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On Adaptive Stabilization of Mode-Observable Switching Linear Systems

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Abstract: This paper addresses the problem of controlling a continuous-time linear system that may switch among different modes taken from a finite set. The current mode of the system is supposed to be unknown. Moreover, unknown but bounded disturbances are assumed to affect the dynamics as well as the measurements. The proposed methodology is based on a minimum-distance mode estimator which orchestrates controller switching according to a dwell-time switching logic. Provided that a certain mode observability condition holds and the plant switching signal is slow on the average, the resulting control system turns out to be exponentially input-to-state stable.

Keywords: Switching systems; Mode observability; Switching control; Adaptive control.

1. INTRODUCTION

Over the last decade, a lot of attention has been devoted to switching systems from both research and industrial areas, as they allow one to represent and investigate the properties of a large class of plants in numerous applications resulting from the interactions of continuous dynamics, discrete dynamics, and logic decisions (see D.Liberzon, 2003). In a switching system, the system dynamics as well as the measurement equations may switch, at each time instant, among different configurations (system modes) taken from a finite set. Under the assumption that an exact knowledge of the current system configuration is available on line without delay, the stabilization of a switching system is now a well understood problem. In fact, necessary and sufficient conditions for the existence of a switching controller that stabilizes the plant under arbitrary swtiching have been derived both in the continuous-time (Blanchini et al., 2009) and in the discrete-time case (Lee and Dullerud, 2006). Similar methodologies can also be exploited in order to address the case of delayed information on the plant configuration (Xie and Wu, 2009).

On the other hand, the case in which the knowledge of the plant configuration is not available, neither in real time nor with delay, still poses many challenges. In this framework, most of the approaches proposed in the literature are based on the idea of estimating the current plant mode on the grounds of the plant state (Cheng et al., 2005; Caravani and De Santis, 2009) or output (Li et al., 2003). However, to the best of our knowledge, the problem of orchestrating the controller switching so as to ensure exponential stability for arbitrary initial conditions and arbitrary noise amplitudes is still an open issue. Further, even if in recent years extensive theoretical studies have been carried out on mode observability and mode estimation (see, for instance, Vidal et al., 2003; Babaali and Pappas, 2005; Alessandri et al., 2005; Baglietto et al., 2007; Alessandri et al., 2007; Di Benedetto et al., 2009; Baglietto et al., 2009, and the references therein), such results have yet to be fully exploited in the context of adaptive stabilization of switching linear systems (with the notable exception of (Caravani and De Santis, 2009)).

Motivated by this, a method is proposed to estimate the plant mode that naturally arises from mode observability considerations. Such a technique is based on the idea of evaluating the distance of the plant input/output data collected over a moving horizon from the subspaces associated to each possible mode. The minimum-distance mode estimator is then embedded in a supervisory unit that orchestrates the switching between the candidate controllers according to a *dwell-time switching logic* (DTSL) (Morse, 1995). Provided that all the candidate controllers are designed so as to satisfy a certain closed-loop mode observability condition, it is shown that the proposed minimum-distance criterion provides a reliable estimation of the current plant mode even in the presence of disturbances. Moreover, the exponential input-to-state of the resulting control system can be proved under the additional assumption that the plant switching signal is sufficiently slow on the average. The proofs are omitted due to space constraints.

Before concluding this section, let us introduce some notations and basic definitions. Given a vector $v \in \mathbb{R}^n$, $|v|$ denotes its Euclidean norm. Given a symmetric, positive semi-definite matrix P, we denote by $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ the minimum and maximum eigenvalues of P, respectively. Given a matrix M, $M^\top$ is its transpose and $\|M\| = \left[\lambda_{\max}(M^\top M)\right]^{1/2}$ its norm. Given a measurable time function $v : \mathbb{R}^+ \in \mathbb{R}^n$ and a time interval $\mathcal{I} \subseteq \mathbb{R}^+$, we denote the $\mathcal{L}_2$ and $\mathcal{L}_\infty$ norms of $v(\cdot)$ on $\mathcal{I}$ as $\|v\|_{2,\mathcal{I}} = \sqrt{\int_{\mathcal{I}} |v(t)|^2 dt}$ and $\|v\|_{\infty,\mathcal{I}} = \operatorname{ess\,sup}_{t \in \mathcal{I}} |v(t)|$ respectively. When $\mathcal{I} = R^+$, we simply write $\|v\|_2$ and $\|v\|_\infty$. Further,

$\mathcal{L}_2(\mathcal{I})$ and $\mathcal{L}_\infty(\mathcal{I})$ denote the sets of square integrable and, respectively, (essentialy) bounded time functions on $\mathcal{I}$.

