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Evaluation of Torque Observer in Automotive Context ⋆

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Abstract:
This paper describes how it is possible to estimate online the individual torque at each wheel of an automotive vehicle by using an unknown input observer together with a simple longitudinal 4 wheels model. The necessary measurements are the usual rotation speed of each wheel plus the vertical load at the center of each wheel. By doing this, this study anticipates the affordability of a new generation of wheel bearing with embedded measurements of transmitted forces. Successful simulated experimentations of the method are shown as well as its present limits.

1. INTRODUCTION

In 2008, the European Commission has proposed to spread Advanced Driver Assistant Systems (ADAS) for active safety on any vehicle sold in the European Union and manufactured after 2014. The concerned ADAS are Electronic Stability Control (ESC), Advanced Emergency Braking Systems (AEBS) an Brake Assist System (BAS). “These measures will reduce fatal casualties in traffic by an estimated 5000 a year” says the report European Commission [2008].

These ADAS build effective automatic control strategies from a vehicle dynamical state whose components are either measured or estimated. Embedded measurements such as wheels rotation speeds, yaw rate or lateral acceleration are now generalized but nowadays economic conditions leads to kill the production costs and at least ban the introduction of new components. Nevertheless, the scientific and technological progression is not stopped and it should be noted that the classical wheel bearings have evolved from a purely mechanical device to a mechatronic one that now integrates rotation speed measurements. Recent works by SNR (Kwapisz et al. [2008], Kwapisz [2008]) and SKF (Holweg [2008]) have shown the feasibility of the integration of forces measurements in a future generation of affordable wheel bearings. The cost per wheel of measuring tire/road forces and torques applied to the wheel would decrease from now hundred thousand euros to a small amount of euros. The present communication anticipates the use of these newly affordable measurements by showing how it is possible to estimate the torques applied to each wheel from the classical measurements set plus the vertical component of the force at each wheel bearing.

Those estimated torques may be used in Active Driveline Torque Management (ADTM - Pyabongkarn et al. [2010]) in the ESC in complement of the existing brake-based actuation with the obvious economic and ecological advantages that energy is not necessarily lost during the actuation. These variables are now issued from procedures that use open-loop calibrated models. For example, the engine torques applied to the wheels are computed from the engine speed by using look-up tables and a powertrain model. In the same way, the braking torque is computed from the measured pressure in the master cylinder and the “Electronic Brake Distribution” function that uses a model of brake disks friction. Some works like Ray [1997] and M’Sirdi et al. [2008] assume that resultant torques applied to the wheels are “measured” although they are actually computed in open-loop. An alternative strategy based upon state observation theory is proposed here with the advantages of the closed loop structure. This technology has been spread up in the industry under the “software sensor” denomination. Examples are the works of Doumiati et al. [2009] who present experimental embedded validations of tire/road force observer and Stéphant et al. [2007] who give a sideslip angle observer.

This work proposes to estimate the resultant torques applied to each wheel of an automobile. The observer uses measurements of wheels rotation speeds and normal forces together with a non-linear model of the vehicle (section 2). The observer theory and application are presented (section 3). A vehicle simulator is used to validate the vehicle model (section 4.3) and the designed observer (section 4.4). It will be shown that the vehicle state observation is accurate in spite of the fact that resultant torques applied to the wheels are unknown. In section 5 the present limits of the proposed observer are discussed. The notations are sumed up in section 7.

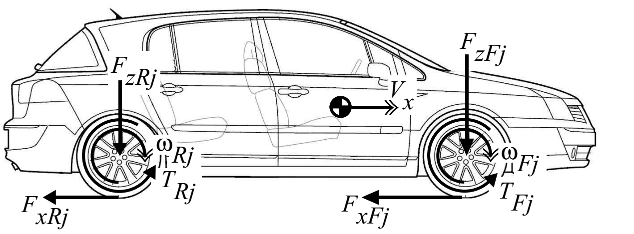

Fig. 1. Vehicle variables ; j = R(Right), L(Left).

