Model Order Reduction of Rotordynamical Systems

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Abstract: This paper presents an approach for model order reduction of linear rotordynamical systems resulting from the accurate modeling of large machines with a rotor. Automatically partitioning the overall model into decoupled sub-models, describing the main components, namely, rotor, bearing and housing, allows these to be separately considered, and if necessary, reduced using a suitable reduction method. For instance, for large-sized systems with Krylov-subspace methods. The resulting subsystems are then re-coupled to deliver a reduced-order model of the overall machine which can be simulated far faster than when compared to the original one. As an example, the method presented here is applied to reduce a simplified finite-element model of a gas compressor drive.

Keywords: Model Order Reduction; Rotordynamics; Electrical machines; Reduced-order models; Decoupling.

1. INTRODUCTION

Today, large rotordynamical machines are widely used in numerous industrial sectors. Ship's engines, large mills with a diameter of several meters, high-power electrical generators, and wind turbines are just a few examples. At the Siemens "Dynamowerk Berlin" (DW), where such customized large drives are engineered and manufactured, one of the main tasks is to analyze the designed machines whose dynamics are mainly influenced by that of the rotor. Until just recently, this suggested that only the rotor be modeled and simulated under different operating conditions. Lately, in order to be able to produce more powerful, reliable and lighter drives, the simulation of the complete machine, or even of the machine together with the surroundings, e.g. the base on which it is mounted, is required. Accurate modeling of these systems, mostly realized using the Finite Element Method (FEM) leads to parametric large-scale dynamical systems with more than 250,000 state variables, involving the rotational speed of the rotor as a parameter in the damping matrix. Hence, the computational cost of solving the underlying system of equations is very expensive, especially if several models have to be coupled and solved simultaneously. For specific applications, this can take up to three weeks.

Model Order Reduction (MOR) techniques offer a solution to this problem, by making it possible to generate accurate reduced-order models that are able to approximate the behavior of the original large-scale ones, while being much faster in the required simulation time. Thus, by generating appropriate reduced-order models of the different components of a system or at least for some of them, the simulation of the complete system can be performed very efficiently. For the class of large-scale systems (a few

hundred thousand), Krylov-subspace methods have proven lately to be one of the leading projection-based approaches (Antoulas (2005); Freund (2003); Grimme (1997)). These numerically efficient methods are based on matching some of the first coefficients of the Taylor series expansion of the transfer functions (the so-called moments) of the original and reduced models. Recently, several studies tried to generalize this approach to parametric systems (Daniel et al. (2004); Panzer et al. (2010); Peng et al. (2005)). However, due to several disadvantages, it is still too early to consider that the problem of model reduction of parametric systems has been solved.

By taking a closer look at the generated model of the rotating machines being considered (Zienkiewicz et al. (2005); Huebner et al. (2001)), it can be observed that: i) the sub-model of the housing and of the baseplate, on which the machine is mounted, is by far the largest, and ii) only the sub-model describing the rotor is a parametric dynamical system with a relatively low order (Yamamoto and Ishida (2001)). Accordingly, the new approach presented in this paper consists of partitioning the automatically generated FE-model into three sub-models, namely, rotor, bearing and housing, and then appropriately reducing that of the housing so that the resulting subsystem can be re-coupled to the other two. This results in a reduced-order model of the overall machine which can be simulated within a much shorter time compared to the original one, but which and still allows the rotational speed to be varied.

The paper is structured as follows: the preliminaries, the modeling of the main components of the rotating machines at Siemens DW and a short overview on the Krylov-based model order reduction are presented in the next two sections. They show the problem with reducing the

Fig. 1. Lateral cross-section of an FE-model of a gas compressor drive showing the main parts of the machine: the rotor (blue), the model of the fluid bearing (red), the bearing housing (green) and the housing (gray).

parameter depending rotordynamical structures. The two main results, the partitioning of the machine model into several sub-models together with its automation and the reduction and re-coupling to a reduced overall model, are presented in Sec. 4 and 5. Finally, the new approach is applied to reduce the model of a gas compressor drive.

2. MODELING OF ROTORDYNAMICAL MACHINES

The large machines produced at Siemens DW are customized drives with a power up to 100 MW and a weight up to 500 t. Therefore, before production, every designed drive must be subject to a thorough simulation under different operating conditions e.g. rotational speeds of the rotor. This is achieved by a complex FE-model involving the main parts of the machine, as illustrated in Figure 1. In this section, a short overview of the modeling of the machine's main parts, namely, the housing, the bearings and the rotor is presented.

