

## Modelling of an Under-Hip Prosthesis with Ankle and Knee Trajectory Control by Using Human Gait Analysis

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**Abstract:** In this paper, depending on bipedal human walking analysis, knee and ankle joint control of special under-hip prosthesis is investigated. After the designing of the prosthesis and its parts by using SolidWorks, strength analyses under various pedestal loads are given, to ensure an optimal and convenient model.

Depending on measurements of hip, knee and ankle motions by using vision techniques, bipedal human gait analysis is investigated. After these measurements using the hip motion as reference, knee and ankle joint angles are derived and calculated. The calculated values are then used to control the motions of knee and ankle joints via DC-motors so that the investigated trajectories for optimal bipedal walking could be realized.

A mathematical model for bipedal walking is then executed as a combination of two serial manipulators, each having two revolute joints, in other words, having two degrees of freedom. Inverse kinematics analysis and recursive Newton-Euler computation methods are given to obtain the dynamic equations, which describe the motion of the walking system. For desired walking characteristics, knee and ankle trajectories are derived.

With this novel method both ankle and knee joint positions in case of upper knee amputees can be determined and controlled for various gait instants.

**Keywords:** Bipedal walking, Feed forward compensation, Inverse kinematics analysis, Recursive Newton-Euler computation, Trajectory planning

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### 1. INTRODUCTION

Walking is one of the most important and composite process of human body. Hip, Knee and Ankle have significant role at stable bipedal human walking and the worth of optimal knee prosthesis is evident. In order to understand the differences in prosthetic knees and their applications, it is first necessary to investigate the human gait analysis. In the following a vision based practical method to measure the hip, knee and ankle angles is given.

Prosthesis is a device designed to replace, as much as possible, the function or appearance of a missing limb or body part ideally, prosthesis must be comfortable to wear, easy to put on and remove, light weight, durable, and cosmetically pleasing. Furthermore, prosthesis must function well mechanically and require only reasonable maintenance. Finally, prosthetic use largely depends on the motivation of the individual, as none of the above characteristics matter if the patient will not wear the prosthesis.

### 2. TRANSFEMORAL AMPUTATION

#### 2.1 Limb Amputation

There are several levels at which the surgeon can amputate a limb (Fig. 1), the most common are:

- Through the foot
- Ankle (Syme)
- Below the knee (transtibial)
- Through the knee (knee disarticulation)
- Above the knee (transfemoral)

Most lower extremity amputations occur in individuals older than 60 years and result from disease complications. Complications of diabetes and peripheral vascular occlusive disease are the leading causes of amputation (65%), followed, among disease-related causes, by complications of thromboembolic disease and vasculitis. Trauma is the second

most common cause of lower extremity amputation (25%) and typically occurs in the young male population. Tumors and congenital malformations less commonly (5% each) result in lower extremity amputation.

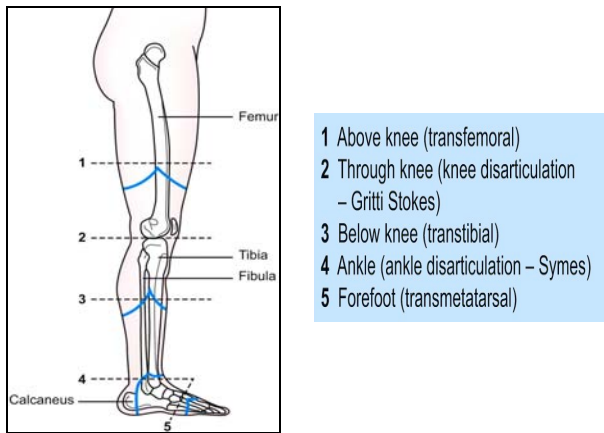


Fig. 1. Common levels of limb amputation.

The level of amputation depends on where there is the greatest blood flow and, therefore, the greatest possibility of healing. The surgeon often attempts to save the knee, because the energy cost of walking with an intact knee is much less than without it. No amputation is “easy” to adapt to, but the transfemoral certainly offers more challenges than amputations in the calf or foot. Generally the higher the amputation level, the more energy needed for walking.

