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A Method for Reactive Navigation of Nonholonomic Robots in the Presence of Obstacles ⋆

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Abstract: We present a method for guidance of a Dubins-like vehicle towards a target in a cluttered maze-like environment. The vehicle is strongly information and memory limited. In particular, it has no knowledge about the environment and is not capable of memorizing its characteristics. The sensor system provides only the distance to the nearest obstacle if this distance is within the given sensor range, and also gives a partial access to the target relative bearing angle. We examine the simple memoryless static local controller that implements the simple pursuit guidance at a large distance from the obstacles and combines this guidance with collision avoidance activity in a vicinity of obstacles. This activity is undertaken when and only when the distance to the obstacle is decreasing and consists in a maximally sharp turn. At the start of this turn, its direction is randomly chosen; evidence in favor of the random choice option is presented. Mathematically rigorous analysis of this law is provided and it is proved that the vehicle necessarily reaches the destination. Convergence and performance of the proposed controller are confirmed by computer simulations.

Keywords: Robot Navigation, Collision Avoidance, Sliding Mode Control, Wheeled Robots

1. INTRODUCTION

Unmanned aerial and ground vehicles have been extensively used in recent past for a variety of applications in hazardous and complex environments or plenary surveillance mainly due to their lightweights, inexpensive components, and low power consumptions, see e.g. Quigley et al. [2005], Ahmadzadeh et al. [2006], Girard et al. [2004], Wang et al. [2008] and references therein. Such applications often involve limitations on communications that require the vehicle to operate autonomously for extended periods of time and distances. In these situations, the unmanned vehicles should be equipped with control systems by which they can move autonomously and safely operate in populated and a priory unknown environments.

In order to operate in a cluttered environment, an autonomous vehicle should be able to detect and avoid the obstacles. Online motion planning is theoretically well understood and practically solved in many settings. Current guidance approaches can be generally classified as global or local path planners Lapierre et al. [2007]. Global sensor-based planners use a priori and sensory information to build a complete model of the environment and then try to find the best solution Belkhous et al. [2005]. Alternatively, local path planners use onboard sensors to locally observe a small fraction of an unknown environment to generate the trajectory Deng et al. [2007], Teimoori and Savkin [2010]. The short calculation time in these strategies allows them to be applied in real-time guidance systems. Marginal situation where the open-loop path planning collapses into infinitesimally short time intervals is represented by local reactive controllers. Examples of such planners, which are alike in flavor to our approach, are the biologically inspired DistBug algorithm Kamon and Rivlin [1997] and Pledge algorithm Abelson and diSessa [1980]. They are members of the Bugs family approaches Lumelsky and Stepanov [1990], Kamon et al. [1991], motivated by bugs behavior on crawling along a wall. Similar to ours, in DistBug algorithm the vehicle directly travels towards the target and bypasses the enroute obstacles by following their boundaries in close range. The Pledge algorithm assumes that the searcher is equipped with a compass and is able to follow the obstacle boundary while counting its turning angles. In practice, however, a problem with these strategies is that kinematic equations of the vehicles and their nonholonomic constraints were not taken into account in these algorithms, which is a severe limitation. Theoretical results and performance guarantees concerning these strategies typically suffer from idealistic assumptions, which fulfillment in a practical setting may at least constitute a separate engineering problem and at most be impossible due to kinematic, dynamic, or sensing constraints. For example, this may concern the instructions like ”follow along the obstacle boundary”, ”on reaching the obstacle, turn right without collision”, etc. At the same time, the implications of real-life kinematic and dynamic constraints were well understood within the classic for the robotics research approach that is to decouple the problem into open-loop path planning and design of a controller to follow the proposed path; see e.g., LaValle and Kuffner [2001], Chakraborty et al. [2009] and the literature therein. However, this approach is computationally more demanding as compared with the local reactive algorithms and typically assumes some a priori knowledge about the environment, which may be a serious limitation for real-time guidance systems.

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This paper deals with reactive guidance of an autonomous vehicle towards a target through an unknown cluttered maze-like and arbitrarily shaped environment, with respecting a pre-specified safety margin. The kinematics of the vehicle are described by the standard model of the Dubins-like car, i.e. a nonholonomic system moving with a constant longitudinal speed along planar paths of upper limited curvature without reversing the direction Dubins [1957]. In the literature, this model is applied to many mechanical systems such as wheeled robots, aerial vehicles, missiles and underwater vehicles; see e.g., Fossen [1994], Ben-Asher and Yaesh [1998], Manchester and Savkin [2004], Low et al. [2007] and references therein. The vehicle is strongly information and memory limited. It has access only to the distance to the nearest obstacle if this distance is within the given sensor range, the rate at which this reading evolves over time, and also to the target relative bearing angle. All measured quantities are related to this frame; in particular, the vehicle has no access to an 'absolute' direction, unlike the classic Pledge algorithm. The vehicle is also unable to put landmarks or memorize the visited parts of the environment, either in full or in the form of their partial characteristics.

