

# Optimal Sensor Placement for Multiple Underwater Target Localization with Acoustic Range Measurements

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## Abstract:

Worldwide, there is increasing interest in the operation of multiple autonomous underwater vehicles (AUVs) to carry out scientific and commercial missions at sea. For some of the envisioned missions, it is crucial that new methods be developed to localize one more of the vehicles simultaneously, based on acoustic range information received on-board a set of autonomous surface vehicles. As a contribution towards meeting this goal, this paper addresses the problem of computing the optimal geometric configuration of a mobile surface sensor network that will maximize the range-related information available for multiple target localization in three-dimensional space. In contrast to what has so far been published in the literature, we address explicitly the localization problem in 3D using a sensor array located at the sea surface (2D). Furthermore, we incorporate directly in the problem formulation the fact that multiple targets must be localized simultaneously. Clearly, there will be tradeoffs involved in the precision with which each of the targets can be localized; to study them, we resort to techniques that borrow from Pareto optimization and estimation theory. Simulation examples illustrate the key results derived.

*Keywords:* Autonomous Underwater Vehicles, Sensor networks, Acoustic Positioning, Multiobjective Optimization, Fisher Information Matrix, Cramer-Rao Bound

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## 1. INTRODUCTION

The last decade has witnessed tremendous progress in the development of marine technologies that are steadily affording scientists advanced equipment and methods for ocean exploration and exploitation. Recent advances in marine robotics, sensors, computers, communications, and information systems are being applied to develop sophisticated technologies that will lead to safer, faster, and far more efficient ways of exploring the ocean frontier, especially in hazardous conditions. As part of this trend, there has been a surge of interest worldwide in the development of autonomous underwater vehicles (AUVs) capable of roaming the oceans freely, collecting relevant data at an unprecedented scale. In fact, for reasons that have to do with autonomy, flexibility, and the new trend in miniaturization, AUVs are steadily emerging as tools par excellence to replace ROVs and also humans in the execution of many demanding tasks at sea. Furthermore, their use in collaborative tasks allows for the realization of complex missions, often with relatively simple systems; see Ghabcheloo (2009).

Many AUV mission scenarios call for the availability of good underwater positioning systems to localize one or more vehicles simultaneously based on acoustic-related range information received on-board a support ship or an autonomous surface system (e.g., a number of autonomous

surface vehicles equipped with acoustic receivers, moving in formation). The info thus obtained can be used to follow the state of progress of a particular mission or, if reliable acoustic modems are available, to relay it as a navigation aid to the navigation systems existent on-board the AUVs. Similar comments apply to a future envisioned generation of positioning systems to aid in the tracking of one or more human divers. For an early reference on related work, the reader may wish to consult Zhang (1995), where a method to determine the optimal two dimensional spatial placement of multiple sensors participating in a robot perception task was introduced. One of the scenarios considered was that of localizing an underwater vehicle.

In this paper we address the problem of computing the optimal geometric configuration of a mobile surface sensor network that will maximize the range-related information available for multiple target localization in three-dimensional space. In contrast to what has so far been published in the literature, we address explicitly the localization problem in 3D using a sensor array located at the sea surface (2D). Furthermore, we incorporate directly in the problem formulation the fact that multiple targets must be localized simultaneously. We assume that the range measurements are corrupted by white Gaussian noise, the variance of which is distance-dependent. The computation of the target positions may be done by resorting to trilateration algorithms. See for example Alcocer

(2007), Alcocer (2009), Bar-Shalom et al. (2001), and the references therein for an introduction to this circle of ideas, covering both theoretical and practical aspects.

Given a single target localization problem, the optimal sensor configuration can be ascertained by examining the corresponding Cramer-Rao Bound (CRB) or Fisher Information Matrix (FIM). See Van Trees (2001) for a lucid presentation of this subject in the context of estimation theory. In the present paper, the FIM is derived considering a distance-dependent variance of the noise model. For this purpose, we use the determinant of the FIM as an indicator of the localization performance that is achievable with a given sensor configuration. Maximizing this quantity yields the most appropriate sensor formation geometry. The single target localization problem was addressed by the authors previously in Moreno et al. (2010), where the optimal geometry of a surface sensor network was determined to maximize the range-related information available for underwater target positioning. This work was in turn greatly inspired by that reported in Martinez & Bullo (2006) on optimal ranging sensor placement to improve the accuracy in the localization of ground robots.

Previous results on single target positioning can be traced back to the work of Abel (1990), where the Cramer-Rao bound is used as an indicator of the accuracy of source position estimation and a simple geometric interpretation of that bound is offered. In the same reference, the authors describe a solution to the problem of finding the sensor arrangements that minimize that bound, subject to geometric constraints. In Levanon (2000), the problem of localization in two-dimensional (2-D) scenarios is examined. The author shows explicitly what the lowest possible geometric dilution of precision (GDOP) attainable from range or pseudo-range measurements to  $N$  optimally located points is and determines the corresponding regular polygon-like sensor configuration. Aranda et al. (2005) study optimal sensor placement and motion coordination strategies for mobile sensor networks. For a target tracking application with range sensors, they investigate the determinant of the FIM with constant covariance error, and compute it in the 2D and 3D cases. They further characterize the global minimal of the 2D case. In Jourdan & Roy (2008), an iterative algorithm that places a number of sensors so as to minimize the position error bound is developed, yielding configurations for the optimal formation subject to several complex constraints. Bishop et al. (2007a) and Bishop et al. (2007b) characterize the relative sensor-target geometry for bearing-only localization and for Time-of-Arrival and Time-difference-of-Arrival in  $\mathbb{R}^2$ . In these two papers the optimal expression for the Fisher Information Matrix is defined for a constant variance error model. Finally, in Isaacs et al. (2009), the authors address the problem of localizing a source from noisy time-of-arrival measurements by seeking an extreme of the FIM for truncated, radially-symmetric source distributions.

