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Adaptive Cruise Control System: Comparing Gain-Scheduling PI and LQ Controllers

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Abstract: Over the recent years, a considerable growth in the number of vehicles on the road has been observed. This increases importance of vehicle safety and minimization of fuel consumption, subsequently prompting manufacturers to equip cars, with more advanced features such as adaptive cruise control (ACC)or collision avoidance and collision warning system (CWS). This paper investigates two control applications design namely the gain scheduling proportional-integral (GSPI) control and gain scheduling Linear Quadratic (GSLQ)control for ACC, covering a high range speed. The control system consist of two loops in cascade, with the inner loop controlling the vehicle speed and the outer loop switching between the cruise control (CC) and the ACC mode and calculating the reference speed. A nonlinear dynamic model of the vehicle is constructed and then a set of operating points is determined and then a of linear models is extracted in operating point. For each operating point, PI and LQ controllers are obtained off-line. An integrated Simulink model including the nonlinear dynamic vehicle model and the ACC controller (either PI or LQ) was used to test the controllers in various traffic scenarios. Comparison results between the two controllers applications is provided to show the validity of the design.

Keywords: linear approximation, gain-scheduling PI controller, adaptive cruise control, longitudinal dynamic of the vehicle

1. INTRODUCTION

Adaptive cruise control (ACC) is the extension of the Cruise Control(CC) system where adaptation law is included. An ACC allows the variation of the velocity of the vehicle depending on the behavior of the other vehicles moving in front of it, by applying the brake and modulating the throttle to produce the necessary power. This system uses a radar or other sensory devices to measure the distance between vehicles (Winner et al. (2003)). An ACC system functions at higher speed range, usually more than 30km/h, and the structure is formed from two control loop, i.e. an inner loop controller (low level controller) or servo loop, and an outer loop controller (upper level controller).

The control parameters computed by the outer loop in order to be sent to servo loop is either the reference torque (Zhou and Peng (2003); Choi et al. (2009)) or the reference speed. Either of these reference value are then sent to the low level controller. Low level controller computes the required throttle opening position and brake pressure by invoking a control design methodology such as conventional PID controller. In a different way, Girard et al. (2001) explained the method in which the desired throttle opening was computed from the desired torque using an engine map. Choi et al. (2009) proposed the model-free control approach for designing the low level controller to obtain the desired torque, so as to reduce the impact of existing nonlinearity in the brake and engine model on the control process. In some research work, the design of the low level controller was not explained (Zhou and Peng (2003)) and mostly focused on designing the outer loop controller. In this paper both control loop are explained in details. Also due to the nonlinearity of the system, the gain scheduling approach is suggested to vary the gains of PI or LQ controller in order to manipulate the throttle position.

In this paper algorithm of an ACC system is developed by classifying into two distinguished levels. First, the outer loop or upper level controller (ULC) of the system known as distance tracking controller aims to control the distance between the ACC equipped vehicle and leading vehicle. This is accomplished by use of a simple PI controller which takes the deviation of detected distance between the vehicles from the desired distance. Therefore, in order for the vehicle to effectively follow other vehicle in front of it by keeping a desired distance, the new reference speed is calculated by outer loop controller. ULC is also capable of switching between the cruise control (CC) and ACC mode depending on the situation in front of the ACC equipped vehicle. Second, the inner loop or low level controller (LLC) is provided to track the reference velocity calculated by ULC. This is implemented through switching between the brake and throttle control systems. The throttle control system modulates the opening position of the throttle

by use of either the gain scheduling proportional-integral (GSPI) or gain scheduling linear quadratic (GSLQ) controller. A PI controller is used to regulate the brake pedal position in order to produce effective braking torque. In this paper the nonlinear dynamic model of the vehicle is linearized around selected operating points. Moreover, the tuning set of the PI and LQ controllers gains is obtained by use of each of the linear models in order to regulate the nonlinear model by applying gain scheduling approach.