Under minor technical assumptions, we prove that despite all constraints and limitations, the problem is solved by a very simple reactive local control law. It combines the pursuit guidance with the simplest obstacle avoidance algorithm. During the pursuit guidance, the vehicle heading is rotated towards the target line-of-sight at the maximal velocity; if the vehicle is headed towards the target, it moves in a straight line. Whenever a threat of collision with an obstacle is detected, this guidance law is replaced by the sharpest turn in a randomly chosen direction; which turn is terminated and the pursuit guidance is again put in use as soon as the threat vanishes. The threat situation is defined as that where the distance to the obstacle does not exceed a given threshold and is decreasing. Overall, the proposed strategy does not involve computationally intensive determination of a route to avoid a pop-up obstacle but instead consists in very simple reactions on the current sensor readings. Unlike many other papers in the area of robotic guidance with obstacle avoidance, mathematically rigorous justification of the proposed strategy is offered; we prove that the vehicle inevitably reaches the target with respecting the safety requirement even in complex maze-like environments. The applicability of this law is confirmed by extensive computer simulations.

The body of the paper is organized as follows. Section 2 offers the system description and problem statement, the main assumptions are given in Section 3. Section 4 presents the proposed control law and the main result of the paper, whereas Section 5 illuminates the performance of the closed-loop system during obstacle avoidance maneuvers and provides an evidence in favor of the random choice option. Simulation results are presented in Section 6, whereas Section 7 offers brief conclusions.

2. SYSTEM DESCRIPTION AND PROBLEM SETUP

We consider a planar vehicle modeled as unicycle. It travels with a constant speed v and is controlled by the angular velocity u limited by a given constant $\overline{u}$. There is a steady point target $\boldsymbol{T}$ and a complex obstacle $D \not\ni \boldsymbol{T}$. The objective is to drive the vehicle to the target through the obstacle-free part of the plane $\mathbb{R}^2 \setminus D$. To accomplish this, the vehicle has access to the current distance $d(t)$ to the obstacle and the rate $\dot{d}(t)$ at which this measurement evolves over time. Here

$$d(t) := \mathbf{dist}_D[\boldsymbol{r}(t)] := \min_{\boldsymbol{r}_* \in D} \|\boldsymbol{r}_* - \boldsymbol{r}(t)\|, \tag{1}$$

$\|\cdot\|$ is the standard Euclidian norm, and $\boldsymbol{r}(t) = \boldsymbol{col}[x(t), y(t)]$ is the vector of the vehicle Cartesian coordinates in the world frame. The measurements d and $\dot{d}$ are available if d does not exceed the sensor range: $d \leq d_{\text{range}}$. The vehicle has access to the angle β from the vehicle forward centerline ray to the target.[1] During the entire maneuver, the distance from the vehicle to the obstacle should exceed the given safety margin $d(t) \geq d_{\text{safe}} > 0 \ \forall t$, where $d_{\text{safe}} < d_{\text{range}}$.

Remark 1. The reading β continuously evolves over time.

We employ the following unicycle-like robot model:

$$\begin{array}{l} \dot{x} = v\cos\theta, \\ \dot{y} = v\sin\theta, \\ \dot{\theta} = u \in [-\overline{u}, \overline{u}] \end{array}, \qquad \begin{array}{l} \boldsymbol{r}(0) = \boldsymbol{r}_0 \notin D \\ \theta(0) = \theta_0 \end{array}, \tag{2}$$

where θ gives the vehicle orientation in the world frame. The minimal turning radius of the vehicle is thus given by

$$R = v/\overline{u}. \tag{3}$$

The problem to be considered is as follows. Find a control law that drives the vehicle to the target $\exists t_* : \boldsymbol{r}(t_*) = \mathfrak{T}$ through the obstacle-free part of the plane so that the safety requirement $d(t) \geq d_{\text{safe}} > R$ is always satisfied.

3. ASSUMPTIONS

A *domain* is the closure of a non-empty open connected set. The domain is said to be *simple* if it is bounded and its boundary ∂D is a simple, connected, and piece-wise analytical curve.

Assumption 2. The complex obstacle D is composed of finitely many disjoint simple domains $D = D_1 \cup \ldots \cup D_k$.