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2. MECHANICAL MODEL

Consider the longitudinal motion of a vehicle. The model used in this paper is a four-wheels one. Its variables are presented on Fig. 1. V_x denotes the speed and M_v the mass. Using the usual subscript meaning ($_i =_F$ for front, $_R$ for rear, $_j =_L$ for left, $_R$ for right), T_{ij} denotes the resultant torque (driving and braking) applied at the *ijth* wheel. Tires converts torques into longitudinal tire/road forces denoted F_{xij}.

Dynamics (1) are derived from Newtonian mechanics. It should be pointed out that the aim of the model is torque estimation which implies a 4 wheels description. Without loss of generality and for didactic purpose, aerodynamics effects and rolling resistance forces are neglected.

$$\begin{cases} \dot{V}_x = (F_{xFL} + F_{xFR} + F_{xRL} + F_{xRR})/M_v \\ \dot{\omega}_{FL} = (-R_{lFL}.F_{xFL} + T_{FL})/I_{wFL} \\ \dot{\omega}_{FR} = (-R_{lFR}.F_{xFR} + T_{FR})/I_{wFR} \\ \dot{\omega}_{RL} = (-R_{lRL}.F_{xRL} + T_{RL})/I_{wRL} \\ \dot{\omega}_{RR} = (-R_{lRR}.F_{xRR} + T_{RR})/I_{wRR} \end{cases} \quad (1)$$

In the rotational expressions, tire *loaded radii* R_l are used. As defined in Gillespie [1992], it is "the distance from the center of tire contact to the wheel center". It is evaluated by (2) where R_0 is the free radius, F_z the vertical load and k_z the tire vertical stiffness. This is illustrated on Fig. 2.

$$R_l \simeq R_0 - \frac{F_z}{k_z} \quad (2)$$

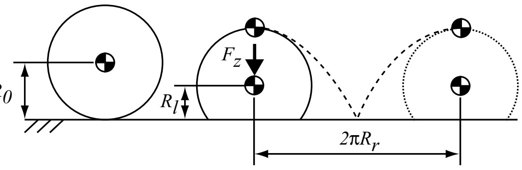


Fig. 2. Wheel and tire radiuses - Free, loaded and rolling radiuses.

In the tire/road contact theory, a longitudinal force (F_x) is defined as a function of longitudinal slip ratio (g_l) which is "the normed skidding speed between the wheel spin velocity (ω) and the equivalent spin velocity of a straight free-rolling tire (V_x/R_r)". It is positive during acceleration and negative during braking phases.

$$g_l = \frac{V_x - \omega.R_r}{V_x} \quad (3)$$

In this expression, the *rolling radii* R_r are defined as in Gillespie [1992] by "the radius of a rigid wheel which covers the same distance covered by a tire, performing an equal number of turns" (Fig. 2). Ellis [1994] has proposed a heuristic estimation (4) of the rolling radius with the same meaning of variables as in (2).

$$R_r \simeq R_0 - 0.28\frac{F_z}{k_z} \quad (4)$$

Among the numerous models of tire force, this study proposes to use a simplified Burckhardt model (Kiencke and Nielsen [2000]) which is altogether compact and thorough for the considered problem. Its expression (5) includes the normal load F_z and a non-linear friction coefficient $\mu(g_l)$ (6) with 3 parameters (c_1, c_2, c_3) depending of road surfaces.

$$F_x = \mu(g_l).F_z \quad (5)$$

$$\mu(g_l) = \mu(\omega, V_x, F_z) = c_1\,(1 - \exp(-c_2 g_l)) - c_3 g_l \quad (6)$$

Parameters (c_1, c_2, c_3) used in the sequel have been calibrated by matching the friction curve with the one obtained from a realistic simulation during a braking phase with locked wheels on a dry asphalt road (Fig. 3).

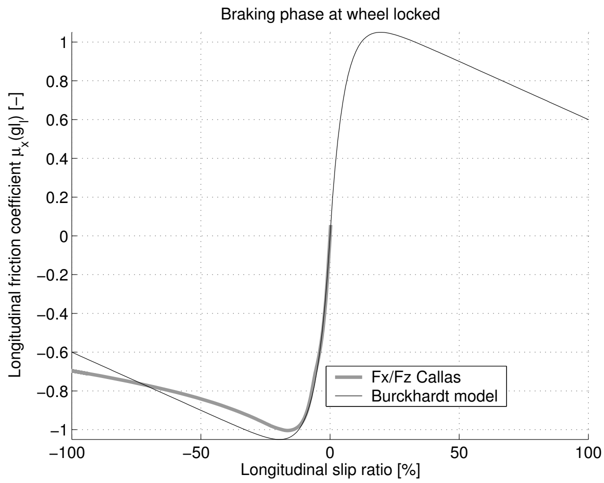


Fig. 3. Model of longitudinal tire/road friction coefficient. Simulator reference and calibrated model.