The shape and shortened length of the residual foot increases the difficulty of fitting it with a partial foot prosthesis that can provide adequate suspension and/or a forefoot lever for ambulation. Successful prosthetic restoration often requires a prosthetic or orthotic design that is more substantial and extends proximal to the ankle. The ideal length is from the proximal one third to the middle of the limb.

The under (lower)-knee prosthesis taken into account in this survey. The main parts and their list are also given (Fig. 2).

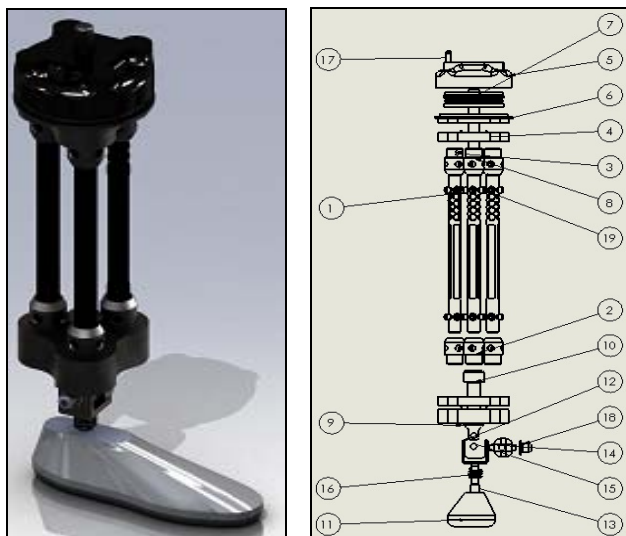


Fig. 2. General view of prostheses and its main parts.

## 2.2 Determinants of a successful outcome with prosthetic use

To insure a successful prosthetic outcome, it is necessary to determine the goals of each individual amputee. This should include the patient's expectations for functional activities with the prosthesis. The prosthesis must be comfortable to wear, easy to put on and remove, light weight and durable, and cosmetically pleasing. Furthermore, the prosthesis must function well mechanically and require reasonably low maintenance. Successful prosthetic intervention should be judged by patient-specific functional outcomes.

## 2.3 Prosthetic fitting and testing

When the suture line has completely healed, fitting for the prosthesis can begin. Each prosthesis must be individually fitted to the patient. Prostheses are either preparatory or definitive. The preparatory prosthesis is fitted while the residual limb is still remoulding. This allows the patient to commence the rehabilitation program, which includes the following activities:

- Training in the donning and removal of the prosthesis
- Transfer training
- Building of wear tolerance
- Attainment of balance
- Ambulation with the prosthesis several weeks prior to final residual limb volume stabilization

Use of a preparatory prosthesis often results in a better fit of the final prosthesis because the preparatory socket can be used to mold the residual limb into the desired shape and stable volume. Because of the materials from which they are constructed, most preparatory prostheses are easily modified. Once these basic requirements are met, stability, ease of movement, energy efficiency, and the appearance of a natural gait are the goals to be achieved with prosthetic training and use. Sometimes, a preparatory prosthesis is not feasible because of financial considerations.

## 2.4 SolidWorks design and analysis of a special lower extremity prosthesis

A SolidWorks model and analysis of the prosthesis (Fig. 2) is made in the study. After the general design, the parts of prosthesis are modelled in detail and different strength analyses against various pedestal loads are then made by using animations. Strain analysis of the prosthesis and upper joint part are given in simulation (Fig. 3).

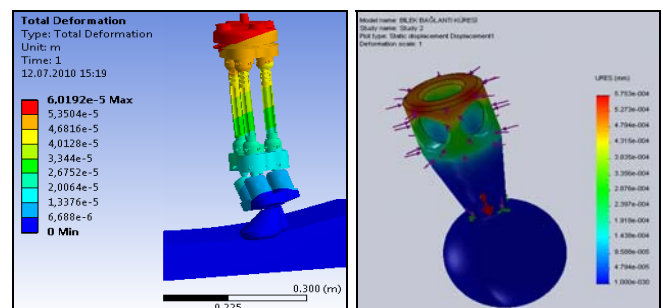


Fig. 3. Strain analysis of the prosthesis and upper joint part.

