An improvement on a method for nonlinear pressure control of self-supplied variable displacement axial piston pumps[1]

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Abstract: A method for pressure control of self-supplied variable displacement axial piston pumps is considered. By analyzing the mathematical model, a crucial part of the nonlinear system is found to be a piecewise linear function. A control strategy composed of an overall feedforward control and a feedback control is given using the smooth hinge function. Compared with current switching feedforward control, the overall feedforward control strategy can avoid problems when switching. Simulation results and stability analysis are also presented in this paper.

Keywords: Axial piston pumps, Switched systems, Nonlinear control, Smooth hinge functions.

1. INTRODUCTION

Electro hydraulic systems are widely used in machines and factories (Merritt, 1967). The difficulties in supply pressure control of electro hydraulic systems are mainly caused by the obvious nonlinear dynamic behaviour of the systems (Grabbel and Ivantysynova, 2005; Manring, 2005). To control the supply volume flow of the hydraulic supply systems, variable displacement axial piston pumps are often used. By changing the displacement of the pump via a swash plate, the supply volume flow will be controlled. There are already many works which deal with the supply pressure control of electro hydraulic systems comprising a variable displacement axial piston pump and a variable load, see Kemmetmüller, Fuchshumer and Kugi (2010) for a review of details.

In Kemmetmüller, Fuchshumer and Kugi (2010) a model-based nonlinear control strategy is given, in which the necessary nonlinearities of the system are considered. The variable displacement axial piston pump studied is self-supplied. This leads a switching mathematical model of the system. A control structure, comprising switching feedforward control and feedback control, is given in the controller design. Two problems would occur when switching the feedforward control and feedback control synchronously. One is that the desired trajectory may not be continuous. Another problem is that sliding motion of the controller may occur.

Work presented in this paper is motivated to solve the aforementioned problems associated with the following facts. The switching part of the pump model can be represented as a continuous piecewise linear function, see more details of piecewise linear function in Wang, Huang and Junaid (2008). Thus an overall control stagey can be designed to replace the switching control to avoid aforementioned problems. One difficulty in designing overall control stagey is that the piecewise linear function is not differentiable. By using smooth hinge function (Wang, Huang and Yam, 2010), an overall feedforward control strategy compared with switching feedforward control strategy (Kemmetmüller, Fuchshumer and Kugi, 2010) is given. Simulation results show that the new method is suitable for control of the system. Qualitative stability analysis is presented.

Though the present work is aimed at a specific control problem, the work can be extended to designing and controlling of more general switching systems. Since smooth hinge function has good approximation capability and is differentiable, new control method may be obtained by replacing switching part of the original system with smooth hinge function.

The paper is organized as follows: In the next section, the mathematical model of the system and the description of the control mission is given. The switching feedforward control is introduced and the overall feedforward control is presented in Section 3. Simulation results are shown in Section 4. Stability analysis is the topic of Section 5. Section 6 gives the conclusions.

2. MODELING AND CONTROL MISSION

Schematic diagram of the electro hydraulic system and the variable displacement axial piston pump along with

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description of the mechanism of the system, also review of works on the mathematical modeling of electro hydraulic systems can be find in Kemmetmüller, Fuchshumer and Kugi (2010).

The model used in this paper is proposed in Fuchshumer (2009) and studied in Kemmetmüller, Fuchshumer and Kugi (2010) . The model is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\varphi_p = -\frac{q_a}{A_a r_a}, \tag{1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}p_l = \frac{\beta}{V_l}\left(k_p\varphi_p - k_l\sqrt{p_l} - \eta(q_a)\right), \tag{2}$$

where φ_p is the swash plate angle, p_l is the load pressure, q_a is the volume flow into the actuator, A_a is the effective area of the actuator, r_a is the effective radius of the actuator, β is the bulk modulus of the oil, V_l is the volume of the load, k_p is the pump coefficient, k_l is the unknown coefficient of the load orifice (load coefficient). In this paper, the load coefficient is assumed to be a known constant. $\eta(q_a)$ describes the volume flow taken from the load in order to tilt the swash plate.