The progression equation of the vehicle model (1) is now:

$$\begin{cases} \dot{V}_x = \dfrac{1}{M_v} \displaystyle\sum_{i=\{FL,FR,RL,RR\}} \mu_i\,(g_{li})\,.F_{zi} \\ \dot{\omega}_{FL} = (-R_{lFL}.\mu_{FL}\,(g_{lFL})\,F_{zFL} + T_{FL})/I_{wFL} \\ \dot{\omega}_{FR} = (-R_{lFR}.\mu_{FR}\,(g_{lFR})\,F_{zFR} + T_{FR})/I_{wFR} \\ \dot{\omega}_{RL} = (-R_{lRL}.\mu_{RL}\,(g_{lRL})\,F_{zRL} + T_{RL})/I_{wRL} \\ \dot{\omega}_{RR} = (-R_{lRR}.\mu_{RR}\,(g_{lRR})\,F_{zRR} + T_{RR})/I_{wRR} \end{cases} \quad (7)$$

Measurements are the usual 4 wheels rotation speeds plus the 4 vertical loads. The model can now be expressed under a state equation (8).

$$\Sigma \begin{cases} \dot{x} = f(x, u) + G\bar{u} \\ y = Cx \end{cases} \quad (8)$$

In this expression $x = (V_x, \omega_{FL}, \omega_{FR}, \omega_{RL}, \omega_{RR})^T$ is the state. The available measurements are split into:
•the progression equation as a known input
$u = (F_{zFL}, F_{zFR}, F_{zRL}, F_{zRR}.)^T$
•the observation equation as an output
$y = (\omega_{FL}, \omega_{FR}, \omega_{RL}, \omega_{RR})^T$.
$\bar{u} = (T_{FL}, T_{FR}, T_{RL}, T_{RR})^T$ is an unknown input.

3. HIGH GAIN UNKNOWN INPUTS OBSERVER (SOFTWARE SENSOR)

An unknown input observer (UIO) estimates simultaneously the state x and the unknown input $\bar{u}$ by using the output y composed of available measurements and the known input u of a state equation (Fig. 4).

3.1 Model formatting

In the unknown inputs observer theory, the state is split into two parts $x = (x_1, x_2)^T$.

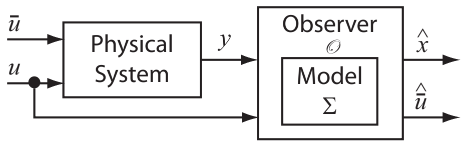


Fig. 4. Unknown inputs observer principle.

For system (8) $x_1 = y = (\omega_{FL}, \omega_{FR}, \omega_{RL}, \omega_{RR})^T$ is the measured one and $x_2 = V_x$ the other one. The formatted system is now written by (9)

$$\Sigma \begin{cases} \dot{x}_1 = f_1(x,u) + G_1 \bar{u} \\ \dot{x}_2 = f_2(x,u) \\ y = x_1 \end{cases} \tag{9}$$

where $G_1 = diag^{-1}(I_{\omega FL}, I_{\omega FR}, I_{\omega RL}, I_{\omega RR})$.

It should be noted that the number of unknown inputs is equal to the number of measures and that there is no input in the measurement equation.

3.2 Observer synthesis

Let us consider the non-linear high gain unknown inputs observer $\mathcal{O}$ (10) proposed by Liu et al. [2006]; θ is the tuning parameter of the observer.

$$\mathcal{O} \begin{cases} \dot{\hat{x}}_1 = f_1(\hat{x},u) + G_1 \hat{\bar{u}} + 2\theta(y - \hat{x}_1) \\ \dot{\hat{x}}_2 = f_2(\hat{x},u) \\ \dot{\hat{\bar{u}}} = \theta^2 G_1^{-1}(y - \hat{x}_1) \end{cases} \tag{10}$$

The convergence of this observer assumes some technical hypotheses that are not detailed here for the sake of brevity.