In the present work, inspired by developments in ground and marine robotics for the problem of single target positioning, we tackle the *multiple target positioning problem*. Clearly, there will be tradeoffs involved in the precision with which each of the targets can be localized; to study them, we resort to techniques that borrow from estimation theory and Pareto optimization. For the latter, the reader

is referred to Khargonekar & Rotea (1991), Da Cunha & Polak (1991), Vincent & Grantham (1991). See also the Appendix for a very short review of some key concepts and results. Stated briefly, we avail ourselves of concepts on Pareto optimality and maximize convex combinations of the logarithms of the determinants of the FIMs for each of the targets in order to compute the Pareto optimal surface that gives a clear image of the tradeoffs involved in the multiobjective optimization problem. We thus obtain a powerful tool to determine the sensor configuration that yields, if possible, a proper tradeoff for the accuracy with which the position of the different targets can be computed. In what follows, and with an obvious abuse of notation, we often refer to Pareto optimal solutions simply as optimal.

It is important to remark that for the multiobjective optimization problem at hand, the logarithms of the determinants of the FIMs will be used instead of the determinants themselves. This makes the functions to be maximized jointly convex in the search parameter space, thus justifying the use of scalarization techniques in the computation of the Pareto-optimal surface, as described in the Appendix. For a discussion of the convexity of the functions adopted, see for example Boyd & Vandenberghe (2004), Chapter 3 and the work in Ucinisky (2005) on the D-optimality criterion. These facts will be exploited in Section 4.

For a multi-target localization problem, the optimal geometry of the sensor configuration depends strongly on the constraints imposed by the task itself (e.g. maximum number and type of sensors that can be used), the environment (e.g. ambient noise), the number of targets and their configuration, and the possibly different degrees of precision with which their positions should be estimated. An inadequate sensor configuration may yield large localization errors for some of the targets. It is interesting to remark that even though the problem of optimal sensor placement for range based localization is of great importance, not many results are available on this topic yet. Even more, the results are only for single target positioning, as far as the authors know.

The key contribution of the present paper is twofold: i) we fully exploit concepts and techniques from estimation theory and multiobjective optimization to obtain a numerical solution to the optimal sensor configuration problem for multiple underwater targets, and ii) in striking contrast to what is customary in the literature, where zero mean Gaussian processes with fixed variances are assumed for the range measurements, the variances are now allowed to depend explicitly on the ranges themselves. This allows us to capture the fact that measurement noise increases in a nonlinear manner with the distances measured.

The document is organized as follows. Section 2 derives the FIMs that are necessary to solve the optimal sensor placement problem under consideration and computes their determinants in the case where the measurement noise is Gaussian, with distance-dependent variance. The gradient optimization algorithm that is used to compute the optimal sensor configuration is summarized in Section 3; in preparation for the section that follows, the examples included refer to a multiple target position problem where

all the targets are weighted equally (in the Pareto optimal sense). In Section 4, the multiple localization problem is studied for the case where the targets have different importance weights along the mission, conditioning the sensor formation. Finally, in Section 5 the maximization of the average value of the logarithms of the FIM determinants is studied when a static fixed sensor network surveys a certain working area or when there is uncertainty in the *a priori* knowledge about the target positions. The conclusions and topics for further research are included in Section 6.

## 2. INFORMATION INEQUALITY WITH DISTANCE-DEPENDENT MEASUREMENT NOISE

Underwater range measurements between two objects are plagued with errors that depend on a multitude of effects: depth-dependent speed of propagation of sound in the water, physical propagation barriers, ambient noise, and degrading signal-to-noise ratio as the distance between the two objects increases, to name but a few. For analytical tractability, it is commonly assumed that the measurement errors can be captured by Gaussian, zero mean, additive noise with constant covariance. See for example Zhang (1995), where different noise covariances are taken for different sensors, but the covariances are constant. Clearly, this assumption is artificial in view of the simple fact that the "level of noise" is distance dependent. In this paper, in an attempt to better capture physical reality, we assume that the measurement noise can be modelled by a zero-mean Gaussian process where the covariance depends on the distance between the two objects that exchange range data. A similar error model is considered in Jourdan & Roy (2008) for its iterative algorithm. Stated mathematically,

$$w = (I + \eta\delta(r^\beta)) \cdot w_0 \quad (1)$$

where  $\omega$  is measurement noise,  $\omega_0$  is a zero mean Gaussian process  $N(0, \Sigma_0)$  with  $\Sigma_0 = \sigma^2 \cdot I$ ,  $I$  is the identity matrix,  $r$  is range, and  $\eta$  and  $\beta$  are the modelling parameters for the distance-dependent noise component. In the above,  $\delta$  is the operator *diag*, that either converts a square matrix into a vector consisting of its diagonal elements, or converts a vector into a square diagonal matrix whose diagonal components are the array elements. With these assumptions, the measurement noise covariance is given by