2. MODE-OBSERVABILITY OF FEEDBACK LINEAR SWITCHING SYSTEMS

Consider a plant $\mathcal{P}_{\sigma(t)}$ described by a continuous-time switching linear system of the form

$$\mathcal{P}_{\sigma(t)} : \begin{cases} \dot{x}(t) = A_{\sigma(t)}\, x(t) + B_{\sigma(t)}\, u(t) \\ y(t) = C_{\sigma(t)}\, x(t) \end{cases} \tag{1}$$

where $t \in \mathbb{R}_+$ is the time instant, $x(t) \in \mathbb{R}^{n_x}$ is the plant state vector, $\sigma(t) \in \mathcal{N} \triangleq \{1, 2, \ldots, N\}$ is the plant mode, $u(t) \in \mathbb{R}^{n_u}$ is the control input, $y(t) \in \mathbb{R}^{n_y}$ is the vector of the measurements. A_i, B_i, and C_i, $i \in \mathcal{N}$, are constant matrices of appropriate dimensions. It is supposed that the unknown and unobserved switching signal $\sigma : \mathbb{R}^+ \to \mathcal{N}$ belongs to the class Σ of all the functions that are piecewise constant, right continuous, and admit no Zeno behavior (i.e., the number of switching instants is finite on every finite interval).

For the plant $\mathcal{P}_{\sigma(t)}$, we consider a linear switching controller $\mathcal{C}_{\hat{\sigma}(t)}$ of the form

$$\mathcal{C}_{\hat{\sigma}(t)} : \begin{cases} \dot{q}(t) = F_{\hat{\sigma}(t)}\, q(t) + G_{\hat{\sigma}(t)}\, y(t) \\ u(t) = H_{\hat{\sigma}(t)}\, q(t) + K_{\hat{\sigma}(t)}\, y(t) \end{cases} \tag{2}$$

where $q(t) \in \mathbb{R}^{n_q}$ is the controller state vector and $\hat{\sigma}(t) \in \mathcal{N}$ is the controller mode. F_i, G_i, H_i, and K_i, $i \in \mathcal{N}$, are constant matrices of appropriate dimensions. The switching signal $\hat{\sigma} : \mathbb{R}^+ \to \mathcal{N}$ is supposed to be known and belonging to Σ. Hereafter, for the sake of simplicity, both the plant $\mathcal{P}_i$ and the controller $\mathcal{C}_j$ will be understood to be controllable and observable for any fixed indices i and j, respectively.

In this section, attention will be devoted to the problem of inferring the plant mode $\sigma(t)$ from observation of the plant input/output data. To this end, it is convenient to consider the following state space realization for the closed loop system $(\mathcal{P}_{\sigma(t)}/\mathcal{C}_{\hat{\sigma}(t)})$ resulting from the feedback interconnection of (1) with (2)

$$(\mathcal{P}_{\sigma(t)}/\mathcal{C}_{\hat{\sigma}(t)}) : \begin{cases} \dot{w}(t) = A^{cl}_{\sigma(t)/\hat{\sigma}(t)}\, w(t) \\ z(t) = C^{cl}_{\sigma(t)/\hat{\sigma}(t)}\, w(t) \end{cases} \tag{3}$$

where

$$w(t) \triangleq \begin{bmatrix} x(t) \\ q(t) \end{bmatrix}, \quad z(t) \triangleq \begin{bmatrix} u(t) \\ y(t) \end{bmatrix},$$

$$A^{cl}_{i/j} \triangleq \begin{bmatrix} A_i + B_i K_j C_i & B_i H_j \\ G_j C_i & F_j \end{bmatrix}, \quad i, j \in \mathcal{N}$$

$$C^{cl}_{i/j} \triangleq \begin{bmatrix} K_j C_i & H_j \\ C_i & 0 \end{bmatrix}, \quad i, j \in \mathcal{N}.$$

Further, let us denote by

$$z_{i/j}(t, t_0, w_0) \triangleq C^{cl}_{i/j} e^{A^{cl}_{i/j}(t-t_0)} w_0$$

the output of (3) at time $t > t_0$ when the initial state at time t_0 is w_0, the controller switching signal is $\hat{\sigma}(\tau) = j$ for any $\tau \in [t_0, t]$, and the plant switching signal is $\sigma(\tau) = i$ for any $\tau \in [t_0, t]$. The following notion of *distinguishability* between two plant modes can now be introduced.

Definition 1. For system (3), two plant modes $i, i' \in \mathcal{N}$ with $i \neq i'$ are said to be *distinguishable* if

$$z_{i/j}(\cdot, t_0, w_0) \neq z_{i'/j}(\cdot, t_0, w_0') \quad \text{a.e. on } [t_0, t]$$

for any t_0, t with $t > t_0$, $j \in \mathcal{N}$, and $w_0, w_0' \in \mathbb{R}^{n_x+n_q}$ with $w_0 \neq 0$ or $w_0' \neq 0$. □

In words, two plant modes are distinguishable when, over every finite interval, they always lead to different input/output data z provided that the initial state is different from zero.

Definition 2. The feedback system (3) is said to be *mode-observable* if any two different plant modes $i, i' \in \mathcal{N}$ are distinguishable. □

Mode observability corresponds to the invertibility of the mapping from plant input/output data $z(\cdot)$ to the plant switching signal $\sigma(\cdot)$. In fact, it is an easy matter to see that, under mode observability, it is possible, at least in principles, to reconstruct the unknown switching signal $\sigma(\cdot)$ from observation of $z(\cdot)$, provided that the initial state $w(0)$ is not null. In what follows, necessary and sufficient conditions for mode observability of (3) will be derived.