2.1 Model of the housing

The housing of the machine consists of a large metal structure that can be easily modeled using the finite element method leading to a second-order system of differential equations of the form

$$\begin{aligned} \mathbf{M}_h \ddot{\mathbf{z}}_h(t) + \mathbf{D}_h \dot{\mathbf{z}}_h(t) + \mathbf{K}_h \mathbf{z}_h(t) &= \mathbf{J}_h \mathbf{u}_h(t) \\ \mathbf{y}_h(t) &= \mathbf{L}_h \mathbf{z}_h(t), \end{aligned} \quad (1)$$

where $\mathbf{M}_h, \mathbf{D}_h, \mathbf{K}_h \in \mathbb{R}^{n \times n}$ are constant matrices representing the mass, damping, and stiffness matrices of the system, respectively. The vector of internal generalized coordinates is $\mathbf{z}_h(t) \in \mathbb{R}^n$, $\mathbf{u}_h(t) \in \mathbb{R}^m$ corresponds to the vector of external forces, $\mathbf{J}_h \in \mathbb{R}^{n \times m}$ is the input matrix, $\mathbf{y}_h(t) \in \mathbb{R}^p$ is the output measurement vector, and $\mathbf{L}_h \in \mathbb{R}^{p \times n}$ is the output matrix. The resulting model is typically non-parametric and of large-scale with at least 100,000 states.

2.2 Model of the rotor

The main idea behind modeling the rotor is to divide it into rotating cylindrical beam sections while considering the

Fig. 2. Illustration of the gyroscopic moment of a rotating disk mounted on a mass-less shaft.

gyroscopic moment generated when rotating (Yamamoto and Ishida (2001), Friswell et al. (2010)). This guarantees a realistic description and therefore more accurate simulation results.

The gyroscopic moment describes the coupling between the lateral rotation and the angle of the rotor element caused by an axial mass moment of inertia. As an example, the gyroscopic moment of a rotating disk mounted on a mass-less shaft is shown in Figure 2. A rotation around the x -axis with $\dot{\varphi}_x = -\Omega$ and a rotation $\dot{\varphi}_y$ along the y -axis, create a moment M_z around the z -axis leading to shaft bending. Hence, the gyroscopic moment M_z around the z -axis of the disk with the polar mass moment of inertia Θ_x around the x -axis is

$$M_z = \Theta_x \Omega \dot{\varphi}_y. \quad (2)$$

Please note that for the machines considered in this paper, incorporating the gyroscopic moment in the model is crucial for its accuracy, as for even small tangential deviations of the large-sized rotor, large mass moments of inertia are present.

By combining the gyroscopic moments for each rotor element in the gyroscopic matrix $\mathbf{G}(\Omega)$, the rotor dynamics can be described by a parametric second-order system of the form

$$\begin{aligned} \mathbf{M}_t \ddot{\mathbf{z}}_t(t) + (\mathbf{D}_t + \mathbf{G}_t(\Omega)) \dot{\mathbf{z}}_t(t) + \mathbf{K}_t \mathbf{z}_t(t) &= \mathbf{J}_t \mathbf{u}_t(t) \\ \mathbf{y}_t(t) &= \mathbf{L}_t \mathbf{z}_t(t), \end{aligned} \quad (3)$$

where the structure of the system matrices are comparable with Eq. (1). The resulting model has typically only the rotational speed Ω as a parameter and does not consist of more than 300 second-order differential equations.

2.3 Model of the fluid bearings

The rotor is coupled to the housing through the bearings, whose type, arrangement, and number strongly depends on the size of the machine. For the drives produced at the Siemens DW, fluid bearings are the most commonly used ones, however with an increasing trend towards employing magnetic bearings. As fluid bearings significantly influence the dynamic interaction between rotor and housing, they will be considered in this paper.

The fluid bearings are modeled in a separate specialized tool which includes the non-linearities and the characteristics of the oils being used (Glienicke et al. (1980), Athavale and Hendricks (1996)) and provides suitable damping and stiffness matrices, so that the model of the fluid bearing

can be replaced by that of a spring-damper device. One of the main characteristics of the resulting matrices is that they have coupling terms between the different degrees of freedom involved¹ leading generally to non-symmetrical system matrices $\mathbf{D}_o$ and $\mathbf{K}_o$.