3.3 Summary for the observer applied to the vehicle

To estimate simultaneously the state (x)and the unknown inputs ($\bar{u}$) of the vehicle model (7), the observer ($\mathcal{O}$) is explicitly given by:

$$\begin{cases} \dot{\hat{\omega}}_{FL} = (-R_{lFL}.\mu_{FL}(\hat{g}_{lFL}) F_{zFL} + \hat{T}_{FL})/I_{wFL} \\ \quad +2\theta(\omega_{FL} - \hat{\omega}_{FL}) \\ \dot{\hat{\omega}}_{FR} = (-R_{lFR}.\mu_{FR}(\hat{g}_{lFR}) F_{zFR} + \hat{T}_{FR})/I_{wFR} \\ \quad +2\theta(\omega_{FR} - \hat{\omega}_{FR}) \\ \dot{\hat{\omega}}_{RL} = (-R_{lRL}.\mu_{RL}(\hat{g}_{lRL}) F_{zRL} + \hat{T}_{RL})/I_{wRL} \\ \quad +2\theta(\omega_{RL} - \hat{\omega}_{RL}) \\ \dot{\hat{\omega}}_{RR} = (-R_{lRR}.\mu_{RR}(\hat{g}_{lRR}) F_{zRR} + \hat{T}_{RR})/I_{wRR} \\ \quad +2\theta(\omega_{RR} - \hat{\omega}_{RR}) \\ \dot{\hat{V}}_x = \dfrac{1}{M_v} \displaystyle\sum_{i=\{FL,FR,RL,RR\}} \mu_i(\hat{g}_{li}).F_{zi} \\ \dot{\hat{T}}_{FL} = \theta^2 I_{wFL}(\omega_{FL} - \hat{\omega}_{FL}) \\ \dot{\hat{T}}_{FR} = \theta^2 I_{wFR}(\omega_{FR} - \hat{\omega}_{FR}) \\ \dot{\hat{T}}_{RL} = \theta^2 I_{wRL}(\omega_{RL} - \hat{\omega}_{RL}) \\ \dot{\hat{T}}_{RR} = \theta^2 I_{wRR}(\omega_{RR} - \hat{\omega}_{RR}) \end{cases} \tag{11}$$

where $\hat{g}_l = \left(\hat{V}_x - \hat{\omega}(R_0 - 0.28F_z/k_z)\right)/\hat{V}_x$.

4. SIMULATION RESULTS-NOMINAL CASE

4.1 Vehicle simulator

Callas is a realistic vehicle simulator software distributed by Oktal company. According to Lechner et al. [1997], it has been validated by car manufacturers and French research institutions including INRETS ("Institut national de recherche sur les transports et leur sécurité"). Callas is a physical based model that takes into account numerous aspects among which vertical dynamics (suspension, tires, road profile), kinematics, elasto-kinematics, tire adhesion, aerodynamics, ... It is a virtual environment closed to the practical experimental one.

For the present study, using a simulator is of paramount importance because validating the estimated torques by experimental measurements requires rare and very expensive devices (some one hundred thousand euros per wheel). Real experiments then must be cautiously prepared.

4.2 Simulation Conditions

For the nominal case study, model and observer are evaluated using the following simulation conditions. The driver simulator is programmed to control a constant deceleration of the vehicle by acting on the brake pedal (Fig. 5).

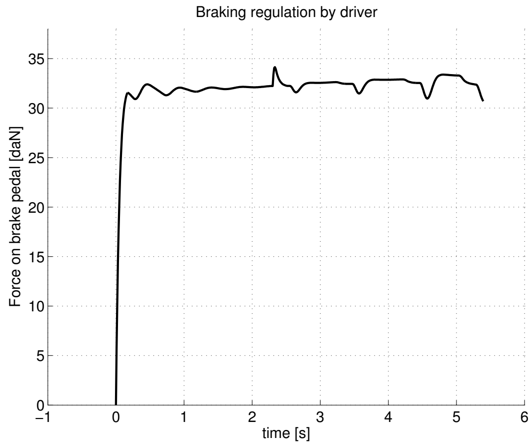


Fig. 5. Force applied on the brake pedal by the driver to control a $6[m.s^{-2}]$ deceleration. Braking start at time $t = 0[s]$. The initial speed of the vehicle is $100[km.h^{-1}]$, giving a braking duration around $6[s]$. It yields an average $32[daN]$ force applied on the pedal.