To this end, some preliminary definitions are needed. Let $O^{(k)}_{i/j}$ be the observability matrix of order k of the feedback system $(\mathcal{P}_i/\mathcal{C}_j)$

$$O^{(k)}_{i/j} \triangleq \begin{bmatrix} C^{cl}_{i/j} \\ C^{cl}_{i/j} A^{cl}_{i/j} \\ \vdots \\ C^{cl}_{i/j} \left(A^{cl}_{i/j}\right)^{k-1} \end{bmatrix}.$$

Further, let

$$\Phi^{cl}_{i/j}(t) = C^{cl}_{i/j} e^{A^{cl}_{i/j} t}$$

be the state-to-ouput transition matrix of the feedback system $(\mathcal{P}_i/\mathcal{C}_j)$ and

$$W_{i/j}(t) \triangleq \int_0^t \left(\Phi^{cl}_{i/j}(\xi)\right)^\top \Phi^{cl}_{i/j}(\xi)\, d\xi$$

its observability Gramian. It is worth pointing out that, since the pairs (A_i, C_i) and (F_j, H_j) are observable by hypothesis, then also $(A^{cl}_{i/j}, C^{cl}_{i/j})$ turns out to be observable, i.e., the observability Gramian $W_{i/j}(t)$ is positive definite for any $t > 0$ and the observability matrix $O^{(k)}_{i/j}$ is full-rank for any $k \geq n_x + n_q$.

The next lemma unveils that the *joint observability matrix*

$$\left[O^{(2n_x+2n_q)}_{i/j} \quad O^{(2n_x+2n_q)}_{i'/j}\right]$$

plays a key role in determining the distinguishability of two plant modes i and i'.

Lemma 1. Two plant modes $i, i' \in \mathcal{N}$ with $i \neq i'$ are distinguishable if and only if their joint observability matrix is full-rank, i.e.,

$$\operatorname{rank}\left[O^{(2n_x+2n_q)}_{i/j} \quad O^{(2n_x+2n_q)}_{i'/j}\right] = 2n_x + 2n_q\,, \quad \forall j \in \mathcal{N}. \tag{4}$$

As a consequence, the feedback system (3) is mode observable if and only if condition (4) holds for any pair of different plant modes $i, i' \in \mathcal{N}$. □

The proof of Lemma 1 is omitted since it is just an adaptation of well-known results about observability of switching linear systems (Vidal et al., 2003; Babaali and Pappas, 2005).

While the previous Lemma 1 provides an answer on the binary question of whether or not the feedback system (3) is mode observable, a measure of the degree of mode observability of (3) can be obtained by analyzing the *joint observability Gramian*

$$W_{i,i'/j}(t) \triangleq \int_0^t \begin{bmatrix} \left(\Phi^{cl}_{i/j}(\xi)\right)^\top \\ \left(\Phi^{cl}_{i'/j}(\xi)\right)^\top \end{bmatrix} \times \begin{bmatrix} \Phi^{cl}_{i/j}(\xi) \ \Phi^{cl}_{i'/j}(\xi) \end{bmatrix} d\xi \,. \tag{5}$$

In fact, since

$$\begin{aligned} &\|z_{i/j}(\cdot,t_0,w_0) - z_{i'/j}(\cdot,t_0,w_0')\|^2_{2,[t_0,t]} \\ &= \begin{bmatrix} w_0^\top & -w_0'^\top \end{bmatrix} W_{i,i'/j}(t-t_0) \begin{bmatrix} w_0 \\ -w_0' \end{bmatrix} \\ &\geq \lambda_{\min}\{W_{i,i'/j}(t-t_0)\} \left(|w_0|^2 + |w_0'|^2\right) \,, \end{aligned}$$

it can be seen that the greater is $\lambda_{\min}\{W_{i,i'/j}(t-t_0)\}$ the more distinguishable are the two plant modes i and i' under controller $\mathcal{C}_j$. Then, a measure of the degree of mode observability of (3) over an interval of length $\tau = t - t_0$ is provided by

$$\omega_{\min}(\tau) = \min_{i,i',j\in\mathcal{N};\, i\neq i'} \lambda_{\min}\{W_{i,i'/j}(\tau)\} \,. \tag{6}$$

As will be clear in Section 4, such a *mode-observability index* plays a crucial role in the presence of disturbances.

Consider now a left polynomial matrix fraction descriptions (LPMFD) of the plant

$$\mathcal{P}_{\sigma(t)} : \ y(t) = P^{-1}_{\sigma(t)}(s)\, Q_{\sigma(t)}(s) u(t) \tag{7}$$

where, for each $i \in \mathcal{N}$, $P_i(s)$ and $Q_i(s)$ are left coprime polynomial matrices of appropriate dimensions with

$$\det P_i(s) = \det\left(sI - A_i\right) \,.$$

Here, equation (7) is intended as a shorthand notation to mean that over each interval of time where $\sigma(t) = i$ is constant, $y(t)$ is the output of a LTI system with transfer matrix $P_i^{-1}(s)\, Q_i(s)$ with the state at the beginning of this interval being initialized according to (1).