Such a modeling approach of the bearing results in a system where the rotor and the housing are coupled to each other via the stiffness and the damping matrices of the substitute spring-damper model. This is reflected in the layout of the overall system matrices $\mathbf{K}$ and $\mathbf{D}$ as shown below² for a simplified model

$$\mathbf{K} = \left(\begin{array}{cccc} k_{t11} & k_{t12} & 0 & 0 \\ k_{t21} & k_{t22} + k_o & -k_o & 0 \\ 0 & -k_o & k_{h11} + k_o & k_{h12} \\ 0 & 0 & k_{h21} & k_{h22} \end{array} \right) \left\{ \begin{array}{l} \text{rotor } \mathbf{K}_t \\ \text{bearing} \\ \text{housing } \mathbf{K}_h \end{array} \right. \quad (4)$$

This exemplary stiffness matrix corresponds to a system with only two degrees of freedom for the rotor and for the housing, which are coupled to each other through one degree of freedom.

2.4 Overall model of the machine

Practically, it is not possible to generate each of the three sub-models separately. The FEM software tool delivers the following model of the complete machine

$$\mathbf{M}\ddot{\mathbf{z}}(t) + (\mathbf{D} + \mathbf{G}(\Omega))\dot{\mathbf{z}}(t) + \mathbf{K}\mathbf{z}(t) = \mathbf{J}\mathbf{u}(t) \quad (5)$$

$$\mathbf{y}(t) = \mathbf{L}\mathbf{z}(t).$$

which consists of few hundred thousands of second-order differential equations. It incorporates all of the three sub-models and their dynamic interactions, however, in a coupled manner that does not allow a straightforward decoupling. Hence, there is a need for an automatic algorithm that first partitions the system (5) into the sub-models listed above, allowing, if necessary, to reduce them separately, and then re-couple the resulting models to conduct a complete system simulation (see Sec. 4 and 5).

3. KRYLOV-BASED MODEL ORDER REDUCTION

Consider the linear time-invariant multi-input and multi-output system

$$\mathbf{E}\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \quad (6)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t),$$

of order $n \in \mathbb{N}$, where $\mathbf{E}, \mathbf{A} \in \mathbb{R}^{n \times n}$, $\mathbf{B} \in \mathbb{R}^{n \times m}$, $\mathbf{C} \in \mathbb{R}^{p \times n}$ are matrices with constant coefficients, $\mathbf{u}(t) \in \mathbb{R}^m$, $\mathbf{y}(t) \in \mathbb{R}^p$, and $\mathbf{x}(t) \in \mathbb{R}^n$, respectively, the input, output and state vectors of the system. The transfer matrix of Eq. (6),

$$\mathbf{H}(s) = \mathbf{C}(s\mathbf{E} - \mathbf{A})^{-1}\mathbf{B}. \quad (7)$$

admits the infinite Taylor series' expansion about s_0

¹ It is assumed here that the bearing does not move in the direction of the rotor's axis.

² The layout of the matrix $\mathbf{D}$ is similar.

$$\mathbf{H}(s) = \sum_{i=0}^{\infty} -\mathbf{C}((\mathbf{A} - s_0\mathbf{E})^{-1}\mathbf{E})^i (\mathbf{A} - s_0\mathbf{E})^{-1}\mathbf{B}(s - s_0)^i$$

$$= \sum_{i=0}^{\infty} \mathbf{m}_{s_0}^i (s - s_0)^i$$

The aim of Krylov-based model reduction is to find a reduced model of order $q \ll n$, whose first few moments $\mathbf{m}_{s_0}^i$ around a certain expansion point s_0 match those of the original one (Freund (2003)). This approach is also known as *moment matching*.