Hence $\partial D = \bigcup_i \partial D_i$ and there is a path from $\boldsymbol{r}_0$ to $\mathfrak{T}$ in the obstacle-free part of the plane.

Our proposed navigation strategy consists in switching between the pursuit guidance towards the target and obstacle avoidance maneuvers. Such a maneuver is performed when the vehicle is in a pre-specified vicinity of the obstacle and is based on the measurements of d. To handle the situation where the minimum distance d is attained at two different points of the boundary, special control decisions are required. We postpone their discussion as a topic of further research by assuming that the avoidance maneuvers can be confined to a $d_\star$-vicinity of the obstacle in which such a situation is not encountered. We also slightly enhance this requirement: the minimum distance cannot be attained at a point that may be viewed as two infinitesimally close minimum distance points. This holds for the so called *focal* locations $\boldsymbol{r} \notin D$, i.e., such that $\exists \boldsymbol{r}_* \in \partial D$: $\boldsymbol{r} = \boldsymbol{r}_* - R_\varkappa(\boldsymbol{r}_*) \cdot N(\boldsymbol{r}_*)$ and $\varkappa(\boldsymbol{r}_*) < 0$. Here $\varkappa(\boldsymbol{r}_*)$ is the signed curvature and $R_\varkappa(\boldsymbol{r}_*) := |\varkappa(\boldsymbol{r}_*)|^{-1}$ is the (unsigned) curvature radius of the boundary ∂D at the point $\boldsymbol{r}_* \in \partial D$, and $N(\boldsymbol{r}_*)$ is the inner unit normal to ∂D at $\boldsymbol{r}_*$. It is assumed that the boundary ∂D is positively oriented, i.e., when traveling on it one always has the curve interior D to the left. So $\varkappa(\boldsymbol{r}_*) > 0$ for convexity points and $\varkappa(\boldsymbol{r}_*) < 0$ for concavity points. It is tacitly assumed here that the point $\boldsymbol{r}_*$ is such that ∂D is smooth in its vicinity. We also put $\varkappa(\boldsymbol{r}_*) = \infty, R_\varkappa(\boldsymbol{r}_*) := 0$ for the outer corner points $\boldsymbol{r}_*$ and assume that $0^{-1} := \infty$.

To state the assumptions concerned with the issue of uniqueness, we introduce the following.

[1] This assumption will be relaxed; see (ii) in Remark 7.

Definition 3. Let $d_\star > 0$ be such that any point $\boldsymbol{r} \notin D$ at the distance $\mathbf{dist}_D[\boldsymbol{r}] < d_\star$ from the obstacle is not focal and $\mathbf{dist}_D[\boldsymbol{r}]$ is attained at only one point $\boldsymbol{r}_* \in D$. The supremum $d_\star(D)$ of all such $d_\star$ is called the *uniqueness distance* of the obstacle D; if there are no such $d_\star$, we put $d_\star(D) := 0$.

It should be noted that $d_\star(D) > 0$ if and only if the following claim is true, which is assumed throughout the paper.

Assumption 4. The obstacle boundary ∂D does not contain inner corner points [2].

For convex domains, $d_\star(D) = \infty$; for non-convex ones,

$$d_\star(D) \leq R_D := \inf_{\boldsymbol{r}\in\partial D:\varkappa(\boldsymbol{r})<0} R_\kappa(\boldsymbol{r}). \tag{4}$$

Our proposed navigation strategy combines the pursuit guidance towards the target and obstacle avoidance activities. The latter are commenced as the vehicle comes close enough to the obstacle: $d \leq d_{\text{trig}}$. The avoidance control law may turn the vehicle through the angle up to π radians. As a result, the distance to the obstacle may appear to be no grater than $d_{\text{trig}} - 2R$. With the safety requirement in mind, it is natural to demand that $d_{\text{trig}} - 2R \geq d_{\text{safe}}$. On the other hand, this maneuver is welcome to be within the obstacle uniqueness and visibility margins, as was discussed: $d_{\text{trig}} < d_\star(D), d_{\text{range}}$. By excluding the controller parameter d_{trig}, we arrive at the following.

Assumption 5. Both $d_\star(D)$ and d_{range} exceed $d_{\text{safe}} + 2R$.

The next assumption is unnecessary and is induced by the paper length limitations.

Assumption 6. The infimum R_D from (4) exceeds $d_{\text{safe}} + 3R$.

By (4) and Assumption 5, $R_D > d_{\text{safe}} + 2R$. Assumption 6 enhances this inequality by adding one more R on the right. The infimum over the empty set is defined to be $+\infty$.