Consider also a LPMFD of the controller

$$\mathcal{C}_{\hat\sigma(t)} : \ u(t) = R^{-1}_{\hat\sigma(t)}(s)\, S_{\hat\sigma(t)}(s) y(t) \tag{8}$$

where, for each $j \in \mathcal{N}$, $R_j(s)$ and $S_j(s)$ are left coprime polynomial matrices of appropriate dimensions with

$$\det R_j(s) = \det\left(sI - F_j\right) \,.$$

Then, the following lemma can be stated that provides an alternative condition for studying mode observability of the feedback system (3).

Lemma 2. Two plant modes $i, i' \in \mathcal{N}$ with $i \neq i'$ are distinguishable if and only if, for any $j \in \mathcal{N}$,

$$\operatorname{rank} \begin{bmatrix} P_i(s) & -Q_i(s) \\ P_{i'}(s) & -Q_{i'}(s) \\ -S_j(s) & R_j(s) \end{bmatrix} = n_u + n_y \,, \quad \forall s \in \mathbb{C} \,. \tag{9}$$

□

Recalling that the characteristic polynomial $\varphi_{i/j}(s)$ of the the feedback system $(\mathcal{P}_i/\mathcal{C}_j)$ is

$$\varphi_{i/j}(s) = \det(sI - A^{cl}_{i/j}) = \det \begin{bmatrix} P_i(s) & -Q_i(s) \\ -S_j(s) & R_j(s) \end{bmatrix}$$

the following result can be readily established.

Proposition 1. Consider two plant modes $i, i' \in \mathcal{N}$ with $i \neq i'$. If, for any $j \in \mathcal{N}$, the closed loop characteristic polynomials $\varphi_{i/j}(s)$ and $\varphi_{i'/j}(s)$ are coprime, then i, i' are distinguishable. □

In general, Proposition 1 provides only a sufficient condition for distinguishability, however in the case of a single-input single-output (SISO) also necessity holds.

Proposition 2. Let the plant be SISO, i.e., $n_u = n_y = 1$. Then, two plant modes $i, i' \in \mathcal{N}$ with $i \neq i'$ are distinguishable if and only if, for any $j \in \mathcal{N}$, the closed loop characteristic polynomials $\varphi_{i/j}(s)$ and $\varphi_{i'/j}(s)$ are coprime. □

Propositions 1 and 2 suggest that mode-observability of the feedback system (3) can be guaranteed by designing each controller $\mathcal{C}_j$ so that, for any pair $i, i' \in \mathcal{N}$ with $i \neq i'$, the closed loop polynomials $\varphi_{i/j}(s)$ and $\varphi_{i'/j}(s)$ have no common roots.

3. CONSTRUCTING A STABILIZING CONTROLLER

In this section, it will be shown that stability of the feedback system (3) can be achieved by means of a suitable choice of the controller switching signal $\hat\sigma(t)$. To this end, it is supposed that the controllers $\mathcal{C}_i$, $i \in \mathcal{N}$ are designed so as to satisfy the following basic assumptions.

- A1. For each index $i \in \mathcal{N}$, the i-th tuned loop $(\mathcal{P}_i/\mathcal{C}_i)$ is asymptotically stable.
- A2. The feedback system (3) is mode-observable.

The choice of the control action to use, among all the available candidate controllers $\mathcal{C}_i$, $i \in \mathcal{N}$, is carried out in real-time by a a data-driven high-level unit called *mode estimator*. At each time $t \in \mathbb{R}_+$, the mode estimator generates an estimate $\hat\sigma(t, z(\cdot)) \in \mathcal{N}$ of the current plant mode on the basis of the plant input/output data $z(\cdot)$ up to the current time t. Such an estimate is then used as the controller switching signal, i.e.,

$$\hat\sigma(t) = \hat\sigma(t, z(\cdot)) \,.$$

As to the generation of the estimates, it is supposed that the mode estimator updates its estimate $\hat\sigma(t, z(\cdot))$ of the plant mode $\sigma(t)$ at discrete-time instants of the type kT where $k \in \mathbb{Z}_+$ and T, a positive real, is the so called *dwell time.* This amounts to assuming the controller switching signal $\hat\sigma(t)$ to be constant over each interval $\mathcal{I}_k \triangleq [kT, (k+1)T)$, i.e.,

$$\hat\sigma(t) = \hat\sigma_k \,, \quad \forall t \in \mathcal{I}_k \,.$$

In other words, the switching between controllers is orchestrated according to a DTSL.