2. VEHICLE MODEL

In this paper the following equations are used for representing the dynamic model of the vehicle (Shakouri et al. (2010)) for the control design purpose:

$$I_{ei}\dot{N}_e = T_e(u_t, N_e) - (\frac{N_e}{K_{tc}})^2 \tag{1}$$

$$m\dot{v} = \frac{1}{r}[\underbrace{R_{tr}R_fC_{tr}(\frac{N_e}{K_{tc}})^2}_{T_{wheel}} - T_b] - \underbrace{\frac{1}{2}\rho AC_dv^2}_{F_{aerodynamic}} - \underbrace{C_rmg\cos(\theta)}_{F_{rolling-resistance}} \pm \underbrace{mg\sin(\theta)}_{F_{gravitational}} \tag{2}$$

$$T_b = K_bP = K_b(150K_cu_b - \tau_{bs}\dot{P}) \tag{3}$$

Equation (1) explains the mathematical relationship of the engine and impeller, with N_e the engine speed measured in rpm, I_{ei} the summation of engine and impeller moment of inertia and T_e the engine torque. The parameters playing an important role in the performance of a torque converter are expressed as follows: the speed ratio $C_{sr} = \frac{N_t}{N_i}$, the torque ratio $C_{tr} = \frac{T_t}{T_i}$, the efficiency $\eta_e = C_{sr} \times C_{tr}$ and the capacity factor (K-factor) $K_{tc} = \frac{N_e}{\sqrt{T_i}}$. The speed ratio can be expressed as a function of the vehicle speed as in Equation (4):

$$C_{sr} = \frac{vR_fR_{tr}}{2\pi N_e} \tag{4}$$

Having the speed ratio helps us finding other parameters of the torque converter by interpolating the graph which illustrates the performance characteristic of a torque converter (Wang (2001)). To present the model of the engine in the simulation, a look up table is applied which defines the amount of engine torque vs. engine rotation speed (rpm) and the percentage of throttle opening position u_t. The engine map was taken from the Simulink/Matlab (MathWorks (1998)). In the model of power train, implementation of the gear shift is done through the shift logic based on the thresholds calculated by the respective blocks for up-shift and down-shift.

Equation (2) is to calculate the velocity of the vehicle by considering the torque produced on the wheel through the power train T_w, the braking torque T_b, aerodynamic force $F_{aerodynamic}$ and the last two terms in Equation (2) defining the rolling resistance $F_{rolling-resistance}$ and the gravitational forces $F_{gravitational}$ respectively. Consequently, the velocity of the vehicle, v can be obtained by integrating the acceleration. Here ρ is the air density, C_d the drag coefficient depending on the body shape, v velocity of vehicle and A is the maximum vehicle cross area, R_{tr} the gear ratio, R_f the final drive ratio, m the total mass of the vehicle, g the gravitational acceleration,

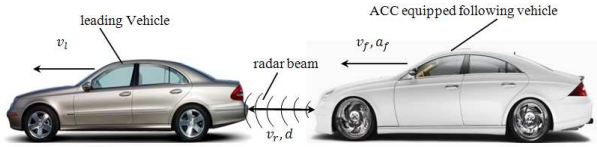

Figure 1. The ACC equipped vehicle following another vehicle in front.

and θ varies depending on inclination of the road and it is so-called road slope.

Finally, Equation (3) calculates the braking torque as u_b is varied from 0 to 1 percentage, where P is the amount of pressure produced behind the brake disk, τ_{bs} is the lumped lag obtained by combining two lags relating to the dynamic of the servo valve and the hydraulic system, K_c is pressure gain. K_b is the lumped gain for entire brake system. The values of the parameters are selected to represent a medium-size passenger car.

3. ACC SYSTEM

ACC operates in two different modes depending on the situation of the traffic ahead; cruise control (CC) mode and ACC control mode, i.e. distance tracking mode. It operates in the CC mode when the road in front of the ACC equipped vehicle is clear, i.e. there is no vehicle within clearance distance. In this situation vehicle travels at the desired cruising speed which is set up by the driver. Once it has approached other vehicles traveling in lower speed it switches to distance tracking mode. In this mode ACC attempts to keep the vehicle within the desired distance headway (Figure 1), by controlling the speed of the vehicle. The distance headway can be customized by the driver taking into account the breaking time (time headway). The transition between the modes is performed automatically by considering the traffic condition ahead and the desired cruising speed.