Because only the longitudinal direction is solicited in the simulated experiments and that the vehicle model in Callas has been defined as symmetrical, only the front left (FL) and the rear right (RR) variables are shown in the sequel.

4.3 Model validation

Model validation and calibration are the first stages of a software sensor design. To do this, model (7) is used with *all inputs supposed to be known.* In particular, torques T_{ij} and vertical loads F_{zij} are extracted from Callas.

In the upper part of Fig. 6 the wheels rotation speeds issued from Callas ("ref" label) and from the model (7) ("BO" label) are shown. In the lower part of Fig. 6, the modelling error is presented. On the rear wheel, the maximum error is $1.8[rad.s^{-1}]$ corresponding to $1.76[km.h^{-1}]$. During a braking phase, front wheels are more solicited than the rear ones because of the load transfer. This implies more slipping and additional difficulties to match the parameters of the longitudinal force model.

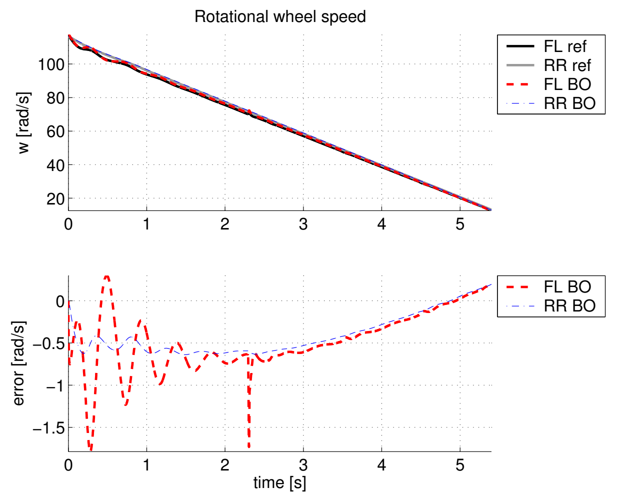


Fig. 6. Wheels rotation speeds computed by the open-loop model with known torques. Callas reference and modelling error.

In the upper part of Fig. 7 the longitudinal tire/road forces issued both from Callas ("ref" label) and from the model ("BO" label) are shown. On the bottom, the modelling error is presented. In average, the absolute value of the force error on all wheels is less than $5[daN]$. A $80[daN]$ peak error appears at time $t = 2.3[s]$ on the front wheel. It is caused by the virtual driver command on the pedal.

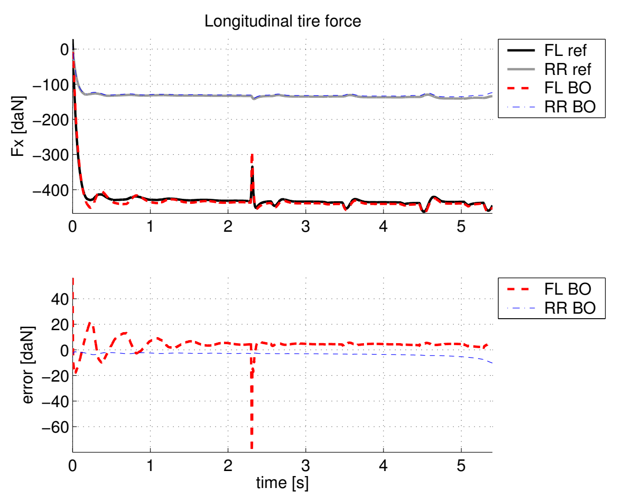


Fig. 7. Longitudinal tire/road force computed by the open-loop model with known torques. Callas reference and modelling error.

This validates the thoroughness of the model (7) whose input are the resultant torques and the vertical forces applied to the wheel.

4.4 Observer validation

For the observer (11) validation, resultant torques applied to wheels are not measured. The observer is designed with a $\theta = 700[s^{-1}]$ tuning.

