous control law can be replaced with continuous one inside the boundary layer (Su et al. (1993) and Oya et al. (2003)). This can be done by adding a boundary layer thickness $\epsilon_{t}$ in the switching function which is given by,

$$
\begin{equation*}
\psi_{j s}=-\beta_{j s} \frac{Y_{j s}^{T} S_{j s}}{\left\|Y_{j s}^{T} S_{j s}\right\|+\epsilon_{t}} \tag{32}
\end{equation*}
$$

It can be seen from the Figs. 14-16 that, the chattering is completely reduced by adding the boundary layer thickness of $\epsilon_{t}=0.75$ and this will lead us to avoid the problems mentioned earlier due to chattering.


Fig. 11. Sliding variable 6 with chattering


Fig. 12. Control torque of joint 1 of manipulator 1 with chattering


Fig. 13. Control torque of joint 2 of manipulator 2 with chattering


Fig. 14. Sliding variable 6 without chattering


Fig. 15. Control torque of joint 1 of manipulator 1 without chattering


Fig. 16. Control torque of joint 2 of manipulator 2 without chattering

## 5. CONCLUSION

In this article, a sliding mode control scheme has been developed for the collaborative manipulators to move a long, rigid object towards its desired position and orientation. The advantages of the proposed controller is that, the upper bounds of the uncertainties for the controller design can easily be determined, which is generally difficult in sliding mode controller designs. The problem of estimating the parameters as in the case of adaptive law is avoided which reduces the calculations. It also does not need persistency of excitation and the convergence of the transient is exponential which is evident from the stability proof presented. In order to avoid chattering, a smooth control law is adopted with the help of boundary layer thickness. Simulation results show that the proposed controller is an effective choice.

## REFERENCES

Ahmad, S. and Zribi, M. (1993). Lyapunov based control design for multiple robots handling a common object. J. of Dynamics and Control, volume 3, 127-157.

Azadi, M., Eghtesad, M., and Ghobakhloo, A. (2006). Robust control of two 5 DOF cooperating robot manipulators. Proc. of the IEEE Int. Workshop on Advanced Motion Control , 653-658.
Caccavale, F., Chiacchio, P., Marino, A., and Villani, L. (2008). Six-DOF impedance control of dual-Arm cooperative manipulators. IEEE/ASME Transactions on Mechatronics, volume 13, 576-586.

Dauchez, P., Fournier, A., and Jordon, R. (1989). Hybrid Control of a Two-Arm Robot for complex Tasks. IEEE J. of Robotics and autonomous Systems, volume 5, 323332.

Goertz, R.C. (1952). Fundamentals of general-purpose remote manipulators. Nucleonics, volume 10, 36-45.
Gueaieb, W., Karray, F., and Al-Sharhan, S. (2003). A robust adaptive fuzzy position/force control scheme for cooperative manipulators. IEEE Transactions on Control Systems Technology, volume 11, 516-528.
Gueaieb, W., Karray, F., and Al-Sharhan, S. (2007). A Robust hybrid intelligent position/force control scheme for cooperative manipulators. IEEE/ASME Transactions on Mechatronics, volume 12, 109-125.
Hayati, S. (1986). Hybrid position / force control of multiarm cooperating robots. Proc. of the IEEE Int. Conf. on Robotics and Automation, 82-89.
Hu, Y.R. and Goldenberg, A.A. (1989). An adaptive approach to motion and force control of multiple coordinated robots. Proc. of the IEEE Int. Conf. on Robotics and Automation, 1091-1096.
Luh, J.Y.S. and Zheng, Y.F. (1989). Constrained relations between two coordinated industrialrobots for motion control. Int. J. of Robotics Research, volume 6, 60-69.
Moosavian, S.A.A. and Papadopoulos, E. (2010). Cooperative object manipulation with contact impact using multiple impedance control. Int.J. of Control, Automation, and Systems, volume 8, 314-327.
Oya, M., Su, C.-Y., and Katoh, R. (2003). Robust adaptive motion/force tracking control of uncertain nonholonomic mechanical systems. IEEE Transactions on Robotics and Automation, volume 19, 175-181.
Raibert, M.H. and Craig, J.J. (1981). Hybrid position/force control of manipulators for motion control. ASME J. of Dynamic Systems, Measurement, and Control, volume 102, 126-133.
Slotine, J.J.E. and Li, W. (1991) Applied Nonlinear Control. Prentice Hall, New Jersey.
Su, C.-Y., Leung, T.P., and Stepanenko, Y. (1993). Realtime implementation of regressor-based sliding mode control algorithm for robotic manipulator. IEEE Transactions on Industrial Electronics, volume 40, 71-79.
Utkin, V. and Lee, H. (2006). Chattering problem in sliding mode control systems. Proc. of the 2006 International Workshop on Variable Structure Systems, 346350.

Uzmay, I., Burkan, R., and Sarikaya, H. (2004). Application of robust and adaptive control techniques to cooperative manipulation. Control Engineering Practice, volume 12, 139-148.
Walker, M.W., Kim, D., and Dionise, J. (1989). Adaptive coordinated motion control of two manipulator arms. Proc. of the IEEE Int. Conf. on Robotics and Automation, 1084-1090.
Yagiz, N., Hacioglu, Y., and Arslan, Y.Z. (2010). Load transportation by dual arm robot using sliding mode control. J. of Mechanical Science and Technology, volume 24, 1177-1184.
Zribi, M., Karkoub, M., and Huang, L. (2000). Modelling and control of two robotic manipulators handling a constrained object. Applied Mathematical Modelling, volume 24, 881-898.


[^0]:    * The first author would like to thank Govt. of India for the financial support.