3.1 A minimum-distance mode estimator

Thanks to the adoption of the DTSL, a simple criterion for the determination of the estimate $\hat\sigma_k$ can be devised. To this end, notice first that, whenever also the plant mode takes on a constant value, say i, over $\mathcal{I}_k$, the evolution of the plant input/output data on $\mathcal{I}_k$ can be written as

$$z(t) = z_{i/\hat{\sigma}_k}(t, kT, w(kT)) , \quad t \in \mathcal{I}_k .$$

Thus the set $\mathcal{S}_{i/\hat{\sigma}_k}(\mathcal{I}_k)$ of all the possible plant input/output data over $\mathcal{I}_k$ associated with a plant mode i and a controller mode $\hat{\sigma}_k$ corresponds to the linear subspace

$$\mathcal{S}_{i/\hat{\sigma}_k}(\mathcal{I}_k) \triangleq \Big\{ \hat{z} \in \mathcal{L}_2(\mathcal{I}_k) : \hat{z}(\cdot) = z_{i/\hat{\sigma}_k}(\cdot, kT, \hat{w}) \text{ on } \mathcal{I}_k , \\ \text{for some } \hat{w} \in \mathbb{R}^{n_x+n_q} \Big\} .$$

It is an easy matter to see that under mode observability the following result holds.

Proposition 3. Under assumption A2, for any two different plant modes $i, i' \in \mathcal{N}$ and any controller mode $\hat{\sigma}_k \in \mathcal{N}$, one has $\mathcal{S}_{i/\hat{\sigma}_k}(\mathcal{I}_k) \cap \mathcal{S}_{i'/\hat{\sigma}_k}(\mathcal{I}_k) = \{0\}$. □

In view of the above considerations, one can see that a particularly convenient approach for estimating the plant mode $\sigma(\cdot)$ on $\mathcal{I}_k$ consists in choosing the index i for which the distance between the observed plant input/output data $z(\cdot)$ on $\mathcal{I}_k$ and the linear subspace $\mathcal{S}_{i/\hat{\sigma}_k}(\mathcal{I}_k)$ is minimal. Then, at the generic time $(k+1)T$ the estimate $\hat{\sigma}_{k+1}$ can be obtained according to the minimum-distance criterion

$$\hat{\sigma}_{k+1} \in \arg\min_{i \in \mathcal{N}} \delta_{i/\hat{\sigma}_k}(z(\cdot), \mathcal{I}_k) \tag{10}$$

where

$$\delta_{i/j}(z(\cdot), \mathcal{I}_k) \triangleq \min_{\hat{w} \in \mathbb{R}^{n_x+n_q}} \left\| z(\cdot) - z_{i/j}(\cdot, kT, \hat{w}) \right\|_{2,\mathcal{I}_k} \tag{11}$$

Notice that, being the pair $(A^{cl}_{i/j}, C^{cl}_{i/j})$ completely observable by hypothesis, the minimization in (11) yields

$$\delta_{i/j}(z(\cdot), \mathcal{I}_k) = \Bigg(\int_{\mathcal{I}_k} \Bigg| z(t) - \Phi^{cl}_{i/j}(t - kT) \left(W_{i/j}(kT) \right)^{-1} \\ \times \int_{\mathcal{I}_k} \left(\Phi^{cl}_{i/j}(\xi - kT) \right)^{\top} z(\xi) \, d\xi \Bigg|^2 dt \Bigg)^{1/2} .$$

The next lemma points out an important property of the minimum-distance criterion (10).

Lemma 3. Suppose that assumption A2 holds, that $w(kT) \neq 0$ and the plant mode is constant on $\mathcal{I}_k$, i.e.,

$$\sigma(t) = \sigma_k , \quad \forall t \in \mathcal{I}_k .$$

Then, if the minimum distance criterion (10) is used, one has $\hat{\sigma}_{k+1} = \sigma_k$. □

Lemma 3 indicates that, under the stated assumptions, the proposed minimum distance criterion always leads to the exact reconstruction of the plant mode provided that no switch occurs in the observation interval. In other words, this implies that an identification error can be incurred only in those intervals characterized by at least one plant mode variation. As will be clarified in the next section, such a property ensures stability of the overall control scheme whenever the plant switching signal is sufficiently slow (on the average).

3.2 Stability under an average dwell-time

Thanks to Lemma 3, it is possible to show that the proposed control system with mode estimator yields an exponentially stable closed-loop system provided that the plant switching signal $\sigma(t)$ is slow on the average, i.e., the number of switches in any finite interval grows linearly with the length of the interval, with sufficiently small growth rate. In this respect, let $N_\sigma(t, t_0)$ be the number of discontinuities of σ in the interval (t_0, t), then the following assumption is needed (Hespanha and Morse, 1999; Hespanha, 2004).

A3. There exist a positive real τ_D, called *average dwell-time*, and a positive integer N_0, called *chatter bound*, such that
$$N_\sigma(t, t_0) \leq N_0 + \frac{t - t_0}{\tau_D}$$
for any $t, t_0 \in \mathbb{R}_+$ with $t > t_0$.

Note now that assumption A1 amounts to the existence of two positive reals μ and λ such that

$$\| e^{A^{cl}_{i/i} t} \| \leq \mu e^{-\lambda t} , \quad \forall t \in \mathbb{R}_+ , \forall i \in \mathcal{N} \tag{12}$$

where $\| \cdot \|$ denotes the matrix norm induced by the Euclidean vector norm $| \cdot |$. Further, since the set $\mathcal{N}$ is finite, one has

$$\| e^{A^{cl}_{i/j} t} \| \leq \theta e^{\rho t} , \quad \forall t \in \mathbb{R}_+ , \forall i, j \in \mathcal{N} \tag{13}$$

for some positive reals θ and ρ. The main stability result of this section can now be stated.