$$\eta(q_a) = \begin{cases} q_a & q_a > 0 \\ 0 & \text{else} \end{cases} = \max\{0, q_a\}, \tag{3}$$

$\eta(q_a)$ is a continuous piecewise linear function, which can be represented as a hinge functions (Breiman,1993). Hinge function is not global differentiable, as in our case $\dot{\eta}(0)$ doesn't exist. Smooth hinge function is proposed in (Wang, Huang and Yam, 2010), which retains the useful features of hinge function while overcoming the drawback of non-differentiability of hinge functions. The next section shows the difficulties in pressure control of self-supplied variable displacement axial piston pumps caused by hinge function's not being differentiable globally and how to use smooth hinge function to solve the problem.

The closed loop system is shown in Fig. 1, where $p_{l,d}$ is the desired trajectory of the load pressure, $q_{a,d}$ is the feedforward control input, $q_{a,c}$ is the feedback control input and $q_a = q_{a,d} + q_{a,c}$. The main difference between the new control strategy and the one proposed in Kemmetmüller, Fuchshumer and Kugi (2010) is in the feedforward part

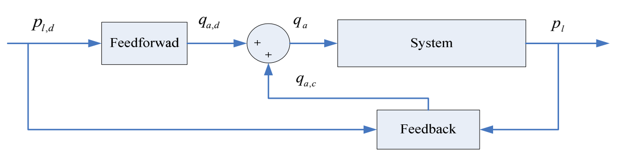

Fig. 1. Diagram of the control system

3. CONTROL DESIGN

3.1 Switching feedforward control and feedback control

This subsection introduces the switching feedforward control proposed in Kemmetmüller, Fuchshumer and Kugi (2010). The system is separated into two systems naturally.

For $q_a \le 0$, we have system Σ^1

$$\Sigma^1: \frac{\mathrm{d}}{\mathrm{d}t}\varphi_p = -\frac{q_a}{A_a r_a}, \tag{4a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}p_l = \frac{\beta}{V_l}\left(k_p\varphi_p - k_l\sqrt{p_l}\right). \tag{4b}$$

And system Σ^2 is valid for $q_a > 0$

$$\Sigma^2: \frac{\mathrm{d}}{\mathrm{d}t}\varphi_p = -\frac{q_a}{A_a r_a}, \tag{5a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}p_l = \frac{\beta}{V_l}\left(k_p\varphi_p - k_l\sqrt{p_l} - q_a\right). \tag{5b}$$

In Kemmetmüller, Fuchshumer and Kugi (2010), the feedforward control for Σ^1 is given by

$$FF^1: q_{a,d} = -\frac{A_a r_a}{k_p}\left(\frac{V_l}{\beta}\ddot{p}_{l,d} + \frac{1}{2}\frac{k_l}{\sqrt{p_{l,d}}}\dot{p}_{l,d}\right), \tag{6}$$

and the desired swash plate angle

$$\varphi_{p,d} = \frac{1}{k_p}\left(\frac{V_l}{\beta}\dot{p}_{l,d} + k_l\sqrt{p_{l,d}}\right). \tag{7}$$

The feedforward control for Σ^2 is

$$FF^2: q_{a,d} = -\frac{V_l}{\beta}\dot{p}_{l,d} + k_p\varphi_{p,d} - k_l\sqrt{p_{l,d}}, \tag{8}$$

$\varphi_{p,d}$ is determined by

$$\frac{\mathrm{d}}{\mathrm{d}t}\varphi_{p,d} = \frac{1}{A_a r_a}\left(-k_p\varphi_{p,d} + k_l\sqrt{p_{l,d}} + \frac{V_l}{\beta}\dot{p}_{l,d}\right). \tag{9}$$