A numerically efficient possibility to calculate such a reduced-order model is applying a projection to the original model,

$$\overbrace{\mathbf{W}^T \mathbf{E} \mathbf{V}}^{\mathbf{E}_r} \dot{\mathbf{x}}_r(t) = \overbrace{\mathbf{W}^T \mathbf{A} \mathbf{V}}^{\mathbf{A}_r} \mathbf{x}_r(t) + \overbrace{\mathbf{W}^T \mathbf{B}}^{\mathbf{B}_r} \mathbf{u}(t), \quad (8)$$

$$\mathbf{y}(t) = \underbrace{\mathbf{C} \mathbf{V}}_{\mathbf{C}_r} \mathbf{x}_r(t),$$

by means of the so-called projection matrices $\mathbf{V}$ and $\mathbf{W}$. For the choice of these matrices, the block Krylov subspace $\mathcal{K}_S$, defined in e.g. (Grimme (1997)) is employed,

$$\mathcal{K}_S(\mathbf{A}, \mathbf{B}) = \text{span}\{\mathbf{B}, \mathbf{A}\mathbf{B}, \dots, \mathbf{A}^{Q-1}\mathbf{B}\},$$

where $\mathbf{A} \in \mathbb{R}^{n \times n}$, $\mathbf{B} \in \mathbb{R}^{n \times m}$, and $S = Q \cdot m$. Specifically, if the projection matrices are chosen such that

$$\mathcal{K}_{q_1}((\mathbf{A} - s_0\mathbf{E})^{-1}\mathbf{E}, (\mathbf{A} - s_0\mathbf{E})^{-1}\mathbf{B}) \subseteq \text{colspan}(\mathbf{V}),$$

$$\mathcal{K}_{q_2}((\mathbf{A} - s_0\mathbf{E})^{-T}\mathbf{E}^T, (\mathbf{A} - s_0\mathbf{E})^{-T}\mathbf{C}^T) \subseteq \text{colspan}(\mathbf{W}),$$

then the first $\left(\frac{q_1}{m} + \frac{q_2}{p}\right)$ moments of the original and reduced models match, and the procedure is called a two-sided Krylov method. In the so-called one-sided method, only one Krylov subspace is used with the common choice $\mathbf{W} = \mathbf{V}$ and only $\frac{q_1}{m}$ or $\frac{q_2}{p}$ moments match (Grimme (1997)). Note that for the matrices $\mathbf{V}$ and $\mathbf{W}$ to have appropriate dimensions, the order of the reduced system should be a common multiple of the number of inputs and outputs.

For the numerical computation of the matrices $\mathbf{V}$ and $\mathbf{W}$ the Lanczos, Arnoldi and two-sided Arnoldi algorithms and their numerous improvements and modified versions are used. For more details see e.g. (Antoulas (2005)) and the references therein.

4. PARTITIONING THE OVERALL MACHINE MODEL

As already discussed in Sec. 2, the overall machine model consists of the model of the rotor coupled to that of the housing by means of the equivalent spring-damper model of the fluid bearings. Hence, the aim now is to separate the rotor from the housing while preserving the complete dynamical characteristics of the overall model.

As it is free from coupling terms, the overall mass matrix $\mathbf{M}$ can be easily divided into a rotor and a housing part, $\mathbf{M}_t$ and $\mathbf{M}_h$, respectively. It is also straightforward to assign the non-zero elements of the overall gyroscopic matrix $\mathbf{G}(\Omega)$ to that of the rotor $\mathbf{G}_t(\Omega)$ as no gyroscopic effects are present within the housing. The input and output matrices $\mathbf{J}, \mathbf{L}$ could also be easily divided into two parts corresponding to the rotor and the housing.

Fig. 3. Schematic of the loop calculation involving the three main sub-models

The decoupling of the overall stiffness and damping matrices $\mathbf{K}$ and $\mathbf{D}$ requires special attention as they express the coupling of the rotor to the housing through the fluid bearings. In order to preserve the overall dynamics, the matrices of the equivalent spring-damper model $\mathbf{D}_o$ and $\mathbf{K}_o$ are first calculated. These matrices are obtained by subtracting the rotor and housing part $\mathbf{K}_t$, $\mathbf{D}_t$ and $\mathbf{K}_h$, $\mathbf{D}_h$ from the coupling terms in the $\mathbf{K}$ and $\mathbf{D}$ matrices, based on the layout of Eq. (4). The input and output matrices of each of the three sub-models are then chosen so that the velocity and position of the interface nodes at the rotor and the housing can be extracted, so that the force interactions with the fluid bearing can be calculated. Such a setting allows a loop calculation as shown in Figure 3.