$$\Sigma = \sigma^2 (I + \eta\delta(r^\beta))^2 \quad (2)$$

Let  $q = [q_x, q_y, q_z]^T$  be the position of an arbitrary target,  $p_s = [p_{s,x}, p_{s,y}, p_{s,z}]$ ;  $s = 1, 2, \dots, n$ , the position of the  $s$ -th acoustic ranging sensor, and  $w_s$  the corresponding measurement noise. Further let  $r_s$  be the distance between target  $q$  and the  $s$ -th sensor. With this notation, the measurement model adopted is given by

$$\hat{r}_s(q) = \|(p_s - q)\| + w_s = r_s + w_s \quad (3)$$

In what follows, we assume that the reader is familiar with the concepts of Cramer-Rao Lower Bound (CRLB) and Fisher Information Matrix (FIM); see for example Van Trees (2001). Stated in simple terms, the FIM captures the amount of information that measured data provide about an unknown parameter (or vector of parameters) to be estimated. Under known assumptions, the FIM is the inverse of the the Cramer-Rao Bound matrix (abbr. CRB),

which lower bounds the covariance of the estimation error that can possibly be obtained with any unbiased estimator. Thus, "minimizing the CRB" may yield (by proper estimator selection) a decrease of uncertainty in the parameter estimation. We therefore focus on the computation of the FIM for the problem at hand. In particular, we examine the determinant of the FIM and seek to determine the optimal acoustic receiver configuration as the one that maximizes it. Following standard procedures, the FIM for an arbitrary target  $q$  is computed from the likelihood function as

$$p_q = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} \exp \left\{ -\frac{1}{2} (y - h(q))^T \Sigma^{-1} (y - h(q)) \right\} \quad (4)$$

where  $n$  is the number of receivers,  $y = [r_1, r_2, \dots, r_n]^T$  consists of  $n$  actual ranges, and  $h(q) = r(q)$  are the measured range. Taking the logarithm of (4), computing its derivative with respect to  $q$ , and making  $\Sigma = B^2$  yields

$$\nabla_q \log p_q = C\delta(r)^{-1} B^2 (r - r(q)) + \eta\beta\sigma C\delta(r)^{-1} \delta(r - r(q)) B^3 \delta(r^{\beta-1}) (r - r(q)) \quad (5)$$

where  $C = (q1_n^T - p) \in \mathbb{R}^{3 \times n}$ ,  $1_n \in \mathbb{R}^{n \times 1}$  is a vector of 1s, and  $p$  is the vector of sensor positions, the latter being defined in  $\mathbb{R}^{3 \times n}$ . The FIM is defined as the expected value of (5) as follows:

$$FIM = E \{ \nabla_q \log p_q \nabla_q \log p_q^T \} = C(\delta(r)\Sigma\delta(r))^{-1} C^T + 3\eta^2\beta^2\sigma^2 C\delta(r)^{-1} \delta(r^{\beta-1}) \Sigma^{-1} \delta(r^{\beta-1}) \delta(r)^{-1} C^T \quad (6)$$

Once the FIM is computed,  $CRB = FIM^{-1}$ . In our case, the FIM is constructed by allowing the measurement error to be distance-dependent. In this context, the optimal sensor placement strategy for a given multiple vehicle localization problem is obtained by maximizing a quantity related to the determinants of the FIM matrices for the different vehicles involved. It is therefore imperative that the FIMs be computed explicitly. To this effect, we start by expanding (6) to obtain

$$FIM = \frac{1}{\sigma^2} \sum_{s=1}^n \begin{pmatrix} (u_{sx})^2 & (u_{sy})(u_{sx}) & (u_{sz})(u_{sx}) \\ (u_{sx})(u_{sy}) & (u_{sy})^2 & (u_{sz})(u_{sy}) \\ (u_{sx})(u_{sz}) & (u_{sy})(u_{sz}) & (u_{sz})^2 \end{pmatrix} \quad (7)$$

where  $u_s = \left( \frac{\partial \|q - p_s\|}{\partial q_x}, \frac{\partial \|q - p_s\|}{\partial q_y}, \frac{\partial \|q - p_s\|}{\partial q_z} \right) \cdot \Gamma_i$ , for  $s \in \{1, \dots, n\}$  and  $\Gamma_s = \frac{\sqrt{1+3\eta^2\beta^2\sigma^2 r_s^{2(\beta-1)}}}{(1+\eta r_s^\beta)}$ . Clearly, the expression of the FIM considering a distance-dependence covariance error is well defined.

## 3. GRADIENT OPTIMIZATION ALGORITHM FOR SENSOR PLACEMENT

As explained, the determinants of the FIMs for each of the targets are used in the computation of an indicator of the performance that is achievable with a given sensor configuration. Maximizing this indicator (which, as a consequence of the Pareto optimality conditions described in the Appendix, is a convex combination of the logarithms of the determinants of the different FIMs) yields the most appropriate sensor formation geometry for the multiple

underwater target positioning problem. The sensors are constrained to lie in the plane  $z = 0$ , at the sea surface. One simple method to find the optimal formation is the gradient optimization method. To use it, we compute the derivative of the logarithm of the FIM determinant of each target with respect to all sensor coordinates (in 2D). For the sake of simplicity, using the notation in Aranda et al. (2005), the determinant of (7) for each target  $b$  can be written as