Theorem 1. Suppose that assumptions A1-A3 holds, that $w(0) \neq 0$ and let the minimum distance criterion (10) be used. Then, the state transition matrix $\Phi(t, t_0)$ of the closed-loop system $(\mathcal{P}_{\sigma(t)}/\mathcal{C}_{\hat{\sigma}(t)})$ can be upper bounded as

$$\| \Phi(t, t_0) \| \leq \beta \, e^{-\alpha \, (t - t_0)} \tag{14}$$

where

$$\alpha = \lambda - [\log \mu + 2 \log \theta + 2(\lambda + \rho) T] / \tau_D \\ \beta = \left(\mu \theta^2 e^{2(\lambda+\rho)T} \right)^{N_0+1} .$$

□

The following corollary follows at once.

Corollary 1. Suppose that assumptions A1-A3 holds and let the minimum distance criterion (10) be used. If the average dwell-time τ_D is such that

$$\tau_D > [\log \mu + 2 \log \theta + 2(\lambda + \rho) T] / \lambda , \tag{15}$$

then the closed-loop system $(\mathcal{P}_{\sigma(t)}/\mathcal{C}_{\hat{\sigma}(t)})$ is exponentially stable for any plant switching signal $\sigma(\cdot)$. □

Clearly, the right-hand side of (15) represents the minimum plant average-dwell time compatible with the stability of the closed-loop system. As it can be seen from (15), such a lower bound can be reduced by decreasing the switching logic dwell-time T or by making the tuned loops $(\mathcal{P}_i/\mathcal{C}_i)$, $i \in \mathcal{N}$ "more stable" by increasing the convergence rate λ.

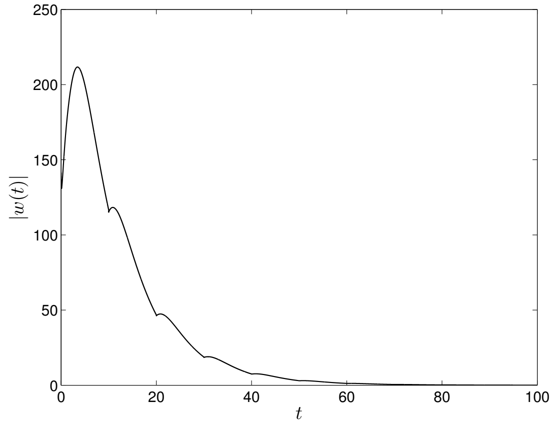

Fig. 1. Time behavior of $|w(t)|$.

3.3 An example

In order to illustrate the effectiveness of the proposed approach for adaptive stabilization of switching plants, let us consider the two-tank system of (Blanchini et al., 2009) which can be modelled as in (1) with $N = 2$ and

$$A_1 = A_2 = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix},$$
$$C_1 = [0 \quad 1], \quad C_2 = [1 \quad 0].$$

It is supposed that for each plant mode a stabilizing controller is available with transfer function $\mathfrak{C}_1(s) = (s + 0.1)/s$ and $\mathfrak{C}_2(s) = -\mathfrak{C}_1(s)$, respectively. The resulting closed-loop characteristic polynomials are

$$\varphi_{1/1}(s) = \varphi_{2/2}(s) = s^3 + 2\,s^2 + s + 0.1$$
$$\varphi_{1/2}(s) = \varphi_{2/1}(s) = s^3 + 2\,s^2 - s - 0.1\,.$$

Since $\varphi_{1/1}(s)$ and $\varphi_{1/2}(s)$ are coprime, mode-observability of the feedback system holds (see Proposition 2). Thus, provided that the plant variations are sufficiently slow-on-the-average, Corollary 1 ensures exponential stability when the controller switching is orchestrated according to the minimum distance criterion (10). This was also confirmed by means of numerical simulations. With this respect, Figure 1 shows the time behavior of the norm of the feedback system state $w(t)$ when the plant mode switches between 2 and 1 every 10 seconds (the dwell-time was set equal to 0.1 seconds; the controller and plant states were initialized to $q(0) = 0$ and $x(0) = [100 \quad 0]^\top$, respectively).

4. STABILITY UNDER PERSISTENT DISTURBANCES

In this section, the effects of persistent disturbances on the stability of the proposed control scheme are analyzed. To this end, suppose that the plant state and measurement equations be affected by additive disturbances $d(\cdot)$ and $n(\cdot)$, respectively, i.e.,

$$\mathcal{P}_{\sigma(t)} : \begin{cases} \dot{x}(t) = A_{\sigma(t)}\, x(t) + B_{\sigma(t)}\, u(t) + d(t) \\ y(t) = C_{\sigma(t)}\, x(t) + n(t) \end{cases} \tag{16}$$

with $d(t) \in \mathbb{R}^{n_x}$ and $n(t) \in \mathbb{R}^{n_y}$. Then, it is an easy matter to verify that a state space representation of the closed-loop system takes the form