Fig. 8 displays the results of the unknown input observer on the vehicle speed estimation $\hat{V}_x$ ("UIO" label). They are compared with those of Callas ("ref" label) and those of the open-loop model with known torques ("BO" label). An error of $0.1[m.s^{-1}] \simeq 0.36[km.h^{-1}]$ shows that the speed estimation is accurate despite the lack of knowledge of torques applied to the wheels. Performances are similar to those reached by the calibrated open-loop model with known torques. Results on wheels rotation speeds are not presented because they are the measured part of the state.

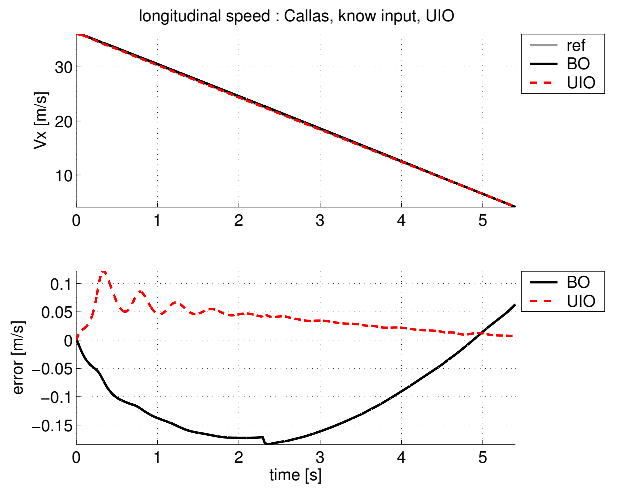


Fig. 8. Vehicle speed computed by the open-loop model and the unknown input observer. Callas reference, modelling error and observation error.

In Fig. 9 the performance of the observer on the longitudinal forces are shown. The observed values are correct (on average $5[daN]$ for front wheels and $-12[daN]$ for the rear ones). This estimation is very accurate if the fact of unknown torque is reminded. These results are comparable to those obtained in the open-loop case (Fig. 7) and are even better since the error appears to vanish.

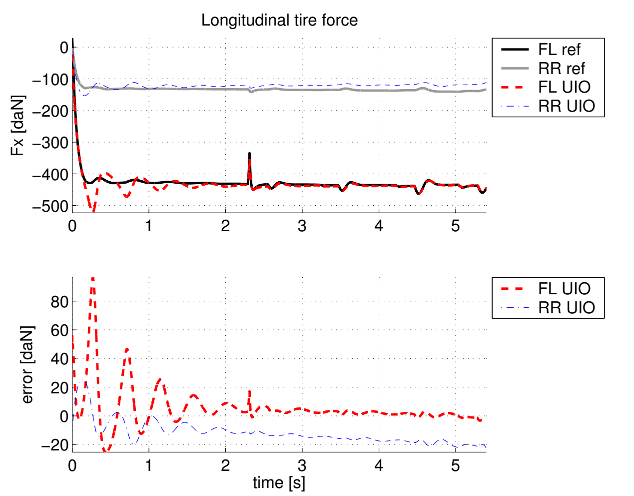


Fig. 9. Longitudinal tire/road forces computed by the unknown input observer. Callas reference and observation error.

Finally, Fig. 10 presents the unknown torque estimated by the observer compared to the "real" resultant torque applied to the wheels by the Callas simulator. On average, the resultant torques estimations is very accurate (a few units compared to the $1200[N.m]$ applied to the front wheels).

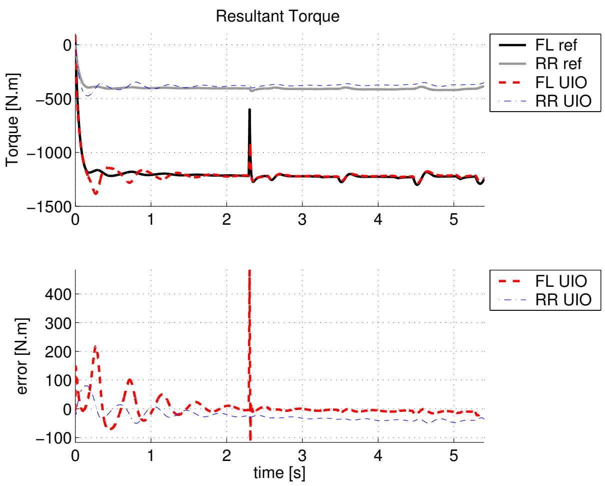


Fig. 10. Resultant torque computed by the unknown input observer. Callas reference and observation error.