$$|FIM|_b = \frac{1}{\sigma^2} \sum_{i \leq j \leq k} \frac{((v_i \times v_j) \cdot v_k)^2 \cdot \Gamma_i^2 \cdot \Gamma_j^2 \cdot \Gamma_k^2}{r_i^2 \cdot r_j^2 \cdot r_k^2} \quad (8)$$

with  $v_s = u_s/\Gamma_s$ , and  $b = 1, \dots, m$ , where  $m$  is the number of targets. Taking the derivatives of the logarithm of (8) with respect to the  $x$  and  $y$  coordinates of sensor  $s$  yields

$$\frac{\partial \log |FIM|_b}{\partial p_{s,x}} = \frac{2q_z^2}{\sigma^2} \sum_{i < j < k} \frac{\Gamma_r^2 \Gamma_t^2 ((v_i \times v_j) \cdot v_k)}{|FIM|_b \cdot r_r^2 \cdot r_t^2} \cdot \left( \frac{\Gamma_s^2}{r_s^2} B_x + ((v_i \times v_j) \cdot v_k) \cdot 2(q_x - p_{s,x}) \cdot \Phi \right) \quad (9)$$

$$\frac{\partial \log |FIM|_b}{\partial p_{s,y}} = \frac{2q_z^2}{\sigma^2} \sum_{i < j < k} \frac{\Gamma_r^2 \Gamma_t^2 ((v_i \times v_j) \cdot v_k)}{|FIM|_b \cdot r_r^2 \cdot r_t^2} \cdot \left( \frac{\Gamma_s^2}{r_s^2} B_y + ((v_i \times v_j) \cdot v_k) \cdot 2(q_y - p_{s,y}) \cdot \Phi \right) \quad (10)$$

where  $\Phi = \frac{(-3\eta^2\sigma^2(\beta^3 - \beta^2)r_s^{2(\beta-2)} + \Gamma_s^2(1 + \eta r_s^\beta)(1 + \eta(\beta+1)r_s^\beta))}{r_s^2(1 + \eta r_s^\beta)^2}$ ;  $s = 1, \dots, n$ ;  $s, r, t = i, j$ , or  $k$ , with  $r \neq s \neq t$ ; and  $B_x = p_{r,y} - p_{t,y}$  and  $B_y = p_{t,x} - p_{r,x}$  with  $r < t$ .

Once the gradients have been computed for each target, they are combined to update the sensor configuration so as to yield an increase in the specified convex combination of the logarithms of the FIM determinants. This computation is recursive, until the optimal position is found. For the single target positioning problem, an adequate initial guess for the solution is for example any regular distribution around the target. Checking that this algorithm behaves well for single target positioning is easy, for an analytical solution to the optimal sensor positions is available in Moreno et al. (2010). For the multiple target localization problem, the initial guess will be a regular distribution around the mass centre of the target group, with all the targets inside the sensor formation. The Armijo rule is used for the sensor placement update phase, yielding the following iterative gradient optimization algorithm.

- (1) For each target, (8) is computed for the current sensor formation at iteration  $k$ , from which  $|FIM|_\lambda$ , the convex combination of the logarithms of the determinants, given by  $|FIM|_\lambda[k] = \sum_{b=1}^m \lambda_b \log |FIM|_b[k]$ , follows for a specific choice of  $\lambda_b$ ;  $b = 1, 2, \dots, m$ ,  $\lambda_1 + \dots + \lambda_m = 1$
- (2) Using (9) and (10) the gradient of  $|FIM|_\lambda[k]$  is computed, yielding  $\nabla_{s,c} |FIM|_\lambda[k]$  with  $c = x, y$  and  $s = 1, \dots, n$ .
- (3) The sensor positions are updated according to the gradients:  $p_{s,c}[k+1] = p_{s,c}[k] + \mu \gamma^{[k]} \nabla_{s,c} |FIM|_\lambda[k]$ , with  $\mu \in (0, 1)$ ,  $\gamma_0 = 1$ , and  $\gamma[k] = \gamma[k-1] + 1$ .
- (4) If  $|FIM|_\lambda[k+1] > |FIM|_\lambda[k]$ , then  $p_s[k+1]$  becomes the new set of sensor positions,  $\gamma[k] = \gamma[k] + 1$ ,

and the iteration goes back to step 1, with  $p_s[k+1] = [p_{s,x}[k+1], p_{s,y}[k+1]]$ .

- (5) If  $|FIM|_\lambda[k+1] < |FIM|_\lambda[k]$ , then there is no improvement in the convex combination of the determinants,  $\gamma[k] = 0$ , the iterative algorithm stops, and  $p_s[k]$  is considered to be the optimal configuration for the current target position.

The above cycle is only run once if the targets are stationary. Notice the unrealistic assumption, also made in many of the publications available in this area, that the positions of the targets are known in advance. This is done to simplify the problem and to first fully understand its solution before the realistic scenario where the positions of the targets are known with error can be tackled. In this respect, see Section 5, which is largely inspired by the work in Isaacs et al. (2009).