4.1 Mathematical model of the decoupled systems

Now, assuming that the overall model of the machine is available in the form (5), the decoupled systems of the rotor and the housing are then,

$$\mathbf{M}_t \ddot{\mathbf{z}}_t + (\mathbf{D}_t + \mathbf{G}_t(\Omega)) \dot{\mathbf{z}}_t + \mathbf{K}_t \mathbf{z}_t = \begin{pmatrix} \mathbf{J}_t & \mathbf{J}_{t,o} \end{pmatrix} \begin{pmatrix} \mathbf{u}_t \\ -\mathbf{F}_o \end{pmatrix} \quad (9)$$

$$\begin{pmatrix} \mathbf{y}_t \\ \mathbf{y}_{t,o} \end{pmatrix} = \begin{pmatrix} \mathbf{L}_t \\ \mathbf{L}_{t,o} \end{pmatrix} \mathbf{z}_t,$$

and

$$\mathbf{M}_h \ddot{\mathbf{z}}_h + \mathbf{D}_h \dot{\mathbf{z}}_h + \mathbf{K}_h \mathbf{z}_h = \begin{pmatrix} \mathbf{J}_h & \mathbf{J}_{h,o} \end{pmatrix} \begin{pmatrix} \mathbf{u}_h \\ \mathbf{F}_o \end{pmatrix} \quad (10)$$

$$\begin{pmatrix} \mathbf{y}_h \\ \mathbf{y}_{h,o} \end{pmatrix} = \begin{pmatrix} \mathbf{L}_h \\ \mathbf{L}_{h,o} \end{pmatrix} \mathbf{z}_h,$$

where $\mathbf{J}_{t,o}$ and $\mathbf{J}_{h,o}$ are the newly inserted input matrices to account for the fluid bearing forces acting on the rotor and housing, respectively. Similarly, matrices $\mathbf{L}_{t,o}$ and $\mathbf{L}_{h,o}$ are the newly added output matrices to allow the forces of the rotor and that of the housing acting on the fluid bearing to be calculated using

$$\mathbf{D}_o(\dot{\mathbf{y}}_{t,o} - \dot{\mathbf{y}}_{h,o}) + \mathbf{K}_o(\mathbf{y}_{t,o} - \mathbf{y}_{h,o}) = \mathbf{F}_o. \quad (11)$$

It is important to point out that the difference between the new added outputs is included in Eq. (11), because the vector of the generated forces $\mathbf{F}_o$ is caused by the relative displacement and velocity of the spring-damper system representing the bearing. Caused by the fact that the velocity is needed, it is necessary to get over to a first order model (see Sec. 5). Please note also that the number of inputs and outputs added to the sub-models depends on the interface between the bearing and the rotor from one side and the bearing and the housing from the other side. Hence, the number of inputs and outputs can be quite large once these interfaces involve a large number of nodes.

4.2 Automation of the partitioning step

As the models being considered are large scale models with several thousands of equations, it is almost impossible to partition their corresponding system matrices manually. Hence, there is a need for a numerically efficient algorithm that automatically extracts the matrices of the three sub-models once those of the overall model are given. This is in fact possible, once the information about the correlation between equation number, node number and degrees of freedom is available. Fortunately, to the author's knowledge, all FEM software tools can deliver such information. It is generally in a text file where each line contains the ordered sequence of node number, degree of freedom and equation number. This information can be expressed as a matrix denoted as $\mathbf{N} \in \mathbb{N}^{n \times 2}$, where $\mathbf{N}(i, 1) = j$ and $\mathbf{N}(i, 2) = k$ represent the i -th equation, node number j and k -th degree of freedom.

The automation of the decoupling of a large-scale rotor dynamical system which is given in the form of Eq. (5) with $\mathbf{N}$ is summarized in the following algorithm:

Algorithm 1 Partitioning Algorithm

Input: $\mathbf{M}, \mathbf{D}, \mathbf{G}, \mathbf{K}, \mathbf{J}, \mathbf{L}$ and $\mathbf{N}$.