### 3. LEG BIOMECHANICS

In the second part of study, bipedal walking model of a human like motion is introduced. Walking is considered as a motion composed of a stance leg phase and a swing leg phase. Double support conditions arise only at the beginning and the end of the motion. The trajectories of hip and ankle joints can be generated for the given hip and ankle motion constraints. These trajectories can then be used for the analysis and control of the bipedal locomotion of the model. After the trajectories are obtained, the inverse kinematics analyses can be made in order to find the joint variables for the given reference motions in both stance and swing leg phases (Fig. 4).

Knee flexion occurs when the leg is bent dorsally (towards the back), whereas extension occurs when the leg is straightened. The muscular insertions responsible for flexing and extending the leg are not present in a transfemoral amputee. Therefore, the amputee must compensate by changing their gait or by means of their prosthetic knee.

When we walk, one foot or the other is always in contact with the ground. Each leg is constantly transitioning, going from standing and supporting our weight to swinging through from behind to in front of us to get ready for the next step. The legs are always transitioning from stance to swing, which is why our walking motion is divided into what we call the “swing phase” and the “stance phase” (Fig. 4).

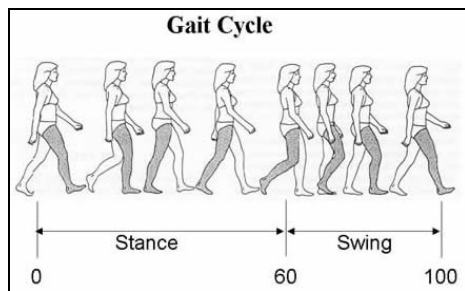


Fig. 4 Gait cycle.

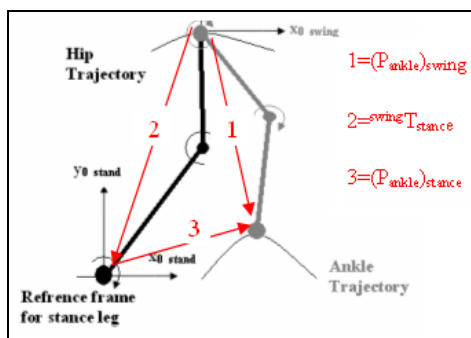


Fig. 5 Bipedal walking Phases

### 4. THE MODEL

The walking model is composed of two serial R-R manipulators with 2 degrees of freedom, namely stance leg and swing leg (Fig. 5). During motion, each leg changes its state from stance leg to swing leg periodically and it is

assumed that the legs can only be in the same phase (stance leg phase) at the beginning and at the end of the motion. Each three joints of the legs can be actuated (hip joint, knee joint and ankle joint) but it is important to note that only two of these three joints are actuated simultaneously according to the phase of walking. This means, in the stance leg phase, the ankle and knee joints are actuated while in the swing leg phase, the ankle joint stands still but hip joint is actuated.

In human walking, hip and ankle joints of walker track certain paths to stabilize bipedal movement. Motion of these joints within the reference frame is periodic and mostly harmonic and at every instant, all the joints of walking mechanism change their positions to maintain these trajectories. Trajectories must be determined by considering two important factors: obtaining a stable motion and avoiding the obstacles on the walking area. Considering these two factors, both hip and ankle positions and orientations on a trajectory must be chosen appropriately.

### 5. INVERSE KINEMATICS OF BIPEDAL WALKER

The joint variables  $\Theta_1$  (for ankle joint in stance leg and for hip joint in swing leg phase) and  $\Theta_2$  (for knee) of the legs can be computed from the given hip and ankle trajectories. For this purpose, the inverse kinematics analysis must be done. For the stance leg phase, we have two trigonometric expressions as:

$$\begin{cases} p_x = a_1 \cdot c\theta_1 + a_2 \cdot c\theta_{12} \\ p_y = a_1 \cdot s\theta_1 + a_2 \cdot s\theta_{12} \end{cases}_{stance} \quad (1)$$

where  $\alpha_1, \alpha_2$  are the length of lower leg and upper leg respectively and  $\{P(p_x, p_y)\}_{stance}$  is the position of hip joint with respect to the reference base frame of stance leg. From (1), the joint variables can be obtained after some manipulations.