4. CONTROLLER STRUCTURE

The ACC consists of two control loops; The inner- loop is the typical cruise controller. This controller works as velocity tracking controller. It takes an advantage of modulating both the brake and, the throttle in order to track the reference speed. Applying the brake is required when the quicker reaction of the system is necessary. This situation occurs when the leading vehicle reduces its speed quickly, then a synchronized switching between the brake and throttle is crucial. The outer loop controller introduces the new reference velocity into the inner-loop controller in order to track the specified driver distance headway (desired distance). The outer loop calculates the reference velocity (Riis (2007)) from the velocity of the following vehicle, the distance from the leading vehicle, the desired cruising speed (driver chosen speed) and the desired distance. In order to attain an accurate interaction between the CC and ACC applications, a switching process between the two modes is implemented. Figure 2 illustrates the block diagram of this switching structure. If the actual distance is less than the desired distance the system will switch to the distance control (ACC) mode , otherwise the following vehicle keeps moving at the cruising speed. Also, if the

leading vehicle increases its speed, the following vehicle keeps tracking it until its speed reaches the desired cruising speed (Zhou and Peng (2003)). The desired distance headway, d_{des} can be computed using the following equation, which is known as Constant-Time Headway policy (Zhou and Zhang (2003); Zhou and Peng (2003)):

$$d_{des} = l + d_s + hv \tag{5}$$

where l is the vehicle length, d_s is the additional distance between two vehicle in order to avoid collision, v is vehicle velocity and h is constant-time headway which is specified by driver. Note that the time headway must be greater than driver reaction time.

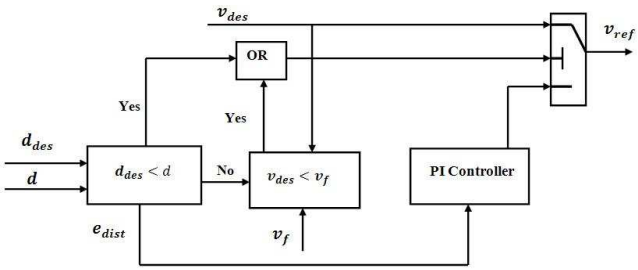

Figure 2. An ACC structure with switching algorithm between cruise control and ACC mode.

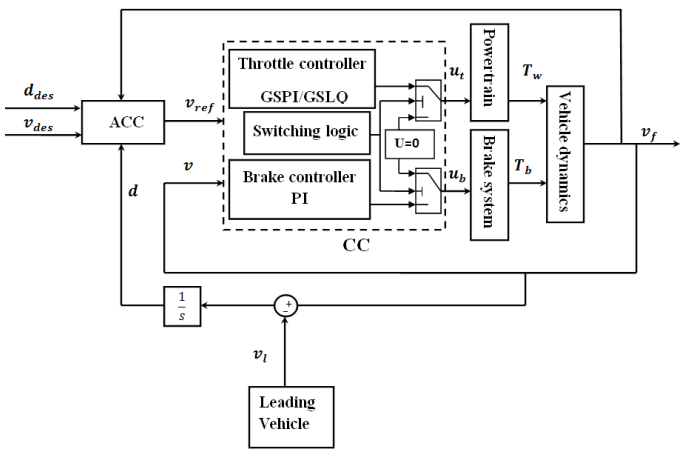

Figure 3. Schematic block diagram of the ACC (inner loop & outer loop).

4.1 Inner loop controller

This level of the system includes two different controllers, i.e. the throttle controller and the brake controller. Throttle controller is active when the reference speed is higher than the measured speed; while the brake controller becomes active when the reference speed is less than the measured speed. A fixed PI controller is used for controlling the brake torque whereas gain-scheduled controllers are used for controlling the throttle. Those gain-scheduled controllers could be either PI or LQ type. The outputs of the brake and throttle controllers are restricted between zero and one. The anti windup is added in the feedback control loop.

The values of the proportional and integral gain of the PI brake controller are given in Table Table 3. The switching between the throttle and the brake controllers is implemented based on the Table 1.

Table 1. Logical condition at which either the brake or the throttle is activated.

d \ v	$> v_{ref}$	$< v_{ref}$
$< x_{min}$	Brake	Brake
$> x_{min}$ & $< x_{max}$	Brake	Throttle
$> x_{max}$	Throttle	Throttle

The quantities x_{min} and x_{max} are the minimum and the maximum distances between the vehicles, respectively. x_{max} is defined as the distance taken by the vehicle to reach standstill from the maximum speed if hard brake applies i.e. braking distance. Braking distance of the vehicle model used in our study is estimated around 536 m which is associated with the maximum speed of 70 m/s. The minimum distance x_{min} is considered equal to desired distance headway d_{des}.