- 1: Identify all equations involving a node which is affected by a gyroscopic moment (rotor nodes) and merge their numbers in the set

$$\mathcal{R} = \{i \in [1, n] \mid \exists a, b \in [1, n] : \mathbf{N}(i, 1) = \mathbf{N}(a, 1) \wedge \mathbf{G}(a, b) \neq 0\}.$$

- 2: Define the set of equations involving the housing nodes as

$$\mathcal{H} = [1, n] \setminus \mathcal{R}.$$

- 3: Knowing $\mathcal{R}$ and $\mathcal{H}$, the mass, stiffness, gyroscopy and damping could be separately stored for rotor $\mathbf{M}_t$, $\mathbf{K}_t^*$, $\mathbf{D}_t^*$, $\mathbf{G}_t$ and housing $\mathbf{M}_h$, $\mathbf{K}_h^*$, $\mathbf{D}_h^*$.
- 4: Identify all elements $\mathbf{K}(i, j) \neq 0$ where i is out of $\mathcal{R}$ and j is out of $\mathcal{H}$. Store the identified stiffness elements in the new bearing stiffness matrix $-\mathbf{K}_o$. Merge the equation numbers in the sets

$$\mathcal{R}_o = \{i \in \mathcal{R} \mid \exists j \in \mathcal{H} : \mathbf{K}(i, j) \neq 0\}$$

$$\mathcal{H}_o = \{i \in \mathcal{H} \mid \exists j \in \mathcal{R} : \mathbf{K}(i, j) \neq 0\}.$$

- 5: Calculate the rotor stiffness matrix $\mathbf{K}_t$ and that of the housing $\mathbf{K}_h$ by subtracting the bearing stiffness $\mathbf{K}_o$ from $\mathbf{K}_t^*$ and $\mathbf{K}_h^*$, respectively.
- 6: Generate the new inputs $\mathbf{J}_{t,o}$, $\mathbf{J}_{h,o}$ and outputs $\mathbf{L}_{t,o}$, $\mathbf{L}_{h,o}$ out of the equation numbers of $\mathcal{R}_o$ and $\mathcal{H}_o$.
- 7: Repeat step 4, 5 and 6 for the damping matrix.

Output: The matrices of the three sub-models (9), (10), and (11).

5. ORDER REDUCTION AND RE-COUPLING THE OVERALL SYSTEM

Now that the three sub-models are on hand in a second-order form, the last step before model reduction would be to transform them into a classical first-order state-space representation of the form Eq. (6). The following step is

the same for the housing and the rotor and for simplicity only shown for the latter.

$$\begin{aligned} \underbrace{\begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_t \end{pmatrix}}_{\mathbf{E}_t} \underbrace{\begin{pmatrix} \dot{\mathbf{z}}_t \\ \mathbf{z}_t \end{pmatrix}}_{\mathbf{x}_t} &= \underbrace{\begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{K}_t & (\mathbf{D}_t + \mathbf{G}_t(\Omega)) \end{pmatrix}}_{\mathbf{A}_t(\Omega)} \underbrace{\begin{pmatrix} \dot{\mathbf{z}}_t \\ \mathbf{z}_t \end{pmatrix}}_{\mathbf{x}_t} \\ &+ \underbrace{\begin{pmatrix} \mathbf{B}_t & \mathbf{B}_{t,o} \\ \mathbf{J}_t & \mathbf{J}_{t,o} \end{pmatrix}}_{\mathbf{B}_t} \underbrace{\begin{pmatrix} \mathbf{u}_t \\ \mathbf{F}_o \end{pmatrix}}_{\mathbf{u}_t}, \\ \underbrace{\begin{pmatrix} \mathbf{y}_t \\ \mathbf{y}_{t,o} \\ \dot{\mathbf{y}}_{t,o} \end{pmatrix}}_{\mathbf{y}_t^*} &= \underbrace{\begin{pmatrix} \mathbf{L}_t & \mathbf{0} \\ \mathbf{L}_{t,o} & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_{t,o} \end{pmatrix}}_{\mathbf{C}_t} \underbrace{\begin{pmatrix} \mathbf{z}_t \\ \dot{\mathbf{z}}_t \end{pmatrix}}_{\mathbf{z}_t}. \end{aligned} \quad (12)$$

At this point, it should be noted that the inverse of the matrix $\mathbf{M}_t$ for large-scale systems should be avoided, which is why a matrix $\mathbf{E}_t$ has been included in the system representation above. Moreover, even though it is not the most common way, this choice of $\mathbf{L}_{t,o}$ is needed to obtain $\dot{\mathbf{y}}_{t,o}$ as an output to calculate the fluid bearing force (see Eq. (11)).