If all parts D_i of the complex obstacle D are convex, $R_D = +\infty$ and there are no focal points. So Assumption 6 is true and $d_\star(D)$ equals half of the minimal distance between two different obstacle parts D_i and $D_j, i \neq j$ (if there is only one such part, $d_\star(D) = \infty$).

Assumptions 5 and 6 imply that $d_{\text{safe}} < d_\star(D), d_{\text{range}}, R_D$. Conversely, if these inequalities hold, Assumptions 5 and 6 can be ensured by adjustment of the vehicle cruise speed v:

$$0 < v < \overline{u} \min\left\{\frac{\min\{d_\star(D); d_{\text{range}}\} - d_{\text{safe}}}{2}; \frac{R_D - d_{\text{safe}}}{3}\right\}.$$

This can be used for tuning or even adaptive adjustment of v, if the cruise speed $v > 0$ is tunable and limited by only an upper bound $v \leq \overline{v}$, and estimates of $d_\star(D)$ and R_D are available.

If not only v but also the safety margin d_{safe} is tunable and limited only by the lower bound $d_{\text{safe}} > R$, Assumptions 5, 6 can be always satisfied by a proper choice of v and d_{safe}. Then they should be viewed as recommendations on this choice.

4. THE NAVIGATION AND GUIDANCE STRATEGY

The proposed control strategy combines obstacle bypasses with the simple pursuit guidance $u = \overline{u} \cdot \operatorname{sgn} \beta$ to the target $\mathfrak{T}$. The latter means that the vehicle heading is rotated towards the target line-of-sight at the maximal velocity. When the vehicle is headed towards the target $\beta = 0$, it moves in a straight line to $\mathfrak{T}$. By the above concise formula for the pursuit law, this is implemented as sliding motion, though the equivalent control $u = 0$ Utkin [1992] can be directly applied.

To avoid collision with the obstacle, maximally sharp turn $u = -\sigma\overline{u}$ is performed, where $\sigma = \pm$ gives its rotational direction. This maneuver is undertaken only if there is a threat of collision: the distance to the obstacle is decreasing.

The above two guidance laws are combined in different ways determined by a controller variable, which is called *mode* and takes two values $\mathfrak{A}$ and $\mathfrak{B}$. In the *pursuit* mode $\mathfrak{A}$, only the pursuit guidance is used. In the *avoidance* mode $\mathfrak{B}$, this guidance is used if it gives the firm guarantee of no collision with the obstacle $\dot{d} \geq 0$; otherwise, the companion control law is put in use. The transition $\mathfrak{A} \mapsto \mathfrak{B}$ occurs whenever the distance d from the vehicle to the obstacle reduces to the pre-specified margin $d = d_{\text{trig}}$. The converse switch holds when the distance increases to d_{trig}. In brief, this control strategy can be expressed as the following discontinuous control law:

$$u = \overline{u} \times \begin{cases} \operatorname{sgn}\beta & \Big| \quad \text{if } d > d_{\text{trig}} \\ \begin{cases} \operatorname{sgn}\beta & \text{if } \dot{d} \geq 0 \\ -\sigma & \text{if } \dot{d} < 0 \end{cases} & \Big| \quad \text{if } d \leq d_{\text{trig}} \end{cases}. \tag{5}$$

Whenever d reduces to d_{trig} and so $\mathfrak{A} \mapsto \mathfrak{B}$, the parameter σ is updated. The updated value is picked randomly and independently of the previous choices from $\{+, -\}$, with the value $+$ being drawn with a fixed probability $p \in (0, 1)$.

When mode $\mathfrak{B}$ is activated, $\dot{d} \leq 0$. Even if $\dot{d} = 0$, at first the controller is probationally set to the 'turn' submode $u := -\sigma\overline{u}$. The parameter d_{trig} is chosen so that

$$d_{\text{safe}} + 2R < d_{\text{trig}} < d_\star(D), d_{\text{range}}, R_D - R. \tag{6}$$

Such a choice is possible thanks to Assumptions 5 and 6.

Remark 7. **(i)** It can be shown that the times when σ is updated do not accumulate.

(ii) The control law (5) does not employ the full knowledge of the target bearing β. Formally, only its sign is needed, i.e., the sensor system should recognize whether the target is on the left or right with respect to the vehicle centerline. However due to Remark 1, the vehicle should also recognize whether the transition from the left to the right target view and vice versa holds in the heading-to-target or heading-off-target orientation. To meet these requirements, it for example, suffices that the vehicle has access to only the quadrant containing the current target bearing.

The control law (5) is discontinuous in the mode $\mathfrak{B}$. The solution of the closed-loop system is in the Fillipov's sense.