The gain scheduling-PI controller was chosen to meet the requirement of the ACC system. In this concept the PI controller parameters change according to the condition in which the system operates by applying a gain scheduling technique. The criteria for switching between each operating points is based on the current speed of the vehicle. These operating points are introduced by the normalized variation of the throttle opening position varying from 0 to 1 with interval of 0.1. It was heuristic approach for which 10 points were selected and tests performed. It was hoped that some similarities would emerged, enabling reduction in the number of models, but this did not happen. Hence 10 models were used. In order to implement the gain scheduling method (Figure 4) the nonlinear dynamic model of the vehicle was linearized in different operating points and each linearized models was used to tune the PI controller parameters.

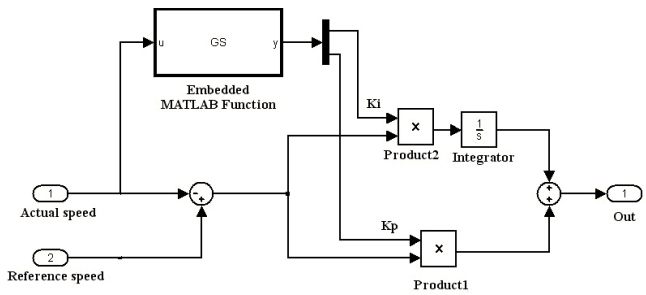

Figure 4. Gain scheduling approach implemented in adapting the PI controller parameter.

In this paper, linearization method is based on small-signal analysis method (Franklin et al. (2010)). To obtain a set of linear models, a set of operating points of the vehicle were calculated for a various opening positions of the throttle, with the brake non active. Linear models have been computed for the specified operating points, through a trimming procedure. The validity of the approximated linear models have been tested through verifying the

responses of both linear and nonlinear models. Figure 5 shows the response of linear model conforms to the nonlinear model. This particular linear model is extracted for a condition at which the velocity of vehicle is around 13.1 m/s and that is equivalent to the throttle opening position of 0.1. This process has been carried out for all the linear models.

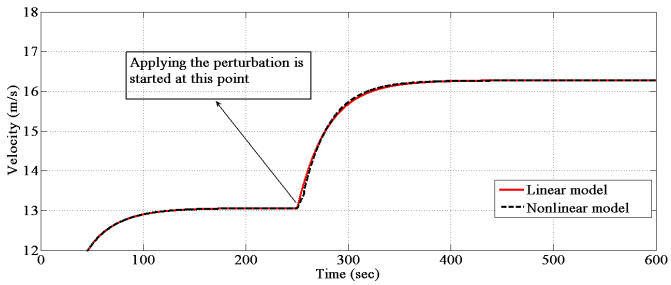

Figure 5. Comparison of the linear and nonlinear model responses associated with throttle position of 0.1 equivalent to vehicle velocity of 13.1 m/s

Tuning of the PI controller was carried out using the Control and Estimation Tools Manager in Simulink/Matlab. The SISO transfer functions of the system corresponding to various operating points are give in Figure 4. The values of PI controller parameters (K_p&K_i) are given in Table 2. The PI controller parameters were re-tuned for nonlinear model.

Table 2. Tuning set of PI controller obtained for controlling the linear and nonlinear model

OP.	Transfer function of PI controller (Lin.)	K_p (Lin. &Nonlin.)	K_i (Lin.)	K_i (Nonlin.)
0.1	$0.01 \times \frac{1+21s}{s}$	0.21	0.01	0.012
0.2	$0.006 \times \frac{1+68s}{s}$	0.41	0.006	0.0065
0.3	$0.007 \times \frac{1+54s}{s}$	0.37	0.007	0.0055
0.4	$0.009 \times \frac{1+34s}{s}$	0.31	0.009	0.00425
0.5	$0.02 \times \frac{1+27s}{s}$	0.54	0.02	0.0045
0.6	$0.03 \times \frac{1+26s}{s}$	0.78	0.03	0.005
0.7	$0.04 \times \frac{1+29s}{s}$	1.14	0.04	0.005
0.8	$0.09 \times \frac{1+27s}{s}$	2.42	0.9	0.006
0.9	$0.22 \times \frac{1+24s}{s}$	5.32	0.22	0.02
1	$0.15 \times \frac{1+23s}{s}$	3.49	0.15	0.01

Table 3. PI control including Anti wind-up in design of outer-loop controller as well as brake controller.