Taking the derivatives of (1), the velocity expressions can be obtained as:

$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \end{bmatrix} = \mathbf{J} \cdot \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}_{stance} \quad (2)$$

where the first term in right hand side represents the Jacobian Matrix J. Joint variables can be obtained by multiplying both sides of (2) by inverse of the Jacobian.

Differentiating (2) with respect to  $\Theta_1$  and  $\Theta_1 + \Theta_2$ , and joint variables with respect to world frame (coincident with stance leg base reference frame), the acceleration term can be found as:

$$\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_1 + \ddot{\theta}_2 \end{bmatrix} = \mathbf{K} \cdot \begin{bmatrix} a_2 c\theta_{12} & a_2 s\theta_{12} \\ -a_1 c\theta_1 & -a_1 s\theta_{12} \end{bmatrix} \cdot \begin{bmatrix} \dot{v}_x \\ \dot{v}_y \end{bmatrix} \quad (3)$$

$$+ \mathbf{K} \cdot \begin{bmatrix} a_1 a_2 c\theta_2 & a_2^2 \\ -a_1^2 & -a_1 a_2 c\theta_2 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_1^2 \\ (\dot{\theta}_1 + \dot{\theta}_2)^2 \end{bmatrix}$$

$$\mathbf{K} = \frac{1}{a_1 a_2 \cdot s\theta_2} \quad (4)$$

where

In swing leg phase, different from the stance leg phase, the reference frame is attached to the hip joint which moves within the hip trajectory. For each time sequence, ankle position is obtained with inverse transformation matrix which represents the coordinate transformation from world frame to swing leg reference frame. This transformation is given in (5).

$$\{p_{ankle}\}_{swing} = {}^{swing}T_{stance} \{p_{ankle}\}_{stance} \quad (5)$$

All the equations for motion are derived according to this manipulation.

## 6. TRAJECTORY PLANNING

Also hip and ankle trajectories can be computed theoretically in world space through following steps:

- First, space curve which passes through all of the desired points in hip trajectories are specified.
- Then, parameters of hip trajectory curve are specified to assure that the hip tracks this curve in the desired fashion (with the desired velocity).
- The corresponding joint variables in time domain are calculated.
- For ankle trajectory, three critical points corresponding to extremum poses of bipedal walker are defined.
- By solving kinematic equations of the bipedal walker, the joint angles of the swing leg are obtained for the given poses.
- Time dependent angular position parameters are calculated in configuration space.

### 6.1 Hip Trajectory Generation

The trajectory curve for hip joint can be chosen as a parabola with the given characteristics:

$$y_h(t) = A \cdot [x_h(t) - a] \cdot [x_h(t) - b] \quad (6)$$

where  $y_h$  and  $x_h$  are the Cartesian coordinate components of hip trajectory and A, a, b are the parameters of the trajectory which are going to be evaluated for the given constraints:

- P1(-L<sub>w</sub>/4,0) → First point constraints
- P2(0,h<sub>max</sub>) → Midpoint constraints
- P3(L<sub>w</sub>/4,0) → End point constraints

where L<sub>w</sub> is the gait length and h<sub>max</sub> is the maximum displacement in y direction.