In what follows, as explained above, we will deal with convex combinations of the logarithms of the determinants of the FIMs for each of the targets, denoted  $|FIM|_\lambda$ . Clearly, in order for the information about the Pareto-optimal configurations to be useful, one must check if the determinants of the individual FIMs for each target meet desired specifications. To this effect, and for comparison purposes, the determinant of the FIMs obtained for a number of hypothetical target points (based on a fixed optimal sensor configuration corresponding to a well-defined multi-target scenario) will at times be computed by allowing these points to be on a grid in a finite spatial region  $\mathcal{D}$ . This will allow us to evaluate how good the sensor formation is in terms of yielding accurate localization of the real targets, in comparison with the performance localization accuracy that is possible for any hypothetical target (different from the real targets) positioned anywhere in  $\mathcal{D}$ . For the sake of clarity, and with an obvious abuse of notation, we will refer to that determinant, viewed as a function of its argument in  $\mathcal{D}$ , simply as  $|FIM|_{\mathcal{D}}$ . In this paper,  $\mathcal{D}$  will always be a rectangle in  $R^2$ .

In a practical situation where the targets are in motion, the surface sensor network must adapt its optimal configuration as the mission unfolds. Clearly, this requires that three different, intertwined processes be activated as follows:

- i) *multiple target position estimation*, albeit with a possibly large error, using the current sensor configuration and resorting to a dedicated nonlinear filter (e.g. Extended Kalman filter);
- ii) *optimal sensor configuration computation*, based on the data provided by the previous process and the algorithm described above or its modification in Section 5;
- iii) *coordinated motion control* to actually drive the moving sensors to the optimal positions determined in ii).

We thus envision the situation where the algorithm described (or its modification in Section 5) is run during each cycle of the positioning system in i). Interestingly enough, we can also think of a situation where the different iterates of process ii) can be used to yield set points for the autonomous sensor network to move to, effectively guiding them collectively to the optimal configuration that is being computed.

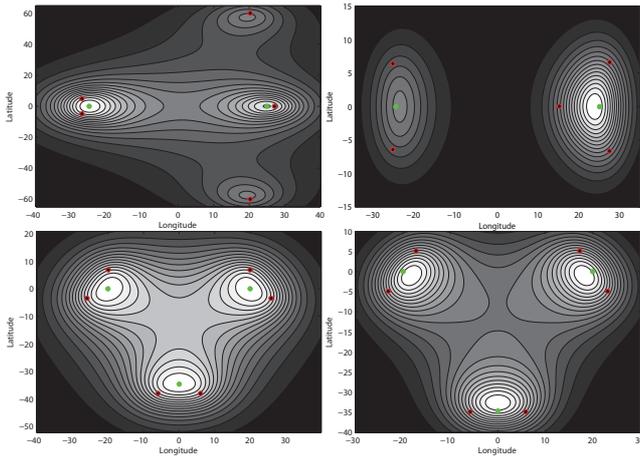


Fig. 1. Plot of  $|FIM|_{\mathcal{D}}$ ,  $\mathcal{D} \in \mathbb{R}^2$ , with sensors in red and targets in green. In a) and c) (left, up and down):  $\eta = 0$ ; in b) and d) (right, up and down):  $\eta = 0.2$ . Lighter regions correspond to larger values of  $|FIM|_{\mathcal{D}}$ .

The advantage of using a gradient optimization method is its simplicity. As will be seen later, based on the simulations done so far, the method has proven to be quite satisfactory. However, should there be a need for a more refined method, the sensor network positions given by the gradient algorithm can be used as initial estimates in the new method. The rest of this section contains the results of simulations that illustrate the potential of the method developed for optimal sensor placement when multiple targets are involved. As an introductory step, only the case where the targets have equal Pareto weights, that is,  $\lambda_1 = \lambda_2 = \dots = \lambda_m$  is considered. The case where the targets have different weights is addressed in the next section.

#### Examples of optimal sensor placement for multiple target positioning

The initial guess for the gradient optimization algorithm is of very importance to avoid local minima or divergence. Experience has shown that a regular formation around the centre of mass of the group of targets, keeping the targets inside the formation, is an appropriate initial guess. Starting from this initial guess, several examples of multi-target positioning are shown in Fig. 1, considering both constant and distance-dependent error covariances, with  $\beta = 1$ .

In Fig. 1(a),  $|FIM|_{\mathcal{D}}$ ;  $\mathcal{D} \in \mathbb{R}^2$  is mapped for a 5 sensor network, 2 targets, and  $\eta = 0$  (constant covariance error). The two maximum values of the function are over the target positions, close to the optimal values obtained for a single target positioning problem, and the sensors are spread around the centre of mass of the targets. In Fig. 1(b) the same example is shown but with the covariance measurement error depending on distance ( $\eta = 0.2$ ). In this case, the position where the maximum of  $|FIM|_{\mathcal{D}}$  occurs is strongly affected by the distance-dependent added error. For the choice of identical weights adopted, the most adequate configuration with 5 sensors and 2 targets is such that the accuracy with which one of the targets can be located is larger than the other because 3 sensors are

closer to it. Should such a solution prove unsuitable, a complete analysis of the tradeoffs involved using the set-up explained in the next section with different target weights would be required.

In Fig. 1(c),  $|FIM|_{\mathcal{D}}$ ;  $\mathcal{D} \in \mathbb{R}^2$  is mapped for a 6 sensor formation and 3 targets, with  $\eta = 0$ . Again, the maximum values of the function are close to the targets and the sensors spread themselves in an organized manner around the targets. In Fig. 1(d) an identical example is shown for the case where  $\eta = 0.2$ . The solution is very similar to the one with  $\eta = 0$ , except that in this case the sensors are closer to the targets to reduce the impact of the distance-dependent measurement noise.