Type of controller	K_i	K_p	K_{AW}
Outer loop	0.1	10	50
Brake	0.0005	5	5

Table 4. Transfer function obtained from linearization corresponding to various operation point

OP.	Transfer function of the plant
0.1	$\frac{55.4912s + 277.4558}{s^3 + 10.1286s^2 + 25.8155s + 0.8625}$
0.2	$\frac{5.5922s + 27.96128}{s^3 + 7.3297s^2 + 11.6938s + 0.2263}$
0.3	$\frac{5.3795s + 26.8976}{s^3 + 7.2683s^2 + 11.4020s + 0.3033}$
0.4	$\frac{6.9343s + 34.6714}{s^3 + 7.4241s^2 + 12.1989s + 0.3923}$
0.5	$\frac{4.9552s + 24.7761}{s^3 + 7.4659 + 12.4230s + 0.4677}$
0.6	$\frac{3.9618s + 19.8090}{s^3 + 7.83s^2 + 14.2663s + 0.5810}$
0.7	$\frac{3.18665s + 15.9330}{s^3 + 7.9614s^2 + 14.9357s + 0.6427}$
0.8	$\frac{1.8432s + 9.2162}{s^3 + 8.2552s^2 + 16.4233s + 0.7359}$
0.9	$\frac{0.98008s + 4.9002}{s^3 + 8.3210s^2 + 16.7581s + 0.7641}$
1	$\frac{0.5657s + 2.8287}{s^3 + 8.7796s^2 + 19.0714s + 0.8813}$

LQ controller-Tracking problem Another control method being utilized is LQ optimal control. The purpose of employing LQ control is to transfer the state of the system to desired region of the state space by finding the optimal control vector, u(k) which minimizes a given quadratic performance index (Ogata (1997)):

$$J = \frac{1}{2}\int_{k=0}^{\infty}(x_k^T Q x + u^T R u)dt \tag{6}$$

R and Q can be manipulated to get the desirable performance of the system.

For the LQ tracking, the derivative error between output and reference value y_{ref} is added to state variables of the system. This variable in continuous-time yields the following equation:

$$\dot{x_j} = y_{ref} - Cx. \tag{7}$$

Therefore, the augmented continuous-time state space model may be described by following equations:

$$\begin{aligned} \underbrace{\begin{bmatrix} \dot{x} \\ \dot{x_j} \end{bmatrix}}_{\bar{\dot{x}}} &= \underbrace{\begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}}_{\bar{A}} \cdot \underbrace{\begin{bmatrix} x \\ x_j \end{bmatrix}}_{\bar{x}} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{\bar{B}} .u + \begin{bmatrix} 0 \\ y_{ref} \end{bmatrix} \\ y &= \underbrace{[\,C \;\; 0\,]}_{\bar{C}} \cdot \begin{bmatrix} x \\ x_j \end{bmatrix} \end{aligned} \tag{8}$$

where $\bar{A}$, $\bar{B}$, $\bar{C}$ are augmented matrices of the closed loop system. The Riccati Algebraic Equation (RAE) for the tracking is as follows:

$$0 = SA + A^T S - SBRB^T S + Q \tag{9}$$

and the controller takes the form:

$$u_k = -K_x x + K_w x_j = -\underbrace{[\,K_x \;\; -K_w\,]}_{\bar{K}} \cdot \begin{bmatrix} x \\ x_j \end{bmatrix}. \tag{10}$$

$$\bar{K} = R^{-1}B^T S \tag{11}$$

The weighting matrices Q and R have been selected thorough iterative trial and error method in order to achieve a good behavior of the controller for each linear model. The schematic block diagram of utilizing the GSLQ for controlling throttle is depicted in Figure 6. This diagram assumes state-feedback control. However, the observer-based control, utilizing Kalman filter is also possible.