Now we are in a position to state the main result of the paper.

Theorem 8. Let Assumptions 2, 4—6 hold and both the vehicle initial location and the target be far enough from the obstacle: $\mathbf{dist}_D[\boldsymbol{r}(0)] > d_{\text{trig}} + 2R, \mathbf{dist}_D[\mathfrak{T}] > d_{\text{trig}} + \varepsilon$. Then for any $p \in (0, 1)$, the vehicle arrives at the target $\exists t_* : \boldsymbol{r}(t_*) = \mathfrak{T}$ with respecting the safety margin $\mathbf{dist}_D[\boldsymbol{r}(t)] \geq d_{\text{safe}}\ \forall t \in [0, t_*]$ with probability 1 if the complex obstacle contains only one part ($k = 1$ in Assumption 2) or all these parts D_i are convex.

The requirement to $\boldsymbol{r}_0$ can be relaxed $\mathbf{dist}_D[\boldsymbol{r}_0] > d_{\text{trig}}$ if the vehicle is initially directed towards the target $\beta(0) = 0$.

The proofs of the results stated in this paper will be given in its full version and are available upon request.

[2] When passing such a point in the positive direction, the vector tangential to the boundary ∂D abruptly turns clockwise.

The control law (5) does not involve a feedback from the distance value d itself. A partial reason for this is that by Theorem 8, the distance at which the vehicle bypasses the obstacle is of minor importance for the overall success. Despite the fact that as will be shown in the next section, the obstacle bypass is basically implemented as a sliding motion, which is widely acknowledged as highly robust to disturbances and noises, the above lack of feedback carries a potential of unacceptable shift to and trespass of the safety margin d_{safe} due to the sensor errors, especially systematic ones. However, unacceptable implications of noises and disturbances do not necessarily hold, as is demonstrated by our simulations; see Section 6. Otherwise modifications of the proposed controller may be required, e.g., $\dot{d} - \nu(d)$ can be put in place of $\dot{d}$ in (5), where $\nu(d)$ may be a positive constant or a barrier function (i.e., $\nu(d) > 0$ for $d \approx d_{\text{safe}}^{+}$ and $\nu(d)$ vanishes at a distance from d_{safe}).

Because of switching between two algorithms, the proposed control law is discontinuous and belongs to the class of sliding mode ones Utkin [1992]. Due to the well-known benefits such as high insensitivity to noises, robustness against uncertainties, good dynamic response, etc. Utkin [1992], the sliding mode approach attracts a growing interest in the area of motion control. The major obstacle to implementation of sliding mode controllers is a harmful phenomenon called ”chattering”, i.e., undesirable finite frequency oscillations around the ideal trajectory due to un-modeled system dynamics and constraints. The problem of chattering elimination and reduction has an extensive literature (see Edwards et al. [2006], Lee et al. [2009] for a survey). It offers a variety of effective approaches, including smooth approximation of the discontinuity, inserting low-pass filters/observers into the control loop, combining sliding mode and adaptive control or fuzzy logic techniques, higher order sliding modes, etc. Whether chattering be encountered in applications of the proposed controller, it can be subjected to treatment under the framework of the above general discipline.

From now on, the assumptions of Theorem 8 are supposed to be valid.

5. OBSTACLE AVOIDANCE MANEUVER

Now we illuminate the performance of the proposed control law during obstacle avoidance. We focus on a particular obstacle *avoidance maneuver* (**AM**), which is defined as the motion within uninterrupted mode $\mathfrak{B}$. Due to (6), only one part D_i of the complex obstacle D is partly visible and it is not altered during the entire **AM**. With a slight abuse of notation, we drop the index $_i$ in D_i, thus making D to stand for D_i.

We start with preliminaries. The *δ-equidistant curve* $C(\delta|D)$ of D is the locus of points $\boldsymbol{r}$ at the distance $\mathbf{dist}_D[\boldsymbol{r}] = \delta$ from D. For $0 < \delta < R_D$, this curve is C^1-smooth and piecewise C^2-smooth Kreiszig [1991]. So for $\boldsymbol{r} \in C(\delta|D)$, the one-sided signed [3] curvatures $\varkappa_{\pm}(\boldsymbol{r}|\delta)$ of $C(\delta|D)$ are well-defined. If $\mathbf{dist}_D[\boldsymbol{r}] < d_\star(D)$ and this distance is attained at $\boldsymbol{r}_* \in \partial D$, we have $\varkappa_{\pm}(\boldsymbol{r}|\delta) = \varkappa_{\pm}(\boldsymbol{r}_*) + \delta$ if $\boldsymbol{r}_*$ is not a corner point; such points contribute circular arcs of the radius δ into $C(\delta)$.