#### 4. TARGETS WITH DIFFERENT PARETO WEIGHTS

In many situations of interest, a number of human divers or AUVs may be required to work scattered over a certain area, executing different tasks or cooperating towards the execution of a common task. It is not hard to envision situations where different "levels of importance" and therefore different localization accuracies are required for the elements in a group of underwater targets. In the case of human divers, for example, in a 2 diver scenario one of the divers may be executing a very demanding and risky task, while the other is carrying out an easy, routine task. In this situation, the surface sensor network should "focus its attention" on the first target, effectively imposing strict requirements on the accuracy with which its position must be estimated, while relaxing the level of localization accuracy required for the second target. This situation may be inverted during the mission, so the formation should be able to reconfigure itself accordingly.

It is obvious that the geometry of the sensor network will impact on the accuracy with which the position of each of the targets can be computed. In the case of multiple targets, improving the accuracy in the estimate of one target may at times be done only in detriment of the accuracy of the other estimates. There are therefore tradeoffs that must be examined carefully. An example of a multi-target localization problem can be briefly described as follows: "given  $m$  targets and  $n$  sensors, determine if possible a geometric configuration for the sensors that will maximize the accuracy with which the position of target  $i$  can be estimated, while keeping the accuracy of the other target estimates above a desired threshold level". It is at this stage that the power of multi-objective Pareto optimization must be brought into the picture. Clearly, in order to fully understand the problem when must compute the corresponding set of Pareto-optimal points and make decisions accordingly. See the presentation in the Appendix. As explained before, this can be done by computing

$$\max |FIM_{\lambda}| \quad (11)$$

over all possible sensor positions, and for all  $\lambda = [\lambda_1, \lambda_2, \dots, \lambda_m]$  such that  $\lambda_1 + \dots + \lambda_m = 1$ . In practice a grid of points is adopted for vector  $\lambda$ . The maximization above is done by resorting to the gradient optimization algorithm introduced before.

For simplicity of explanation, a 2-target positioning problem with 6 sensors is studied although the procedure would

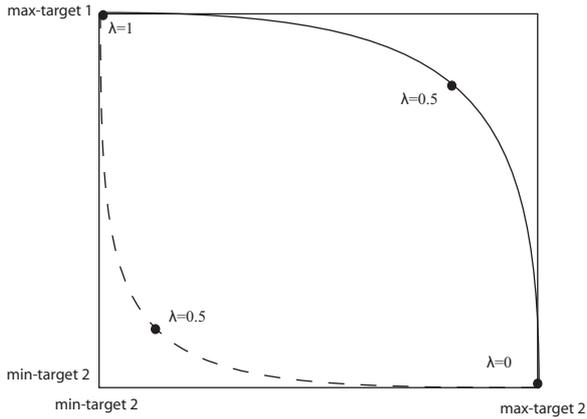


Fig. 2. Pareto curve (solid line) for a 2 target localization problem, using 6 sensors for  $\eta = 0.2$ , and the corresponding FIM determinants (dotted line) for different values of the Pareto scalarization weights in  $\lambda$ .

be the same for more targets and a different number of sensors. Because only two targets are involved, the Pareto optimal curve is parameterized by a single parameter  $\lambda \in [0, 1]$ . For simplicity of notation, we use the same symbol for this scalar as well as vector  $\lambda$ . The meaning will be clear from the context. We assume that  $\lambda_1 = \lambda$  and  $\lambda_2 = 1 - \lambda$ . When  $\lambda$  varies from 0 to 1, the weight on one of the targets changes accordingly. Thus, in the extreme cases of 0 and 1 the solutions degenerate into two those of the single target localization problems for target 2 and 1, respectively. Two normalized curves that show the tradeoffs in the determinants of the Fisher information matrices for each of the targets (with the sensor geometry obtained by running the gradient optimization algorithm) are plotted in Fig. 2. The solid line corresponds to the Pareto curve for the maximization of  $|FIM_\lambda|$ , whereas the dotted line shows the corresponding evolution of the FIM determinants. The two curves are normalized between 0 and 1.

Notice in Fig. 2 how the cost function  $|FIM_\lambda|$  provides a concave Pareto curve (solid line), as expected for a maximization problem. As explained before, this is a consequence of the fact that in this case the criterion for each target is indeed concave. The dotted line shows the corresponding evolution of the FIM determinants. Notice that the curve is not concave, thus supporting the statement that the determinants of the FIMs are not adequate criteria to be maximized jointly (in the Pareto-optimal sense).

Fig. 2 shows how the accuracy of the measurements changes for different values of  $\lambda$ . At this point it is important to remark that if the measurement error does not depend on the distance between targets and sensors, that is,  $\eta = 0$ , it is possible to obtain sensor locations for which the accuracies obtained for each of the targets simultaneously are close to the optimal ones that would be obtained if the targets were operating in isolation. This follows from the shape of the Pareto curve when  $\eta = 0$ , not shown here. For example, with  $\lambda = 0.5$  the performance achievable in the localization of targets 1 and 2 simultaneously does not degrade substantially when compared to the best performance achievable for the two targets isolated. Of course the level of degradation