$$(\mathcal{P}_{\sigma(t)}/\mathcal{C}_{\hat\sigma(t)}) : \begin{cases} \dot{w}(t) = A^{cl}_{\sigma(t)/\hat\sigma(t)}\, w(t) + B^{cl}_{\sigma(t)/\hat\sigma(t)}\, v(t) \\ z(t) = C^{cl}_{\sigma(t)/\hat\sigma(t)}\, w(t) + D^{cl}_{\sigma(t)/\hat\sigma(t)}\, v(t) \end{cases} \tag{17}$$

where $v(t) \triangleq [d(t)^\top \quad n(t)^\top]^\top$ and

$$B^{cl}_{i/j} \triangleq \begin{bmatrix} I & B_i\,K_j \\ 0 & G_j \end{bmatrix}, \quad D^{cl}_{i/j} \triangleq \begin{bmatrix} 0 & K_j \\ & \\ 0 & I \end{bmatrix}, \quad i, j \in \mathcal{N}\,.$$

The main complication arising in this case concerns the effects of the disturbances on the quality of the estimate computed via the minimum distance criterion (10). In fact, even supposing that the plant mode $\sigma(t)$ takes a constant value, say σ_k, on a certain interval $\mathcal{I}_k$, the presence of the disturbance $v(t)$ prevents one from applying Lemma 3 given that neither the plant input/output data $z(\cdot)$ need to belong to $\mathcal{S}_{\sigma_k/\hat\sigma_k}(\mathcal{I}_k)$ nor the distance $\delta_{i/\hat\sigma_k}(z(\cdot), \mathcal{I}_k)$ needs to take its minimum value for $i = \sigma_k$.

However, the mode observability property ensures that an estimate $\hat\sigma_{k+1}$ computed as in (10) becomes reliable provided that the input/output data $z(\cdot)$ contains a sufficient level of excitement. To see this, let us introduce the following definitions

$$\kappa_A \triangleq \max_{i,j\in\mathcal{N}} \|A^{cl}_{i/j}\|\,, \quad \kappa_B \triangleq \max_{i,j\in\mathcal{N}} \|B^{cl}_{i/j}\|\,,$$
$$\kappa_C \triangleq \max_{i,j\in\mathcal{N}} \|C^{cl}_{i/j}\|\,, \quad \kappa_D \triangleq \max_{i,j\in\mathcal{N}} \|D^{cl}_{i/j}\|\,.$$

Then, Lemma 3 can be replaced by the following.

Lemma 4. Suppose that assumption A2 holds and that the plant mode is constant on $\mathcal{I}_k$, i.e.,

$$\sigma(t) = \sigma_k\,, \quad \forall t \in \mathcal{I}_k\,. \tag{18}$$

Then, if the minimum distance criterion (10) is applied to the noisy feedback system (17), one has

$$\hat\sigma_{k+1} = \sigma_k$$

provided that

$$|w(kT)| \geq \frac{2\,\psi(T)\,\|v(\cdot)\|_{\infty,\mathcal{I}_k}}{\sqrt{\omega_{\min}(T)}} \tag{19}$$

where $\omega_{\min}(T)$ is the mode-observability index (6) and

$$\psi(T) \triangleq \sqrt{T}\left(\kappa_B\,\kappa_C\,\theta\,\frac{e^{\rho\,T}-1}{\rho} + \kappa_D\right). \tag{20}$$

□

Thus, under the stated assumptions and provided that the initial state at the beginning of the observation interval $\mathcal{I}_k$ is "far enough" from the origin, the minimum-distance criterion (10) leads to the exact identification of the plant mode even in the presence of disturbances. Further, it can be seen that condition (19) becomes less stringent the smaller are the disturbances and the greater is the mode-observability index. This state of affairs can be understood by recalling that under assumption (18), for any $t \in \mathcal{I}_k$, the input-output data $z(t)$ can be decomposed as $z(t) = z^{(\mathrm{n})}(t) + z^{(\mathrm{f})}(t)$ where $z^{(\mathrm{n})}(t)$ is the natural response

$$z^{(\mathrm{n})}(t) = \Phi^{cl}_{\sigma_k/\hat\sigma_k}(t - KT)\, w(kT)$$

and $z^{(\mathrm{f})}(t)$ is the forced response

$$z^{(\mathrm{f})}(t) = \int_{kT}^{t} \Phi^{cl}_{\sigma_k/\hat\sigma_k}(t-\tau) B^{cl}_{\sigma_k/\hat\sigma_k} v(\tau)\, d\tau + D^{cl}_{\sigma_k/\hat\sigma_k} v(t)\,.$$

As pointed out in the previous sections, when mode observability holds the plant mode can be reconstructed by observing the natural response (in fact, when there are no disturbances, $z(t) = z^{(\mathrm{n})}(t)$). Then the main idea behind Lemma 4 is that, when the forced response due to the disturbances becomes "negligible" with respect to the natural response (in the sense of condition (19)), everything goes as in the noise-free case and the plant mode can be uniquely determined. A formal proof of Lemma 4 is given in the Appendix.

An important consequence of Lemma 4 is that, when the disturbances are bounded in the $\mathcal{L}_\infty$ sense, their effect on the mode estimator disappears as soon as the system state exceeds a certain threshold. In view of this result, it is possible to show that the feedback control system (17) is exponentially input-to-state stable. More specifically, the following theorem can be stated.