By picking $\delta > 0$ small enough, expanding D to the domain encircled by $C(\delta|D)$, and correcting the measurements and controller parameters $\mathfrak{d} := \mathfrak{d} - \delta$ for $\mathfrak{d} := d, d_{\text{safe}}, d_{\text{trig}}, d_{\text{range}}$, we keep all assumptions true and do not alter the operation of the closed-loop system. Hence without any loss of generality, we can assume that ∂D is C^1-smooth in our analysis.

Let s denote the curvilinear abscissa of the point on the boundary ∂D; s ascends in the positive direction. This abscissa is cyclic: s and $s + L$ correspond to a common point, where L is the length of ∂D. We also introduce the natural parametric representation $\boldsymbol{\rho}(s)$ of ∂D and notationally identify s and $\boldsymbol{\rho}(s)$; so $\varkappa(s) := \varkappa[\boldsymbol{\rho}(s)]$, etc. For any $\boldsymbol{r} \notin D$ within the uniqueness margin $\mathbf{dist}_D[\boldsymbol{r}] < d_\star(D)$, the symbol $s(\boldsymbol{r})$ stands for the curvilinear abscissa of the point $\boldsymbol{r}_*$ closest to $\boldsymbol{r}$; we also put $s(t) := s[\boldsymbol{r}(t)]$, where $\boldsymbol{r}(t)$ is the vehicle location at time t.

Writing $f(\eta_* \pm^{\approx} 0) > 0$ means that there exists $\delta > 0$ such that $f(\eta) > 0$ if $0 < \pm(\eta - \eta_*) < \delta$. The similar notations, e.g., $f(\eta_* \pm^{\approx} 0) \leq 0$, are defined likewise.

5.1 Vehicle Performance during Avoidance Maneuvers

Now we are in a position to state the main result of the section.

Proposition 9. Let the vehicle be driven by the control law (5) and mode $\mathfrak{B}$ be activated with zero target bearing $\beta(t_*) = 0$ at a time t_*. Then the following statements hold:

(i) There exists $\tau \geq t_*$ such that the vehicle moves with the maximal steering angle $u \equiv -\overline{u}\sigma$ and the distance to the obstacle decreases $\dot{d} \leq 0$ until τ, and at $t = \tau$, the sliding motion along the equidistant curve (**SMEC**) $C\left(\mathbf{dist}_D[\tau]|D\right)$ [4] is started with $\sigma\dot{s} > 0$ and $\beta\dot{s} > 0$;

(ii) SMEC holds until β arrives at 0 at a time when $\varkappa[s(t) + \sigma^{\approx}0] > 0$, which sooner or later holds and after which a straight move to the target (**SMT**) [5] is commenced;

(iii) During SMT, the vehicle first does not approach the obstacle $\dot{d} \geq 0$ and either the triggering threshold d_{trig} is ultimately trespassed and so mode $\mathfrak{B}$ is switched off, or a situation is encountered where $\dot{d}(t) = 0$ and $\varkappa[s(t) + \sigma^{\approx}0] < 0$. When it is encountered, the vehicle starts SMEC related to the current distance;

(iv) There may be several transitions from SMEC to SMT and vice versa, all obeying the rules from (ii), (iii);

(v) The number of transitions is finite and finally the vehicle does trespass the triggering threshold d_{trig}, thus terminating the considered avoidance maneuver;

(vi) Except for the initial turn described in (i), the vehicle maintains a definite direction of bypassing the obstacle: $\dot{s}$ is constantly positive if $\sigma = +$ (counterclockwise bypass) and negative if $\sigma = -$ (clockwise bypass).

By (5), AM is commenced with $\dot{d}(t_*) \leq 0$. In the marginal case where $\dot{d}(t_*) = 0$, the initial turn may have the zero duration $\tau = t_*$, as is specified in the following.

Remark 10. If $\dot{d}(t_*) = 0$, the initial turn has the zero duration if and only if $\sigma\dot{s}(t_*) > 0$. Then the following claims are true:

(1) If $\varkappa[s(t_*) + \sigma \cdot^{\approx} 0] < 0$, SMEC is immediately started;

(2) If $\varkappa[s(t_*) + \sigma \cdot^{\approx} 0] \geq 0$, the duration of SMEC is zero, and SMT is continued.