Suppose FB^1 and FB^2 are feedback control for system Σ^1 and Σ^2 . It is proposed in Kemmetmüller, Fuchshumer and Kugi (2010) that switching the feedforward control FF^1 and FF^2 based on $q_a = 0$ yields discontinuities in the desired value of the swash plate angle. Meanwhile sliding motion of the controller would take place. For example, if FF^1 + FB^1 yields $q_a = 0$, this does not necessarily imply that switching to $FF^2 + FB^2$ also obeys $q_a = 0$. The independent switching of the feedforward , the feedback control and the system can solve these problems, but will increase the difficulties in proof of the stability of the closed-loop system. Those problems make the choice of a suitable switching criterion for feedback control quite complicated. In Kemmetmüller, Fuchshumer and Kugi (2010) a simple feedback control law of the form

$$q_{a,c} = \lambda_p e_p + \lambda_\varphi e_\varphi \;\; \lambda_p, \lambda_\varphi > 0 \qquad (10)$$

where $e_p = p_l - p_{l,d}$, $e_\varphi = \varphi_l - \varphi_{l,d}$,is given.

3.2 Overall feedforward control and feedback control

Take (1) into the second derivative of the desired load pressure, we can get

$$\ddot{p}_{l,d} = \frac{\beta}{V_l}\left(-\frac{k_p}{A_a r_a} q_{a,d} - \frac{1}{2}\frac{k_l}{\sqrt{p_{l,d}}}\dot{p}_{l,d} - \dot{\eta}\left(q_{a,d}\right)\dot{q}_{a,d}\right), \qquad (11)$$

where the $\dot{\eta}\left(q_{a,d}\right)$ is a step function that do not have value at $q_{a,d} = 0$. This will cause the difference on the degree of the system between Σ^1 and Σ^2 , which leads to using different feedforward control FF^1 and FF^2 for system Σ^1 and Σ^2 .

This problem can be avoided by approximating $\eta\left(q_{a,d}\right) = \max\left\{0, q_{a,d}\right\}$ with $\hat{\eta}\left(q_{a,d}\right) = \frac{1}{\alpha}\ln\left(1 + e^{\alpha q_{a,d}}\right)$ $\alpha > 0$, which is differentiable at $q_{a,d} = 0$. In Wang, Huang and Yam (2010), it is indicated that α controls the precision of the approximation (see Fig.2). The approximation is more precise with a larger α .

Take $\dot{\hat{\eta}}\left(q_{a,d}\right) = \frac{1}{1 + e^{-\alpha q_{a,d}}}$ instead of $\dot{\eta}\left(q_{a,d}\right)$ into $\ddot{p}_{l,d}$, we can get

$$\dot{q}_{a,d} = -\left(1 + e^{-\alpha q_a}\right)\left(\frac{V_l}{\beta}\ddot{p}_{l,d} + \frac{1}{2}\frac{k_l}{\sqrt{p_{l,d}}}\dot{p}_{l,d} + \frac{k_p}{A_a r_a} q_{a,d}\right), (12)$$

as the overall feedforward control. By using overall feedforward control it can avoid the problems caused by switching between FF^1 and FF^2 . Using overall feedforward control can also cut the trouble calculating $\varphi_{p,d}$. In this work $q_{a,c} = \lambda_p e_p \;\; \lambda_p > 0$ is used as the feedback control.

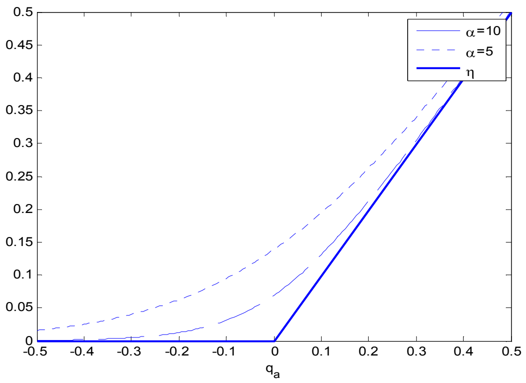

Fig. 2. Approximation of $\eta\left(q_a\right)$

4. SIMULATION RESULTS

In the simulation, the parameters are set as follows:

$A_a = 300\text{mm}^2$

$r_a = 50\text{mm}$

$\beta = 1.6 \times 10^9\,\text{Pa}$

$V_l = 1.5\,\text{l}$

$k_p = 2.23 \times 10^{-3}\,\text{m}^3/\text{s}$

$k_l = 90 \times 10^{-9}\,\text{m}^3/\text{s}\,\sqrt{\text{Pa}}$

First, the desired trajectory of the load pressure is chosen to take the form $p_{l,d}\left(t\right) = A + Be^{-\alpha t} + Cte^{-\alpha t}$. Coefficients $A,\ B,\ C$ are chosen to satisfy

$$\begin{cases} p_{l,d}(0) = D \\ \dot{p}_{l,d}(0) = 0 \end{cases},$$

where D is the initial value of the load pressure.

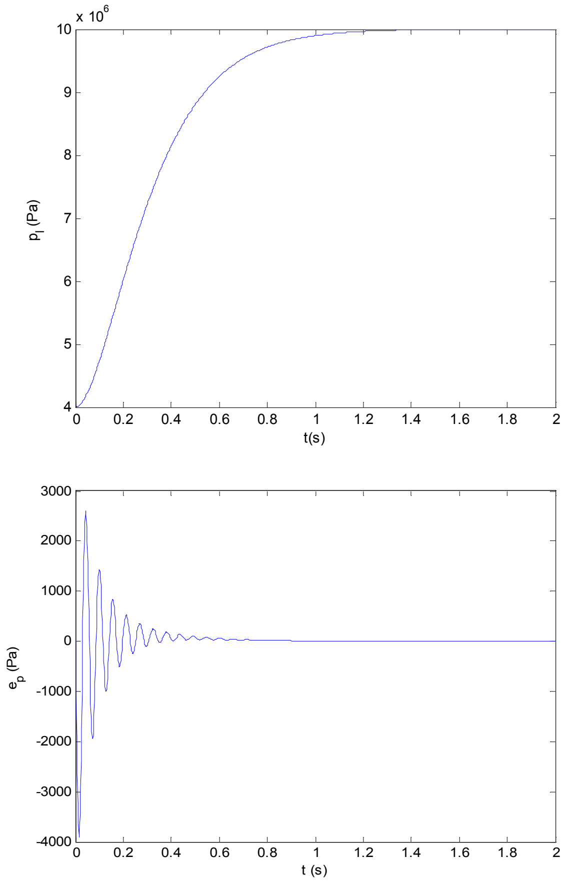

Fig. 3. Simulation result for $p_{l,d}\left(t\right) = A + Be^{-\alpha t} + Cte^{-\alpha t}$

The trajectory of the load pressure p_l and the trajectory of the load pressure error $e_p = p_l - p_{l,d}$ are given in Fig.3. Notice that p_l and e_p are not of the same order of magnitude.

Both overall feedforward control and switching feedforward control requires $\dot{p}_{l,d}$ being continuous to achieve good performance because the right hand of (2) is continuous. The next simulation result shows that with the help of the feedback control, the overall feedforward control and feedback control strategy is suitable when the desired trajectory of the load pressure is of the form $p_{l,d}(t) = A + Be^{-\alpha t}$ with a discontinuous $\dot{p}_{l,d}$, i.e. $\dot{p}_{l,d}(0) \neq 0$. The trajectory of the load pressure p_l and the trajectory of the load pressure error $e_p = p_l - p_{l,d}$ are given in Fig.4.