Theorem 2. Suppose that assumptions A1-A3 holds and let the minimum distance criterion (10) be used. If the average dwell-time τ_D satisfies inequality (15), then the noisy closed-loop system (17) is exponentially input-to-state stable in that, for any $t_0, t \in \mathbb{R}^+$ with $t \geq t_0$ and for any plant switching signal $\sigma(\cdot)$,

$$|w(t)| \leq \beta\, e^{-\alpha\,(t-t_0)}\, |w(t_0)| + \gamma\, \|v(\cdot)\|_{\infty,[t_0,t]} \tag{21}$$

with α and β the same as in Theorem 1 and

$$\gamma = \frac{\beta}{\alpha}\left[\kappa_B + \frac{4\,\theta^2\,\psi(T)\,\kappa_A}{\sqrt{\omega_{\min}(T)}} e^{2\,\rho\,T} + \frac{2\,\theta^2\,\kappa_A\,\kappa_B}{\rho}(e^{2\,\rho\,T} - 1)\right].$$

□

It is worth pointing out that Theorem 2 provides a quite strong stability result in that inequality (21) holds regardless of the magnitude of the disturbances $v(\cdot)$ and of the initial state $w(t_0)$.

5. CONCLUSIONS

The problem of stabilizing a switching linear plant has been addressed under the assumption that the plant switching signal is not available, nor in real-time neither with delay. The proposed methodology is based on a supervisory unit that periodically switches the controller operating mode. The controller switching signal is generated by resorting to a minimum distance criterion, for the estimation of the plant mode, that naturally arises from mode observability considerations. It has been shown that, even in the presence of persistent disturbances, the proposed control scheme yields a stable feedback system provided that the plant switching signal is sufficiently slow on the average.

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REFERENCES

Alessandri, A., Baglietto, M., and Battistelli, G. (2005). Receding-horizon estimation for switching discrete-time linear systems. *IEEE Trans. on Automatic Control*, 50(11), 1736–1748.

Alessandri, A., Baglietto, M., and Battistelli, G. (2007). Luenberger observers for switching discrete-time linear systems. *International Journal of Control*, 80(12), 1931–1943.

Babaali, M. and Pappas, G.J. (2005). Observability of switched linear systems in continuous time. In M. Morari and L. Thiele (eds.), *Hybrid Systems: Computation and Control, Lecture Notes in Computer Science 3414*, 103–117. Springer-Verlag, Berlin Heidelberg.

Baglietto, M., Battistelli, G., and Scardovi, L. (2009). Active mode observation of switching systems based on set-valued estimation of the continuous state. *International Journal of Robust and Nonlinear Control*, 19(14), 1521–1540.

Baglietto, M., Battistelli, G., and Scardovi, L. (2007). Active mode observability of switching linear systems. *Automatica*, 43(8), 1442–1449.

Blanchini, F., Miani, S., and Mesquine, F. (2009). A separation principle for linear switching systems and parametrization of all stabilizing controllers. *IEEE Transactions on Automatic Control*, 54(2), 279–292.

Caravani, P. and De Santis, E. (2009). Observer-based stabilization of linear switching systems. *International Journal of Robust and Nonlinear Control*, 19(14), 1541–1563.

Cheng, D., Guo, L., Lin, Y., and Wang, Y. (2005). Stabilization of switched linear systems. *IEEE Transactions on Automatic Control*, 50(5), 661–666.

Di Benedetto, M.D., Di Gennaro, S., and D'innocenzo, A. (2009). Discrete state observability of hybrid systems. *International Journal of Robust and Nonlinear Control*, 19(14), 1564–1580.

D.Liberzon (2003). *Switching in Systems and Control*. Birkhäuser.

Hespanha, J. (2004). Uniform stability of switched linear systems: extensions of lasalle's invariance principle. *IEEE Transactions on Automatic Control*, 49(4), 470–482.

Hespanha, J. and Morse, A. (1999). Stability of switched systems with average dwell-time. In *Proceedings of the 38th IEEE Conference on Decision and Control*, volume 3, 2655–2660.

Lee, J.W. and Dullerud, G.E. (2006). Uniform stabilization of discrete-time switched and markovian jump linear systems. *Automatica*, 42(2), 205–218.

Li, Z.G., Wen, C.Y., and Soh, Y.C. (2003). Observer-based stabilization of switching linear systems. *Automatica*, 39(3), 517–524.

Morse, A.S. (1995). Control using logic-based switching. In A. Isidori (ed.), *Trends in Control: An European perspective*, 69–113. London: Springer.

Vidal, R., Chiuso, A., Soatto, S., and Sastry, S. (2003). Observability of linear hybrid systems. In F. Wiedijk, O. Maler, and A. Pnueli (eds.), *Hybrid Systems: Computation and Control, Lecture Notes in Computer Science 2623*, 523–539. Springer.

Xie, D. and Wu, Y. (2009). Stabilisability of switched linear systems with time-varying delay in the detection of switching signals. *IET Control Theory Applications*, 3(4), 404–410.