The model of the rotor is always a parametric dynamical system with the gyroscopic matrix depending on the rotational speed, as shown in Sec. 2. Thus, a parameter-preserving model reduction of this sub-model is reasonable, however, unnecessary for the class of machines considered in this work, where the model of a rotor normally consists of only 50–100 nodes, i. e. involving between 200 to 600 state equations. The housing, in contrast, is a parameter-free system with more than 100,000 state variables. Hence, it is advisable and advantageous to only reduce the housing model with a classical MOR method and re-couple it to that of the rotor instead of reducing the overall model with a parameter-preserving approach.

The reduction of the housing sub-model by a two-sided Krylov-subspace method as shown in Sec.3, leads to the following reduced-order model

$$\begin{aligned} \mathbf{E}_h^{\text{red}} \dot{\mathbf{x}}_h^{\text{red}} &= \mathbf{A}_h^{\text{red}} \mathbf{x}_h^{\text{red}} + (\mathbf{B}_h^{\text{red}} \quad \mathbf{B}_{h,o}^{\text{red}}) \begin{pmatrix} \mathbf{u}_h \\ \mathbf{F}_o \end{pmatrix} \\ \tilde{\mathbf{y}}_h^* &= \begin{pmatrix} \tilde{\mathbf{y}}_h \\ \tilde{\mathbf{y}}_{h,o} \\ \dot{\tilde{\mathbf{y}}}_{h,o} \end{pmatrix} = \begin{pmatrix} \mathbf{C}_h^{\text{red}} \\ \mathbf{C}_{h,o}^{\text{red}} \end{pmatrix} \mathbf{x}_h^{\text{red}}. \end{aligned} \quad (13)$$

One advantage of the partitioning/re-coupling approach presented in this work is that it connects the sub-models through their inputs and outputs, which are quantities that can not be eliminated by the MOR step, unlike the state variables. A block diagram illustrating the re-coupling of the overall system using the reduced housing is shown in Figure 3. To avoid the numerical and implementation disadvantages occurring from loop calculations, the reduced system Eq. (13) is re-coupled to the unmodified models of the rotor and bearing. This is done by combining Eqs. (9), (11) and (13). Nevertheless, the reduction introduces a small error in $\tilde{\mathbf{y}}_h$, that propagates through the state vector $\tilde{\mathbf{x}}_t$ to the output $\tilde{\mathbf{y}}_t$.

Fig. 4. Simplified FE-model of the gas compressor drive consisting of: rotor (blue), bearing housing (green), the model of the fluid bearing (red) and spring-damper devices (gray).

The final result of partitioning, reducing and re-coupling is expressed by Eq. (14). Note that the inversion of the reduced matrix $\mathbf{E}_h^{\text{red}}$ and that of $\mathbf{M}_t$ is numerically possible and not time consuming, as both quadratic matrices are non-singular and only have a few hundred rows.

6. TECHNICAL EXAMPLE

To illustrate the approach of this paper, a simplified FE-model of the gas compressor drive shown in Figure 1 is considered. This machine, with a power of 16 MW, was built for an offshore platform in Norway. Its rotor has a length of nearly 6 m and the housing has a quadratic base with a length of 4 m. The simplification of the model involves just considering the rotor, the fluid bearing, and their housings while approximating the rest of the housing using several spring-damper-devices as shown in Figure 4. By taking the constraints of several degrees of freedom into account, the FE-modeling leads to a second order system of the form Eq. (5) with 4204 states. The considered machine has two fluid bearings which are represented by spring-damper-devices and each assumed to be connected to the rotor at a given node.

The resulting system is first partitioned with the proposed algorithm 1 within 1 s, leading to the rotor system with 364 states and the simplified housing system with 3840 states. Both systems' terminal matrices are augmented by 4 new inputs and outputs which allows the interaction force transmitted through the bearings to be calculated. The resulting multi-input and multi-output system of the housing is then transformed into a first-order system with 7680 states and reduced down to 540 states by the Krylov-subspace method introduced in Section 3. The expansion point was chosen as $s_0 = 56 \text{ rad/s}$, leading to an excellent approximation of the transfer function within the bandwidth of interest (see Figure 5).