Since the motion of hip is time dependent, time dependent constraints must also be chosen in order to obtain the desired path. These constraints are X<sub>hip</sub>(0) = -L<sub>w</sub>/4 and X<sub>hip</sub>(t<sub>gait</sub>) = L<sub>w</sub>/4 for the positions and X'<sub>hip</sub>(t<sub>gait</sub>) = 0 for velocity of the walker as the time interval for walking is (0,t<sub>gait</sub>). Time dependent variation of X<sub>hip</sub>(t) will be:

$$x_{hip}(t) = A_t \cdot \sin^2(b_t \cdot t) + C \quad (7)$$

Where A<sub>t</sub>, b<sub>t</sub>, C are the time domain parameters and they are going to be found for the given time dependent constraints. After x component of the trajectory is obtained, y component can be calculated from (6).

### 6.2 Ankle Trajectory Generation

Different then the hip trajectory, trajectory of the ankle joint in swing leg motion is generated in the configuration space. Here, the critical points are the extremum points of the trajectory and they define the start point (-L<sub>w</sub>/2, 0), midpoint (0, L<sub>w</sub>/2) and the end point constraints (L<sub>w</sub>/2, 0). L<sub>lw</sub> represents the lower leg length. These constraints are given in task space in such manner that they satisfy the poses correspond to ankle joint positions of swing leg when stance leg has certain pose to track hip trajectory.

Algorithm used to generate the ankle trajectory can be given as follows:

- Reference swing leg position with respect to moving reference frame attached to hip joint is calculated by using the inverse stance leg transformation given in (5).

For the given critical points, inverse kinematics equations are solved. Hence, configuration space constraints are obtained from the world space coordinates.

Since joint variables are time dependent, polynomial given in (8) and (9) are used to obtain desired motion.

$$\theta_{hip}(t) = a_{hip} \cdot t^2 + b_{hip} \cdot t + c_{hip} \quad (8)$$

$$\theta_{knee}(t) = a_{knee} \cdot t^2 + b_{knee} \cdot t + c_{knee} \quad (9)$$

Parameters in (8) and (9) are selected properly to satisfy the configuration space constraints.

## 7. RECURSIVE NEWTON-EULER COMPUTATION

Lastly to determine the generalized torques recursive Newton-Euler method can be applied to the walking model. The most significant aspect of Newton-Euler formulation is that the computation time of the applied torques can be reduced significantly to allow real time control. Since the joint variables and their derivatives are obtained from the inverse kinematics analysis, the generalized torques can be evaluated. At the end of the Newton-Euler recursive computation, the dynamics of the stance leg of the biped is derived as follows:

$$\tau = \mathbf{M}(\theta)\ddot{\theta} + \mathbf{V}(\theta, \dot{\theta}) + \mathbf{G}(\theta) + \mathbf{F}(\theta) + \tau_e \quad (10)$$

where, M(θ) is the manipulator inertia matrix, v(θ, θ') is the vector of centrifugal and coriolis forces, G(θ) is the vector of gravitational forces, F(θ) is the end effector forces and τ<sub>e</sub> is the end effector moment. End effector forces and moments arise due the weight and the movements of the upper body.

In swing leg phase, it is assumed that there are no external forces acts on the system, since the forces which affect the motion of the ankle arise from the swing leg dynamics only. When the manipulations given by (5) are made, According to

above manipulations, swing leg dynamics can be obtained from (10) as:

$$\tau = M_{swing}(\theta)\ddot{\theta} + V_{swing}(\theta, \dot{\theta}) + G(\theta) \quad (11)$$

where  $M_{swing}(\Theta)$  and  $V_{swing}(\Theta, \Theta')$  are obtained by using Recursive Newton-Euler computation method.

The motion of the ankle joint through a DC Servo motor can be modelled in state space form.  $\tau=T_{La}$  represents the torque disturbance acting at the joints and the torques which are necessary for tracking the given trajectories