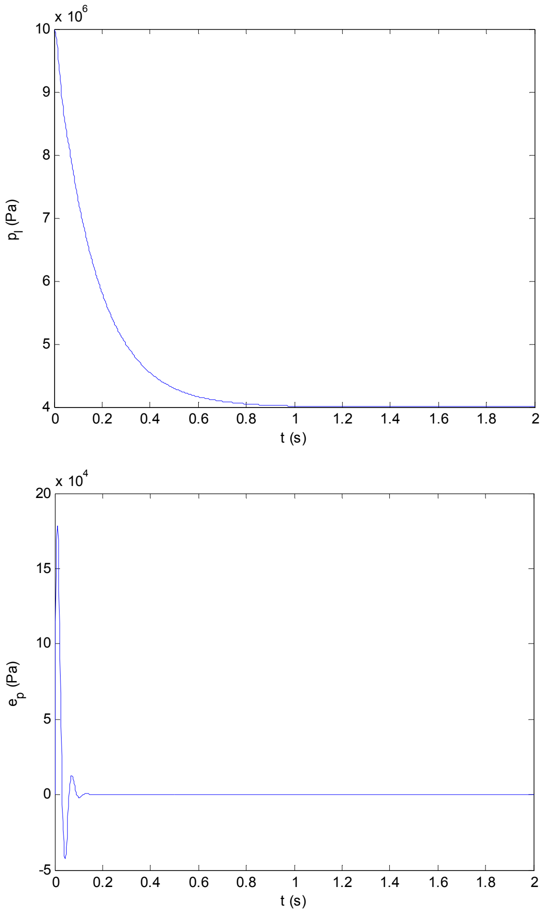

Fig. 4. Simulation result for $p_{l,d}(t) = A + Be^{-\alpha t}$

According to Fig. 3 and Fig. 4, both simulation results shows that control strategy composed of an overall feedforward control and a feedback control achieved acceptable tracking performance.

5. STABILITY ANALYSIS

The stability of the system comprising a switching feedforward control and a feedback control is proved in Kemmetmüller, Fuchshumer and Kugi (2010). The overall feedforward control is obtained through approximating $\eta(q_{a,d}) = \max\{0, q_{a,d}\}$ with $\hat{\eta}(q_{a,d}) = \frac{1}{\alpha}\ln\left(1 + e^{\alpha q_{a,d}}\right)$. And Fig. 2 shows that the main difference between $\eta(q_{a,d})$ and $\hat{\eta}(q_{a,d})$ happens around $q_{a,d} = 0$, while $\eta(q_{a,d})$ and $\hat{\eta}(q_{a,d})$ are nearly the same when $|q_{a,d}|$ is large. Thus the closed loop system comprising an overall feedforward control and a feedback control may have the same stability as the system comprising a switching feedforward control and a feedback control when $|q_{a,d}|$ is large enough. For $|q_{a,d}|$ is small, the feedback control $q_{a,c}$ will be the main part of q_a, the closed loop system may still be stable. So the closed loop system with overall feedforward control is supposed to be stable.

As for complex feedback design, using the overall feedforward control may also makes the stability analysis easier than using the switching feedforward control. If FB^1 and FB^2 are different feedback control for system Σ^1 and Σ^2, meanwhile using switching feedforward control FF^1 and FF^2 .Eight combination of feedforward control, feedback control and system state, such as (FB^2 , FF^1 , Σ^2), must be considered when analyze the stability. By using overall feedforward control, only four combination of feedback control and system state need to be considered, may make the stability analysis easier.

The proof of the stability of the system with a switching feedforward control in Kemmetmüller, Fuchshumer and Kugi (2010) requires $\dot{p}_{l,d}$ being continuous. According to simulation results aforementioned, system with overall feedforward is suitable for $p_{l,d}$ even without a continuous $\dot{p}_{l,d}$.

6. CONCLUSIONS

In this work, an improvement on a method for nonlinear pressure control of self-supplied variable displacement axial piston pumps has been made. Smooth hinge functions are used to approximate the switching part of the mathematical model. By doing this an overall feedforward control is proposed. Using overall feedforward control can avoid the problems occurred when using switching feedforward control. Simulation results show that the performance of the new method is acceptable. The present work is aimed at a specific control problem. For more general switching systems, by replacing switching part of the original system with smooth hinge function, new control method may be obtained. More comparison between the two types of feedforward control and the complete proof of the stability will be carried out in the future work.

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