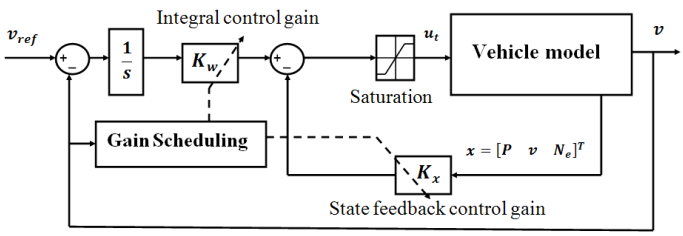

Figure 6. Schematic block diagram of controller, illustrating the gain scheduling LQ controller for modulating the throttle position.

The stability of gain scheduling controllers is ensured by establishing the existence of Common Lyapunov Functions (CLFs) based on the method introduced by HARIS et al. (2007). For the problem described here it has been investigated and proved by solving the Linear Matrix Inequality (LMI), implemented via LMI Control Toolbox in Matlab (Gahinet et al. (1995)), that the overall gain-scheduled system is stable.

4.2 Outer loop controller

This level of controller, known as ACC, computes the new reference speed, $v_{ref}(t)$. It is implemented as a PI controller with distance error as an input. As depicted in Figure 2, the outer loop controller contains the switching function which enables switching between the ACC and the CC mode as explained before by validating various parameters, i.e. the desired distance, the actual distance, the desired speed and the speed of the following vehicle. The parameters of the PI controller are given in Table 3.

5. TESTING SCENARIO

The control algorithm is tested using the following scenarios:

Velocity tracking mode: In this mode, depending on desired speed chosen by driver, the system endeavors to follow the reference speed. This model was examined using two different scenarios (7).

Distance tracking mode: The purpose of this scenario is to examine the performance in following the leading vehicle within a safe distance (Figure 8).

Switching mode: This is a combination of two previous scenarios, which examines the switching performance between "speed tracking" and "distance tracking" modes(Figure 9).

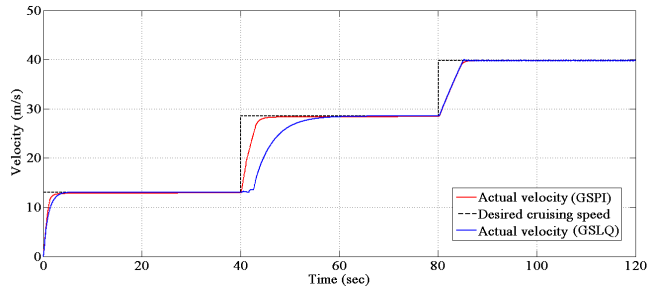

Figure 7. performance of the CC system through the changing of the cruise control speed

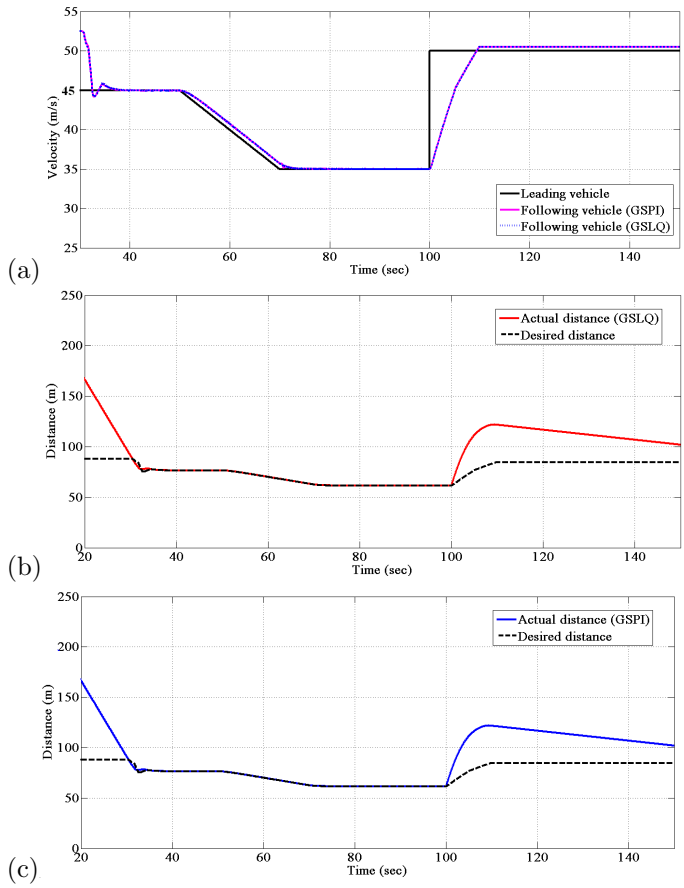

Figure 8. (a) Velocity of the ACC equipped vehicle which is adapted in such a way to achieve desired headway distance, (b) Adapting the distance headway between follower and target vehicle using GSLQ, (c) Adapting the distance headway between follower and target vehicle using GSPI,.