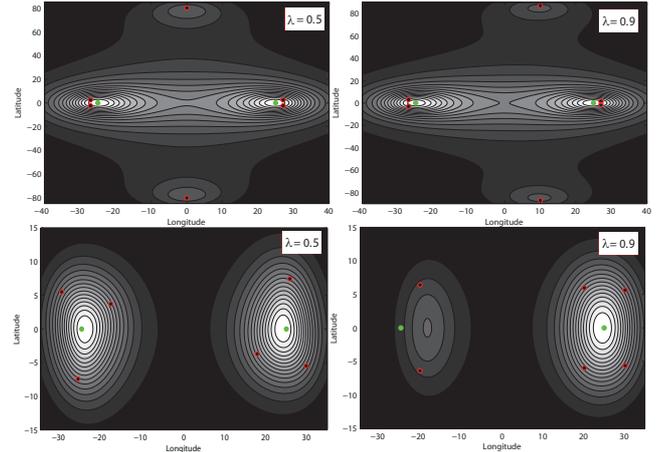


Fig. 3. Plot of  $|FIM|_{\mathcal{D}}$ ,  $\mathcal{D} \in R^2$ . Optimal sensor placement with identical and different weights  $\lambda_b$  for the targets, for constant, a) and b) (up, left and right), and distance-dependent error covariance c) and d) (down, left and right).

in performance that is acceptable is problem dependent. When the measurement error is distance-dependent, the situation changes drastically because of the "steepness" of the Pareto curve. For example, when  $\lambda = 0.5$  the performance that can be simultaneously achieved for both targets degrades substantially. The tradeoffs involved are clear.

Fig. 3 shows some examples for the 2 target positioning problem. Notice how the performance achievable and the sensor geometric configuration formation change for different values of  $\lambda$ . In Fig. 3 (a) and (b) the solution for two different values of  $\lambda$  with  $\eta = 0$  and 6 sensors are shown. In these two examples, the FIM determinants of both targets are very similar and close to the optimal ones for each target. In Fig. 3 (c) and (d) the solutions for the same cases but with  $\eta = 0.2$  are shown. The difference is obvious respect to the two previous examples. In the latter, the formations end up being more compact to combat the effects of the distance-dependent covariance error and because the error grows fast when the optimal formations for single target positioning are left.

## 5. OPTIMAL SENSOR PLACEMENT WITH UNCERTAIN TARGET LOCATION

In the previous sections, the optimal sensor configuration was determined by assuming that the positions of the targets are known *a priori*. We now tackle the situation where the target locations are uncertain. Inspired by the work in Isaacs et al. (2009), we assume the uncertainty in the target locations is described by a given probability distribution function and seek to maximize a convex combination of the logarithms of the determinants of the FIMs for each target, denoted  $|FIM_\lambda|$ . Stated mathematically,

$$\max_D \int |FIM_\lambda| \cdot \varphi(q) dq \quad (12)$$

where  $D$  is the region where the targets are known to be present and  $\varphi(q)$  is a probability density function for the target positions inside the working area. In a real situation,  $\varphi(q)$  will depend on the type of mission

carried out by the targets. If the targets operate mostly in the center of the working area, the function can be for example a truncated, radially-symmetric probabilistic Gaussian distribution centered at the origin. On the other hand, if only the working area is known and the targets can operate anywhere inside it,  $\varphi(q)$  can be taken as the unity function. In the most general set-up, the region  $D$  must be taken as the union of a number of disjoint regions  $D_b; b = 1, 2, \dots, m$ , where  $D_b$  is the work area of target  $b$ .

Conceptually, the procedure to determine the optimal sensor configuration is similar to that explained in the previous sections, that is, one must compute the derivatives of (12) with respect to the sensor coordinates and solve for

$$\frac{\partial}{\partial p_{ij}} \int_D |FIM_\lambda| \phi(q) dq = 0 \quad (13)$$

with  $j = x, y, i = 1, \dots, n$ , where  $n$  denotes the number of sensors and  $i, x$  and  $i, y$  are the  $x$  and  $y$  coordinates, respectively of sensor  $i$ .

To proceed with the computations, the integral and derivative operations are interchanged: the derivatives are determined explicitly first and the integration over region  $D$  is performed afterwards. The derivative can be computed in a recursive way using (9) and (10) for any number of targets. In what regards the computation of the triple integral over the region  $D$  of interest, however, this is virtually impossible to do analytically. For this reason, the integral is computed numerically. Finally, the solution of (12) is obtained using the gradient optimization method detailed in Section 3. Again, to overcome the possible occurrence of local maxima or the divergence of the algorithm, the initial guess in the iterative algorithm must be chosen with care. In the examples that we studied we found it useful and expedite to adopt as an initial guess the solution for the single target positioning problem described in Moreno et al. (2010), with the hypothetical single target placed at the center of mass of the work area. It is important to stress that the solution to (12) depends strongly on the probability density function adopted for the target position  $q$  (e.g., a truncated, radially-symmetric probabilistic Gaussian distribution or a radially-symmetric step distribution, Isaacs et al. (2009)).

As an example, in Fig. 4  $|FIM|_D$  is mapped for two different cases with  $\eta = 0.3$ . Figure 4(a) corresponds to the optimal sensor configuration for single target positioning, with the target placed at the origin. Figure 4(b) corresponds to the solution when only the working area is known and the formation maximizes the average determinant in a square working area of  $10 \times 10$  meters. It is easy to check from these figures how the optimal formation in the second scenario has a larger size, although the regular distribution around the origin is preserved. Similar examples can be given for the multiple target case; this will not be done here due to space limitations.