The assumption $\beta(t_*) = 0$ of Proposition 9 holds for the first AM due to the assumption $\mathbf{dist}_D[\boldsymbol{r}(0)] > d_{\text{trig}} + 2R$ from Theorem 8: since $\mathbf{dist}_D[\boldsymbol{r}_0] > d_{\text{trig}} + 2R$, the pursuit guidance law turns the vehicle towards $\mathfrak{T}$ earlier than the threshold d_{trig}

[3] The orientation of $C(\delta|D)$ is induced by the positive orientation of ∂D.

[4] In the system state space, this is sliding motion along the surface $\dot{d} = 0$

[5] In the system state space, is sliding motion along the surface $\beta = 0$

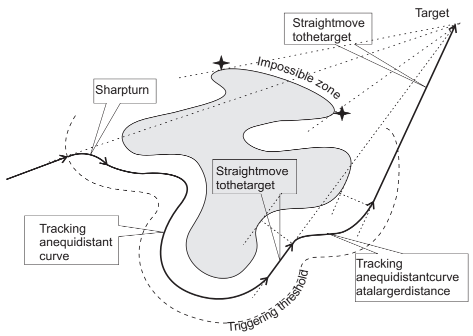


Fig. 1. Obstacle avoidance.

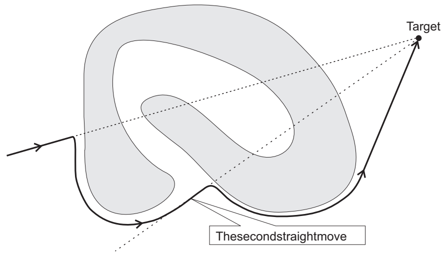
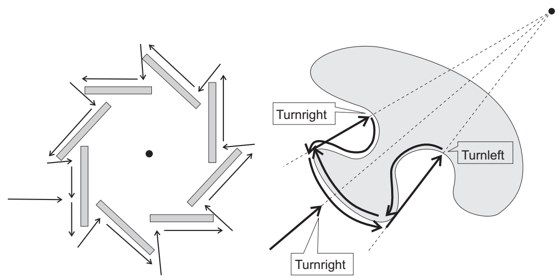


Fig. 2. Obstacle bypass by means of two avoidance maneuvers.



(a) (b)

Fig. 3. Insufficiency of (a) only-right-turns and (b) cycle-left-and-right-turns options

for activation of AM is encountered. It also holds for all other AM's since any AM ends in course of SMT by Proposition 9.

The behavior described in Proposition 9 is illustrated in Fig. 1.

Bypass of a given obstacle may include several AM's. This is illustrated in Fig. 2: during the second SMT, the vehicle trespasses the uniqueness distance $d_\star(D)$ to the obstacle and thus the threshold d_{trig} by (6). So mode $\mathfrak{B}$ is switched on and off twice. At the same time, Proposition 9 guarantees that if the entire obstacle is convex, the vehicle performs at most one AM.

5.2 *The Importance of Being Random*

It follows that in the case of a single convex obstacle, the target is reached irrespective of the choice of $\sigma = \pm 1$. However this may not be the case if the obstacle boundary is concave at some points or the obstacle is complex, e.g., is composed of several convex parts. This is illustrated in Fig. 3 for some particular scenarios. Moreover, insufficient is any deterministic finite-memory algorithm of updating σ:

$$\begin{aligned} \sigma(t_i^+) &:= \Sigma\left[m(t_i^-), \{\beta^{(j)}(t_i^-)\}_{j=0}^p, \{d^{(j)}(t_i^-)\}_{j=0}^q\right], \\ m(t_i^+) &:= \mathcal{M}\left[m(t_i^-), \{\beta^{(j)}(t_i^-)\}_{j=0}^p, \{d^{(j)}(t_i^-)\}_{j=0}^q\right]. \end{aligned} \tag{7}$$

Here $f(t^\pm)$ stands for the one-sided limit, $f^{(j)}$ is the jth derivative, the element m of a finite *memory alphabet* M represents the current state of the memory device (its initial state $m(0) = m_0$ is given), and t_i are the times when the switch $\mathfrak{A} \mapsto \mathfrak{B}$ is implemented. The integers $p \geq 0, q \geq 1$ are given; $p > 0$ or $q > 1$ means that even more sensing capabilities are assumed as compared with our basic assumptions. The algorithm (7) is coupled with the basic control law (5).

The aforementioned insufficiency is justified by the following.