6. SIMULATION

Figure 7-(a) illustrates good tracking of the desired speed when in CC mode. The aim of this scenario is to examine how accurately the system can follow the desired cruising speed. The response shows suitable characteristic of the controller, i.e. no overshoot, no steady state error, stable, suitable rising and settling times. When using GSLQ, rise time is a little slower between instants 40-60 sec, compared with GSPI. However, In other instants similar result can be observed.

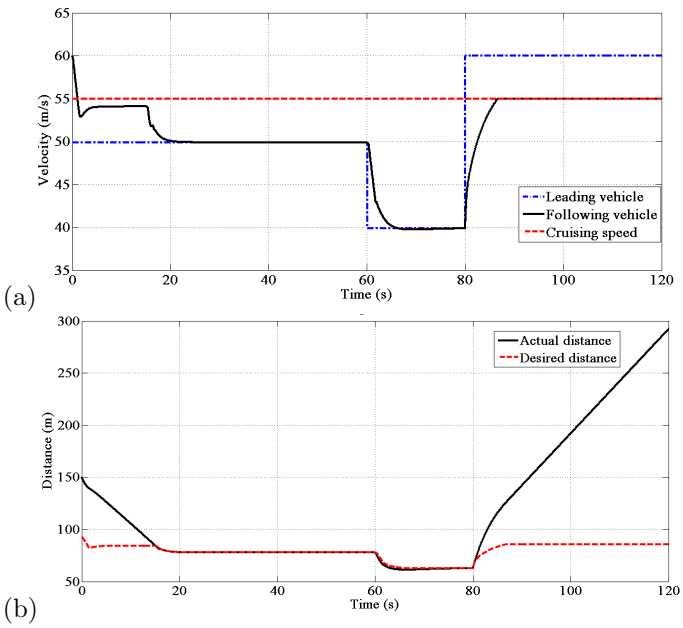

Figure 9. (a) Switching between the ACC and CC mode when the desired speed is less than target vehicle speed, (b) Adapting the distance between follower and target vehicle

Figure 8 shows the response of the system in distance tracking mode. In the test, there is an initial distance of $160m$ between the vehicles and the follower travels initially at the speed of $52m/s$. After the distance reduce vehicles (Figure 8-b), the ACC equipped vehicle follows the leading vehicle at the desired distance which in turn it causes both vehicles to reach the same speed (Figure 8-a). The desired distance is proportional to the velocity of the Following vehicle. The proportionality constant, called time headway, in simulation equals 1.5s. The results obtained by employing GSPI and GSLQ controllers were compared and the results are similar.

Figure 9 shows the result for the switching condition. In this test the initial distance between vehicles is $150m$ and the initial speed of the vehicle equipped with ACC (following vehicle) is $60m/s$. The desired cruising speed is set up at $55m/s$. As it can be observed, initially the following vehicle sets on its cruising speed, until such time when the distance between the vehicles decreases to the value of the desired distance - this happens at approximately $18s$. From this moment, until $80s$, the ACC system controls the distance between vehicles, hence assuring that the speeds are equal. At $80s$ the lead vehicle increases its speed and hence the following vehicle reverts to Cruise Control. This test was carried out by use of GSPI controller.

7. CONCLUSION

This paper has been presented an ACC simulation model utilizing the either GSPI or GSLQ controller in order to control the throttle opening position. Controlling the Brake and throttle is implemented separately. To perform the controller design task, several operating points were obtained for a various characteristic of the system depending on the variation of throttle opening positions. Several linear models have been approximated around those operating point in order to obtain tuning sets of the PI and LQ controller gains. The obtained tuning sets were then used to control the nonlinear model through applying gain scheduling approach. The performance of ACC was examined by introducing various traffic scenarios. The results of the simulation tests have exhibited good performance of the proposed ACC model. Furthermore, performance of the GSLQ and GSPI controllers was compared.

8. ACKNOWLEDGMENT

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