## 6. CONCLUSIONS AND FUTURE WORK

We studied the problem of determining the optimal configuration of a sensor network that will, in a well defined sense, maximize the range-related information available for multiple underwater target positioning. To this effect, we assumed that the range measurements were corrupted by

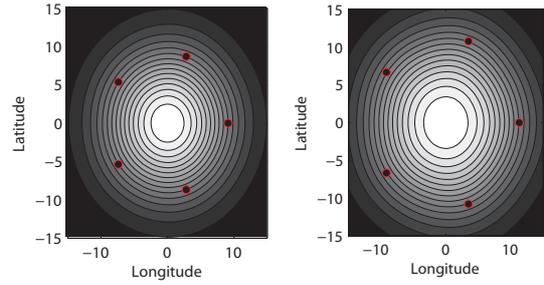


Fig. 4. Plot of  $|FIM|_D$  for single target positioning. In a) (left): benchmark - target position known without uncertainty; b) (right): position of the target is unknown, anywhere in a work area of  $10 \times 10$  meters.

white Gaussian noise with distance-dependent covariance. In contrast to what has so far been published in the literature, we addressed explicitly the localization problem in 3D using a sensor array located at the sea surface (2D). Furthermore, we incorporate directly in the problem formulation the fact that multiple targets must be localized simultaneously. At the core of the techniques used are key concepts and methods from Pareto optimization and estimation theory. From a mathematical standpoint, the key problem that we solved was that of maximizing, by proper choice of the sensor geometric configuration, convex combinations of the logarithms of the determinants of the Fisher Information Matrices corresponding to estimation problems for each target separately. This was done by resorting to an iterative optimization algorithm. The methodology developed allows for an in depth study of the tradeoffs that are inherent to a multiple target localization problem. Simulation examples show clearly how the optimal sensor location depends on the size of the area in which the targets operate, the type of measurement noise, and the "level of importance" attached to each of the targets; the latter aims to capture the fact that tradeoffs are inevitable, and therefore different levels of accuracy may be required in the localization of the different targets.

Future work will aim at: i) extending and applying the methodology developed to a real multiple vehicle mission scenario and ii) studying the performance of the algorithms for optimal sensor configuration computation developed here, together with selected algorithms for target tracking and cooperative sensor motion control.

## 7. APPENDIX

For the sake of completeness, this Appendix contains a very brief introduction to some key concepts and results on multiobjective optimization and Pareto-optimality. The exposition is largely based on the summary in Khargonekar & Rotea (1991).

Because we are interested in the problem of multi-target positioning, we are naturally led to adopt a multiobjective optimization strategy. We adopt the concept of Pareto optimality introduced below. Let  $\mathcal{X}$  be an arbitrary nonempty set and let  $f_i : \mathcal{X} \rightarrow \mathcal{R}_+ : i = 1, 2, \dots, n$  be  $n$  nonnegative functionals defined on  $\mathcal{X}$ . A point  $x^0 \in \mathcal{X}$  is said to be Pareto optimal with respect to the vector-valued criterion  $f := (f_1, f_2, \dots, f_n)$  if there does not exist  $x \in \mathcal{X}$  such that

$$f_i(x) \leq f_i(x^0) \text{ for all } i = 1, 2, \dots, n$$

and

$$f_k(x) < f_k(x^0) \text{ for some } k \in 1, 2, \dots, n$$

From the above it follows that if one wishes to find points  $x \in \mathcal{X}$  such that, in some sense,  $x$  jointly minimizes all the components of  $f$ , then one must examine the Pareto optimal points. It is interesting to point that in the literature on economics a Pareto optimal outcome is one such that *no person could be made better off without having someone else worse off*.

When Pareto optimal solutions do exist, in general they are not unique. The determination of the Pareto optimal set for a given multiobjective problem plays a key role in that it allows for a thorough study of the tradeoffs involved in the problem at hand. The next scalarization result in Da Cunha & Polak (1991) is of crucial importance in characterizing this set.

*Scalarization result.* Suppose that  $\mathcal{X}$  is a normed linear space and that each component of  $f := (f_1, f_2, \dots, f_n)$  is a convex function on  $\mathcal{X}$ . Let

$$\Lambda := \{\lambda \in \mathcal{R}^n : \lambda_i \geq 0, \lambda_1 + \lambda_2 + \dots + \lambda_n = 1\}$$

and for each  $\lambda \in \Lambda$  consider the following scalar-valued optimization problem:

$$\inf\{\lambda^T f(x) : x^0 \in \mathcal{X}\} \quad (14)$$

Suppose that  $x^0 \in \mathcal{X}$  is Pareto optimal with respect to the vector-valued criterion  $f := (f_1, f_2, \dots, f_n)$ . Then, there exists  $\lambda \in \Lambda$  such that  $x_0$  is a solution to the scalar optimization problem above. Conversely, given  $\lambda \in \Lambda$  if the scalar optimization problem has at most one solution  $x^0 \in \mathcal{X}$ , then  $x^0$  is Pareto optimal with respect to  $f(x)$ .

The above result yields a powerful methodology to compute all Pareto optimal points. In this paper, the scalar functions  $f_i$  are related to the logarithms of the determinants of the Fisher Information Matrices corresponding to each of the targets being localized (notice that we wish to maximize the determinants jointly, rather than minimize them; in this case, however, an obvious modification of the result above applies).

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