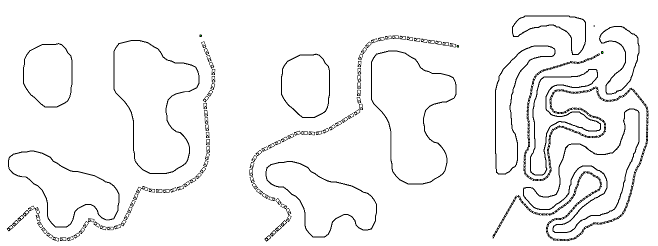
Lemma 11. For any algorithm of the form (7), there exist a complex obstacle D composed of finitely many convex parts D_i, the target location $\mathfrak{T}$, and the vehicle initial location $\boldsymbol{r}(0)$ for which the vehicle driven by this algorithm does not reach the target. Specifically, since some time instant it becomes involved in a periodic motion along a path not containing the target. This obstacle and locations can be chosen so that Assumptions 2, 4—6 are satisfied and the distances from D to $\mathfrak{T}$ and $\boldsymbol{r}(0)$, as well as from $\mathfrak{T}$ to the above periodic path are as large as desired.

By Theorem 8, randomization carries a potential to overcome the discussed insufficiency of deterministic algorithms, especially if physical generator of randomness is employed. Randomization aids to cope with uncertainty about complex scenes. If the scene is relatively simple, even the simplest deterministic algorithms may be successful Matveev et al. [2011].

6. SIMULATIONS

To verify the proposed control law, simulations were carried out in a range of scenarios. For all of them, the control was updated every $0.1\ s$ and the following parameters were chosen: $d_{trig} = 12\text{m}$, $u_{max} = 1\text{rads}^{-1}$, $v = 3\text{ms}^{-1}$.

In Fig. 4(a), σ was chosen three times, every time it was the counter-clockwise direction around the obstacles. In other words, σ successively took the values $+1, +1, +1$. Note that the distance to the obstacle changes after the vehicle passes its concavity. Due to the random choice of σ, many other paths are possible. Fig. 4(b) depicts another path through the same set of obstacles, where σ ran through the sequence $[-1, +1, -1]$. Fig. 4(c) shows that the randomized decision making ensures that the vehicle eventually finds the target in a more complex and unknown environment. In this case, $\sigma = +1, -1, -1, +1$.



(a) (b) (c)

Fig. 4. Navigation with a fairly simple scenario; (b) An alternative path; (c) More complex environment.

Though Theorem 8 deals with the steady target, the proposed controller is still able to achieve good results when the target moves. This is illustrated in Fig. 5. In both cases, the target was modeled as unicycle whose speed and maneuverability are less than those of the pursuer. Furthermore, the target does not react to the pursuer. The dotted path corresponds to the target, whereas the path of the pursuer is filled with small rectangles.

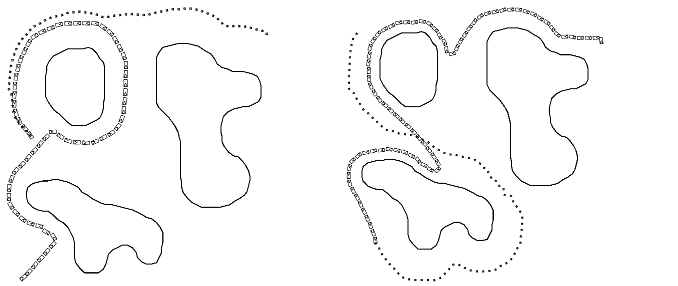


Fig. 5. Capturing a moving target.

A random Gaussian white noise was also added as both the sensor error and system disturbance. The standard deviations of the noises in $\dot{d}$ and $\dot{\theta}$ are $1.8m/s$ and $0.7rad/s$, respectively. These are relatively large errors, which would be unlikely to be met in the real world. It can be seen in Fig. 6 that the control law still satisfactory guides the vehicle. Various paths in this figure correspond to various realizations of the noises.

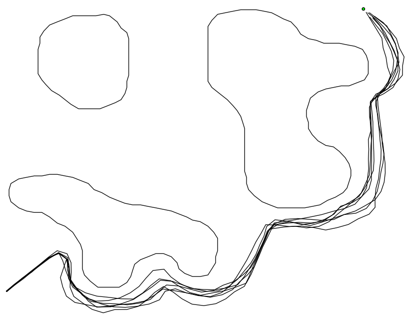


Fig. 6. Performance under random noises.

7. CONCLUSIONS AND FUTURE WORK

A sliding mode based method for a unicycle-like vehicle control has been proposed and justified both theoretically and by computer simulations. Future work includes theoretical analysis of the sensor noise and external disturbance implications with elaboration of modified and more robust versions of the proposed control law. Extension of this law on the case where the minimal distance to the obstacle is attained at not necessarily unique point is on the agenda. Detailed analysis of the performance of the closed-loop system for dynamic environments and moving target is also on the way.

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