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Time Domain Tuning of a Fractional Order PI$^{\alpha}$ Controller Combined with a Smith Predictor for Automation of Water Distribution in Irrigation Main Channel Pools

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Abstract: This paper proposes a new technique for designing fractional order controllers applied to the automation of main canal pools of a water irrigation systems. As large variation of all the plant parameters is present in such systems, a fractional order PI^{α} controller combined with a Smith Predictor is designed using time domain specifications. The designed controller is compared with the standard Smith Predictor combined with *PI*. All these controllers are tuned to fulfil the same time specifications in the case of canal nominal dynamics. In some canals the plant parameters may experience large changes that diminish the performance, and even can unstabilize the closed-loop system. Then the PI^{α} controller is designed to maximize the robustness to variations in these parameters. Simulated results show the robustness improvements achieved with this controller compared with a conventional *PI* controller[1].

Keywords: Fractional order control, irrigation main canal pool control, robust control, time domain dynamics, frequency specifications.

1. INTRODUCTION

Water distribution is a worldwide problem that requires efficient use of water. A considerable amount of water in irrigation main canals is wasted because of the lack of an effective control. Nevertheless, automatic control is a powerful tool for improving the efficiency of water distribution in irrigation systems. Therefore, the introduction of automatic control in irrigation main canals has been increasingly considered in recent years (Clemmens, 2006; Litrico et al., 2006).

Designing a control strategy that leads to an efficient controller is difficult, since irrigation canals are complex systems which are distributed over long distances, with significant time delays and dynamics that change with the different regimes of the channel (Malaterre et al., 1998; Rivas-Perez et al., 2003).

The case of study presented in this paper is about the first pool of the *Aragon Imperial Main Canal* (AIMC) that exhibits large variations in its characteristic parameters.

On the other hand, in recent years, fractional operators have been applied with satisfactory results to model and control processes with complex dynamic behaviour, most of which are distributed parameter processes (Machado, 1997; Podlubny, 1999; Petras, 2002; Chen et al., 2004; Oustaloup et al., 2006). Fractional calculus is the field of mathematics that involves differentiation and integration of a non-integer order. This is a generalization of the standard concepts of differentiation and integration (Oldham et al., 1974; Podlubny, 1999).

Due to the time delay term that irrigation main open canals models presents, a Smith predictor (Smith, 1957) based control scheme was presented in (Deltour *et al.*, 1998) as a suitable technique. This work proposes a Smith predictor scheme combined with a fractional PI^{α} controller is proposed.

A method, previously developed in (Castillo *et al*. 2010), that guarantees the fulfilment of the time specifications in the case of the standard closed-loop control for first order plants, has been applied to tune the PI^{α} controllers combined with a Smith predictor.

Once the nominal time behaviour is ensured, independently of the value of α, (non integer order of the integral action of the PI^{α} controller), this additional design parameter is used to

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fulfil the strong robustness requirements of the AIMC first pool.

This paper is organized as follows. Section 2 introduces the main irrigation channel pool. Section 3 presents the proposed control scheme and the controller. In this section the time domain and frequency domain tunings are also developed. Section 4 compares the robustness of both controllers, *PI* and PI^{α}. Finally, Section 5 resumes the main conclusions obtained during the development of this work.

2. IRRIGATION MAIN CHANNEL POOL MODEL

The study presented in this paper is about the first pool of the *Aragon Imperial Main Canal*. A dynamic model of this pool has to be obtained in order to design its control laws.

Data and results reported here have been obtained from this first pool, known as the *Bocal*, which is part of the *Ebro Hydrographical Confederation* (Spain). This canal water supply comes from the Ebro River thanks to the elevation produced by the *Pignatelli* dam. The *Bocal* main characteristic parameters are resumed in Table 1.

Table 1. Bocal characteristic parameters

pool length	8 km
cross section	trapezoidal
depth	3.1-3.7m
width	15.0-26.9 m
design discharge	30m^3/s

This canal pool is operated by using the distant downstream operation method (Malaterre et al., 1998). The downstream end water level is controlled by means of a undershoot gates set located in the 'Gates House' at the beginning of the pool. Figure 1 shows an upper view of this pool, including gates house.

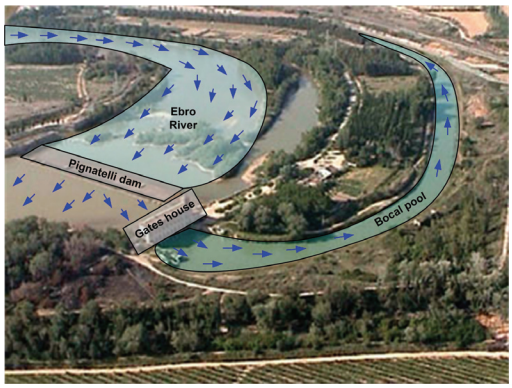

Fig. 1. Upper view of *Bocal* pool.