5. PRESENT LIMITS OF THE METHOD

It is important to emphasize here that the unknown input computed by the UIO observer is *not* the sum of braking and driving torques. Indeed, this unknown input is a combination of different phenomena. All modelling errors are affected to the unknown inputs.

To illustrate these effects, an assumption of worn tires (or slippy surface type) on the front axle has been done. To do that, coefficient c_1 from (6) has been set to 70% of his nominal value. The case study is the same as that presented in the previous section (constant deceleration controlled by pedal). Results are shown on Fig. 11.

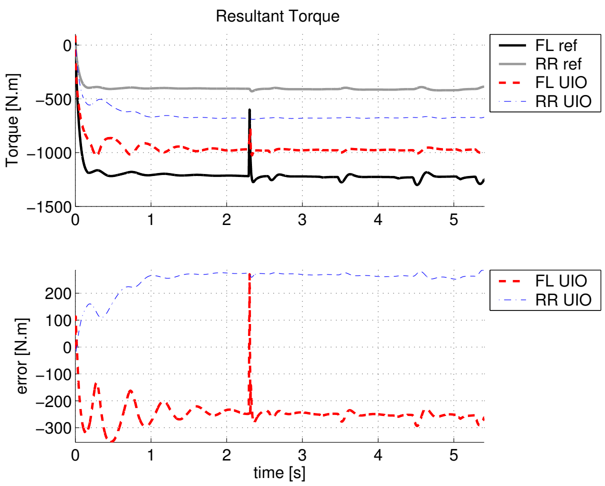
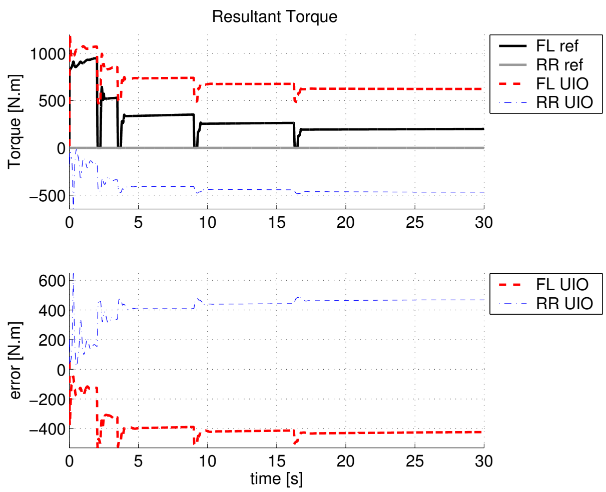


Fig. 11. Effect of unmatching road/tire friction coefficient on unknown torque estimation. Callas reference and observation error.

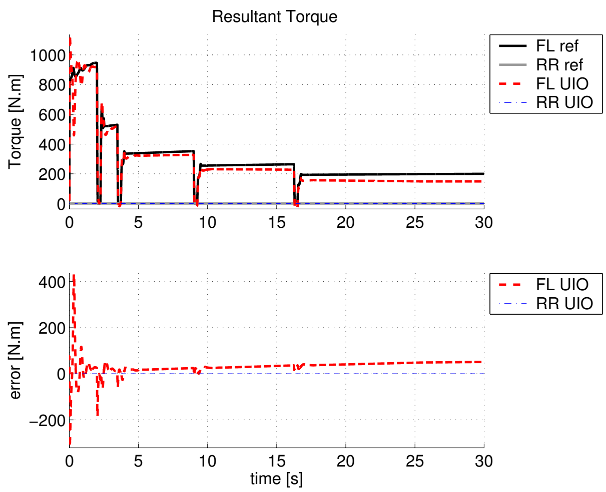
The effect of this underestimation of the friction coefficient is an underestimation of the observed torque. The error on the front wheels is approximatively $5[N.m]$ for $1250[N.m]$ of effective braking torque for the nominal case ("FL" labels on Fig. 10). By comparison, the assumption of worn tires causes an error of $250[N.m]$ for the same level of "reference" torque. This is a "classical" effect of non robustness of high-gain observers with respect to errors on parameters. This kind of behaviour can also be noted in the case of underestimation of wheel radiuses.