From the electrical and mechanical characteristic of DC motor, the state space representation of the actuator can be obtained as follows:

$$\frac{d}{dt} \begin{bmatrix} i_a \\ \omega_a \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_{va}}{L_a} \\ \frac{K_{ta}}{J_a} & -\frac{B_a}{J_a} \end{bmatrix} \begin{bmatrix} i_a \\ \omega_a \end{bmatrix} + \begin{bmatrix} 1/L_a & 0 \\ 0 & -1/J_a \end{bmatrix} \begin{bmatrix} V_a \\ T_{La} \end{bmatrix} \quad (16)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_a \\ \omega_a \end{bmatrix} \quad (17)$$

where,  $i_a$ ,  $R_a$ ,  $L_a$ ,  $K_{va}$ ,  $\omega_a$ ,  $V_a$ ,  $K_{ta}$ ,  $J_a$ ,  $B_a$ ,  $T_{La}$  are the armature current, armature resistance, inductance of armature coil, velocity constant, rotational velocity of the armature, armature voltage, torque constant, inertia of the rotor, damping coefficient of mechanical system and torque of mechanical load respectively. The effect of torque produced at knee joint is considered as a disturbance for the actuator at the ankle joint.

When actuator dynamics is considered analytically it is seen that equivalent moment of inertia of the ankle is a time dependent parameter. This fact turns the equation of mechanical part of the actuator transfer function into a time variant system.

$$J_a = \frac{M_1}{3} L_1^2 + \frac{M_2}{3} (L_1^2 + L_2^2 + L_1 L_2 \cos \theta_2) \quad (22)$$

where  $\Theta_2 = \Theta_2(t)$ .

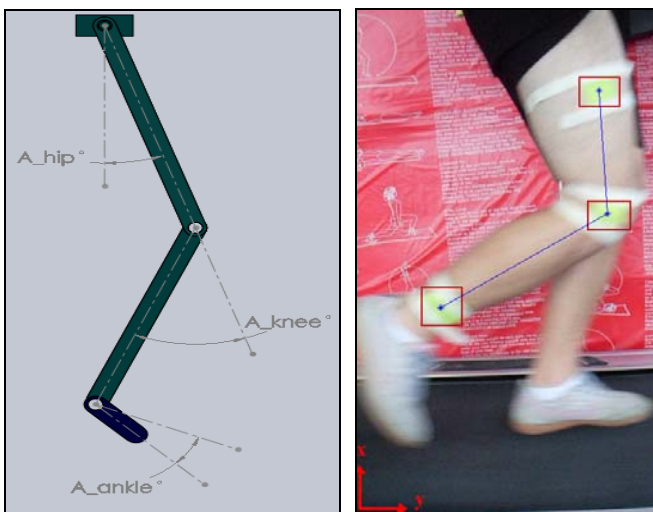


Fig. 6 Hip, Knee and Ankle Joint angles and their vision based measurements

## 8. APPLICATION OF CONTROL

The control problem for the bipedal locomotion considered here is the problem of determining the time history of joint inputs required to cause an optimal gait.

For this purpose by developing a vision based practical method, the hip, knee and ankle motions of the leg are investigated by using sticky marks (Fig.6 ). As the person walks, the changes of the positions of the marks are analyzed by using computer vision techniques, and logged.

The time dependent changes of hip, knee and ankle angles (Fig. 7) are very important for gait analysis and stable walking. The measured Simultaneous time dependent change of joint angles during gait period from a healthy leg during a gait period are given (Fig. 8).