Figure 2 represents an equivalent scheme of the *Bocal* pool, where $y_i(t)$ is the water level measured by the sensor system, $u_i(t)$ is the gates position, $Q_i(t)$ the discharge regime of each pool and $q_i(t)$ the unknown offtake discharge.

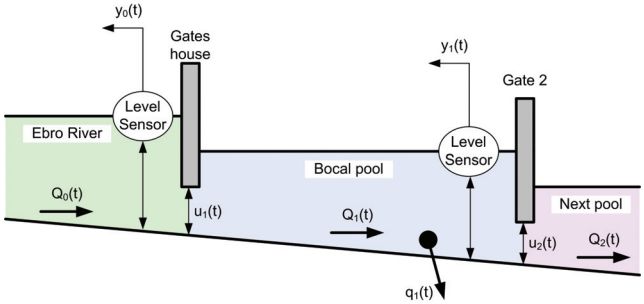

Fig. 2. Equivalent scheme of *Bocal* pool.

Based on the response to a step-like input signal experiments have been carried out in order to obtain a mathematical model which describes the dynamic behaviour of this irrigation main canal pool.

The resulting experiments output that the irrigation main canal pool dynamics behaviour can be represented by a second order system with a time delay:

$$G(s) = \frac{Y_1(s)}{U_1(s)} = \frac{K}{(T_1 s + 1)(T_2 s + 1)} e^{-Ls}, \qquad (1)$$

where K, T_1, T_2 and L are the static gain, time constants and time delay, respectively.

Table 2 resumes the identified parameters of the plant (1) under nominal discharge regime of the pool.

Table 2. Nominal parameters of $G(s)$

K_0	0.04010
T_{10}	880.8
T_{20}	81.27
L_0	600

On the other hand, a change in the pool discharge regime means a wide variation of dynamical parameters K, T_1, T_2 and L in the following ranges:

$$\begin{array}{rcccl} 0.0125 & \leq & K(t) & \leq & 0.125 \\ 500 & \leq & T_1(t) & \leq & 15000 \\ 10 & \leq & T_2(t) & \leq & 3000 \\ 200 & \leq & L(t) & \leq & 600 \end{array}. \qquad (2)$$

3. CONTROL SCHEME

3.1 Control objectives

Firstly, the nominal time response provided by the controller must fulfil the following restrictive goals in terms of the settling time in the ±5% band, t_s, overshot, M_p and steady state error, e_{ss}:

$$\begin{array}{l} t_s \approx 1500\ s \\ M_p \approx 5\% \\ e_{ss} = 0 \end{array}. \qquad (3)$$

Secondly, the controller must ensure a stable controlled system along the parameters variation ranges (2).

3.2 Smith predictor

One of the main advantages of a Smith predictor based control scheme (Smith, 1957) is the suppression of the time delay term of the closed loop transfer function denominator. Previous works (Feliu et al., 2009) demonstrated that the robustness to parameters variations can be improved using this control strategy in comparison to the conventional unity feedback one.

Expressing (1) as:

$$G(s) = G'(s)e^{-Ls}, \tag{4}$$

Figure 3 represents the diagram block of the Smith predictor, supposing that $G_e'(s)$ and L_e correspond to the estimated plant. Notice that that input to the process (gate opening) is $u(t)$.

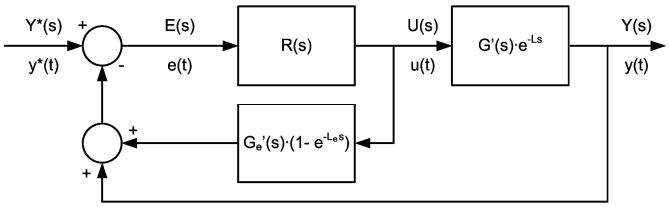

Fig. 3. Smith predictor based control scheme.

Under the assumption of a predictor perfect tuning, $G_e'(s)=G(s)$ and $L_e=L_0$, the closed loop transfer function between the reference, $Y^*(s)$ and the output, $Y(s)$ yields as:

$$M(s) = \frac{Y(s)}{Y^*(s)} = \frac{R(s)G'(s)}{1 + R(s)G'(s)}e^{-Ls}. \tag{5}$$

Thus, the scheme of Smith predictor shown in Fig. 3 can be represented as a unity feedback scheme using $G'(s)$, plus a time delay term (Fig. 4).

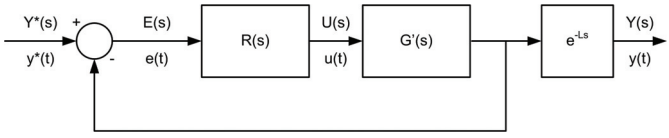

Fig. 4. Smith predictor equivalent block diagram (perfect tuning).