Other precautions are taken to use the proposed model. An acceleration case is presented on Fig. 12. For this test, the virtual driver has to perform a full acceleration. Discontinuities on the torques are due to the gear shifting done by the driver. Each torque level is a function of the gear ratio. The vehicle modeled in the Callas simulator is a front wheel driving car.

On Fig. 12(a) the observer expression (11) is applied without taking any precaution. It can be seen that the observer computes a torque on the rear wheels ($400[N.m]$ at the end of the test) although it should be nul. To keep the correct vehicle speed, the observer overestimate torques applied on the front wheels.



(a) Assumption of four wheels driving/braking model



(b) Assumption of front wheels driving/braking model

Fig. 12. Acceleration test, effect of modelling error on unknown torque observation.

Now the technology of the powertrain with only front driving wheels is taken into account by omitting torques on rear wheels in the referenced model. Fig. 12(b) shows that the driving torques applied to the front wheels are now correctly computed. At time $20[s]$, when the fifth gear ratio is engaged, the estimation error is limited to $50[N.m]$.

6. CONCLUSION

This paper details the estimation of the individual resultant torques on each one of the 4 wheels of an automotive vehicle. The estimation is performed by a non-linear unknown inputs observer and it assumes that both the wheels rotation speeds and the vertical loads are available. The measurement of wheel speeds is now generalized and, after the development of successful prototypes, the affordable integration of forces measurements in wheel bearings can be forecast in a next future giving opportunities for the proposed technique. Another possibility for low cost sensing of the vertical loads is the use of the suspension deflection sensors available on several cars or using a quasi-static approximation using accelerometers measurements. But, these model approaches cannot be so accurate than a dedicated sensor.

Having presented and validated the model of knowledge on a controlled longitudinal braking phase using a realistic simulator, the proposed observer has been evaluated in the same conditions. It was shown that, if the model is well calibrated, the observer is accurate. In a second step, some limitations of the method are presented. One is the non robustness of the method with respect to the parameters. This case is illustrated using a worn tire model. In a third step, it was shown that some cautions have to be taken. The unknown input estimated can be called "Torque" only if the model is sufficiently representative of the system in his environment. The computed unknown input is made up of the effective resultant torque but also of errors on wheel radiuses, friction coefficient and non modelled phenomena. This last case is illustrated using an acceleration test and an hypothesis of a 2WD vehicle.

Those estimated torques may be used in ADTM in the ESC in complement of the existing brake-based actuation with the obvious economic and ecological advantages that energy is not necessarily lost during the actuation. Using the observed torques in redundancy with other estimation methods can help to draw conclusions about tire/road friction coefficient. It should be noticed that the wheel bearing sensor developed in Kwapisz [2008] can give 3 forces components and 2 torques components. The 3rd one is unmeasurable and the observer developed here can estimate it.

The present longitudinal study can be generalized into lateral plus longitudinal motions as it has been shown in Ouahi et al. [2010]. For future studies, a sensitivity analysis can be done to quantify the effects of friction coefficient assumptions. An experimental validation is conceivable using the XLIM experimental vehicle (NOE). It requires preliminary studies of the impact of measurement noise in performance of the observer and procedures for experimental tests including tire/road friction coefficient calibration like braking in curve on the test track with locked wheels.

7. NOTATIONS

F_x: Longitudinal tire/road force $[N]$
F_z: Normal Force $[N]$
g_l: Longitudinal slip ratio $[-]$
I_w: Wheel inertia $[kg.m^2]$
k_z: Tire vertical stiffness $[N.m^{-1}]$
M_v: Total vehicle mass $[kg]$
R_0: Tire nominal radius $[m]$
R_r: Tire rolling radius $[m]$
R_l: Tire loaded radius $[m]$
T: Resultant torque applied to the wheel $[N.m]$
V_x: Vehicle longitudinal velocity $[m.s^{-1}]$
μ: Longitudinal friction coefficient $[-]$
ω: Wheel rotation speed $[rad.s^{-1}]$
θ: Observer tuning parameter $[s^{-1}]$
$\hat{.}$: Observed variable $[-]$

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