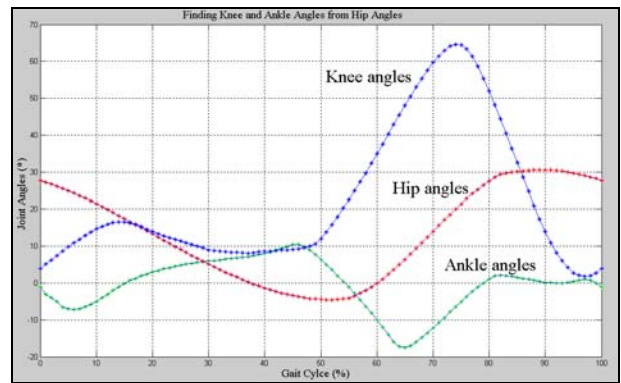


Fig. 7. Database of Hip, Knee and Ankle angles per Gait Cycle

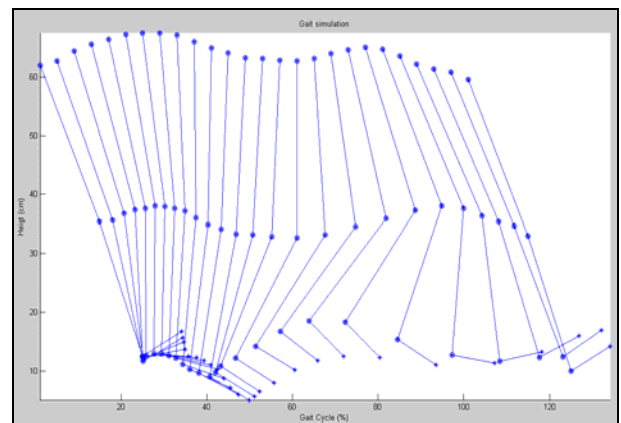


Fig. 8. Simultaneous time dependent change of joint angles during gait period

A novel methodology for ankle and knee joint position control is applied in this survey. According to the measured values of relevant joint angles, their instant positions (Fig. 8) can be calculated and given in form of a database. For a given hip position, the relevant time instant position of ankle and knee joint can then be calculated through this database and these instant values can be used as reference for ankle besides knee servo motor position which is again feedback controlled.

A basic control application scheme for joint ankle position is given (Fig. 9). Also knee joint angle can be calculated and controlled in the same way.

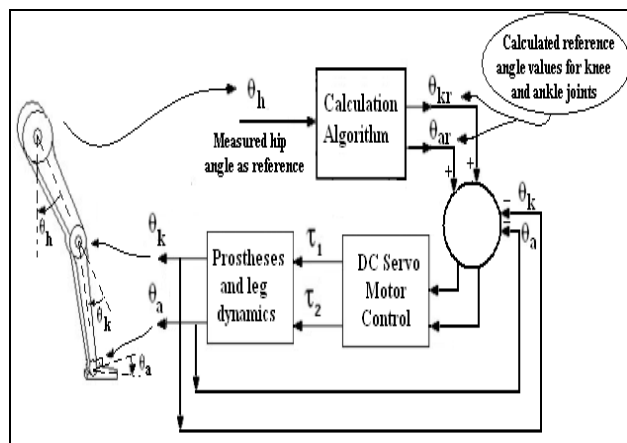


Fig. 9. Ankle joint Control application Scheme.

## 9. CONCLUSION

In this paper, we have described our proposed method to control the walking dynamics of a bipedal walker. Generation of hip and ankle motions varies in the manner that, the constraints for hip trajectories are set due to the motion of hip in world space, while that of the ankle trajectories are defined in configuration space in relevance with specified walking poses for three critical situations.

Dynamical analysis of biped walker is done as a combination of two 2 DoF manipulator. Operation conditions for each mechanism are defined as follows: One in the first quadrant (stance leg and hip trajectory) and the other is in the last quadrant (swing leg and ankle trajectory).

Resultant kinematics data are used as references for control and calculated torques are taken into account as disturbance effect. The control paradigm is based on developing feed-forward load torque compensator to cancel the deviations from given velocity profiles in a PI velocity controlled loop. Simulation results (Fig. 10) verify that with given control scheme (Fig. 9) satisfies walking control of biped.

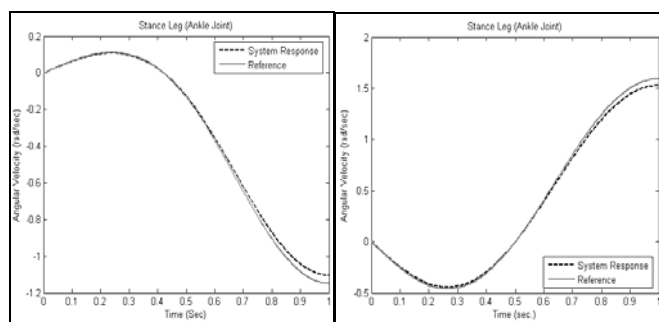


Fig. 10. Control simulation of knee and ankle angular velocity for stance phase.

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