In order to analyze smooth running of the machine, it is necessary to observe the vibration amplitude at several nodes of the housing for different rotor excitations. As the rotational speed of the rotor constitutes the main vibration source, the bandwidth of interest for these excitations is commonly chosen as twice the nominal rotational speed of the rotor. In the amplitude plot of a given housing

$$\begin{pmatrix} \dot{\tilde{\mathbf{x}}}_t \\ \dot{\tilde{\mathbf{x}}}_h^{\text{red}} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_t(\Omega) - \begin{pmatrix} \mathbf{0} \\ \mathbf{M}_t^{-1} \mathbf{B}_{t,o} \end{pmatrix} (\mathbf{K}_o \ \mathbf{D}_o) \begin{pmatrix} \mathbf{C}_{t,o} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{t,o} \end{pmatrix} & \begin{pmatrix} \mathbf{0} \\ \mathbf{M}_t^{-1} \mathbf{B}_{t,o} \end{pmatrix} (\mathbf{K}_o \ \mathbf{D}_o) \mathbf{C}_{h,o}^{\text{red}} \\ \begin{pmatrix} \mathbf{E}_h^{\text{red}} \end{pmatrix}^{-1} \mathbf{B}_{h,o}^{\text{red}} (\mathbf{K}_o \ \mathbf{D}_o) \begin{pmatrix} \mathbf{C}_{t,o} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{t,o} \end{pmatrix} & \begin{pmatrix} \mathbf{E}_h^{\text{red}} \end{pmatrix}^{-1} (\mathbf{A}_h^{\text{red}} - \mathbf{B}_{h,o}^{\text{red}} (\mathbf{K}_o \ \mathbf{D}_o) \mathbf{C}_{h,o}^{\text{red}}) \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{x}}_t \\ \mathbf{x}_h^{\text{red}} \end{pmatrix} + \begin{pmatrix} \begin{pmatrix} \mathbf{0} \\ \mathbf{M}_t^{-1} \mathbf{B}_t \end{pmatrix} & \mathbf{0} \\ \mathbf{0} & \begin{pmatrix} \mathbf{E}_h^{\text{red}} \end{pmatrix}^{-1} \mathbf{B}_h^{\text{red}} \end{pmatrix} \begin{pmatrix} \mathbf{u}_t \\ \mathbf{u}_h \end{pmatrix}, \quad \begin{pmatrix} \tilde{\mathbf{y}}_t \\ \tilde{\mathbf{y}}_h \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \mathbf{C}_t & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_h^{\text{red}} \end{pmatrix} \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{x}}_t \\ \mathbf{x}_h^{\text{red}} \end{pmatrix} \quad (14)$$

Fig. 5. Comparison of the *original* model (8408 states) and the *reduced* model (1268 states); Vibration amplitude of a housing node depending on different excitation frequencies of the rotor node at a rotational speed $\Omega = 3000$ rpm

node presented in Figure 5, the nominal rotational speed is chosen to be around 3000 rpm, corresponding to an analyzed frequency interval of between 0 rad/s and 630 rad/s. The relative error resulting from reducing the model of the housing and re-coupling it with the rotor is smaller than 0.01 % up to 740 rad/s.

The calculation of the frequency response of the original system of the housing for a single rotational speed takes around 121s with optimized tools, while that of its corresponding reduced system only takes 10s. A simulation of the complete re-coupled system including reduction requires around 24s. Hence, for a critical-speed map, i.e. a simulation with 15 different rotational speeds, the complete simulation with the original model would need about half an hour while with the reduced model it would take only 174s including the decoupling, reduction, and re-coupling steps. This corresponds to a relative time saving of over 90 % for this relatively small problem. When considering the whole machine shown in Figure 1, first simulations have shown that model reduction slashes the calculation time for a critical-speed map from about three weeks down to just a few hours!

7. CONCLUSION

In this paper, the problem of order reduction of large rotordynamic systems occurring in industry has been considered. Under the common assumption that only the complete model of the machine can be obtained from the FEM software tool, an algorithm for partitioning the large drive system into the rotor, housing, and bearing sub-models has been introduced. Model order reduction

has then been applied, when necessary, to each of these systems allowing a drastic time saving in the simulation of the re-coupled overall system. However, a number of open questions has still to be solved to optimize the presented approach, including model reduction of systems with a large number of inputs and outputs, the consideration of the nonlinear effects in the equivalent model of the fluid bearings describing the dependency of the oil film on the rotational speed, and the analysis of the error dynamics within the re-coupled system caused by the reduction step.

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