3.3 Controllers

Into the Smith predictor scheme, a conventional *PI* controller and a fractional PI^{α} controller are compared, expressed as follow:

$$R^i(s) = K_p^{\ i} + \frac{K_i^{\ i}}{s} \tag{6}$$

and

$$R^f(s) = K_p^{\ f} + \frac{K_i^{\ f}}{s^{\alpha}}, \tag{7}$$

where K_p and K_i are the proportional and integral gains of the controllers and α the non-integer order of the integral action in the case of the fractional controller. Expression (6) is a particular case of (7) when α=1. Hereinafter, the superindex $_^i$ and $_^f$ mean the *i*nteger and *f*ractional case.

3.4 Frequency Domain Tuning

Due to the integral action of both controllers if the controlled system is stable, a zero steady state error is ensured. In the case of *PI* controller, a frequency tuning can be proposed by means of the gain crossover frequency, ω_c, and the phase margin, ϕ_m, in order to control the settling time and the overshot of the time response. Under the assumption of a predictor perfect tuning, the complex equation that allows obtaining the controller parameters from ω_c and ϕ_m can be expressed as:

$$R(j\omega_c)G'(j\omega_c) = e^{-j(\pi-\phi_m)}. \tag{8}$$

Equation (8) provides the controller parameters that guarantee the fulfilment of the frequency specifications as function of the nominal parameters of the plant, K_0, T_{10} and T_{20}.

The tuning equations for the *PI* controller are:

$$K_p^{\ i} = \frac{\omega_c^{\ 2}T_{10}T_{20} - 1}{K_0}\cos\phi_m + \frac{\omega_c\left(T_{10} + T_{20}\right)}{K_0}\sin\phi_m \tag{9}$$

and

$$K_i^{\ i} = \frac{\omega_c}{K_0}\left[\left(1 - \omega_c^{\ 2}T_{10}T_{20}\right)\sin\phi_m + \omega_c\left(T_{10} + T_{20}\right)\cos\phi_m\right] \tag{10}$$

The fractional *PI* controller parameters also depend of α and yield as:

$$K_p^{\ f}(\alpha) = \frac{\omega_c^{\ 2}T_{10}T_{20} - 1}{K_0}\left(\cos\phi_m + \frac{\sin\phi_m}{\tan\left(\frac{\pi}{2}\alpha\right)}\right) + \frac{\omega_c\left(T_{10} + T_{20}\right)}{K_0}\left(\sin\phi_m - \frac{\cos\phi_m}{\tan\left(\frac{\pi}{2}\alpha\right)}\right) \tag{11}$$

and

$$K_i^{\ f}(\alpha) = \frac{\omega_c^{\ \alpha}}{K_0\sin\left(\frac{\pi}{2}\alpha\right)}\left[\left(1 - \omega_c^{\ 2}T_{10}T_{20}\right)\sin\phi_m + \omega_c\left(T_{10} + T_{20}\right)\cos\phi_m\right], \tag{12}$$

so we can use α to increase the robustness performance of the PI^{α} controller in comparison to the *PI* controller.

In order to obtain the time requirements (3) the following frequency specifications have been set:

$$\omega_c = 1.717 \cdot 10^{-3}\ rad/s \text{ and } \phi_m = 70.5^{\circ} \quad (13)$$

The resulting *PI* controller is:

$$R^i(s) = 32.2569 + \frac{0.05544}{s} \quad (14)$$

Figure 5 shows the time response that (13) provides using a sample time h=60 s.

Note that the *PI* controller fulfils perfectly the desired time specifications (3), therefore the assumption of $G_e'(s)=G'(s)$ and $L_e=L_0$ allows to use (8) as tuning equation.

Figure 6 represents the time responses that a PI^{α} controller combined with Smith predictor provides for different values of α, $\alpha\in[0.8,1.2]$. The non-integer integral action has been implemented using the Grünwald-Letnikov approximation and a sample time, h=60 s.

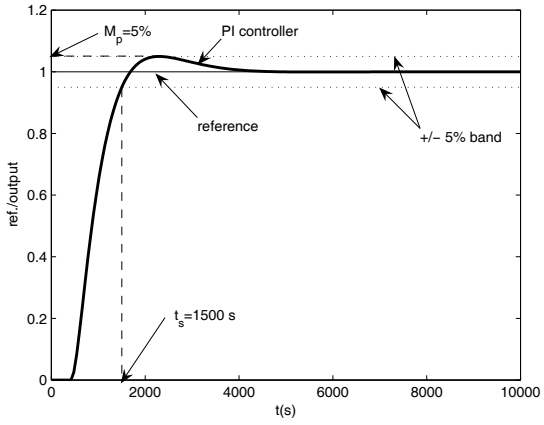

Fig. 5. *PI* combined with Smith predictor time response.

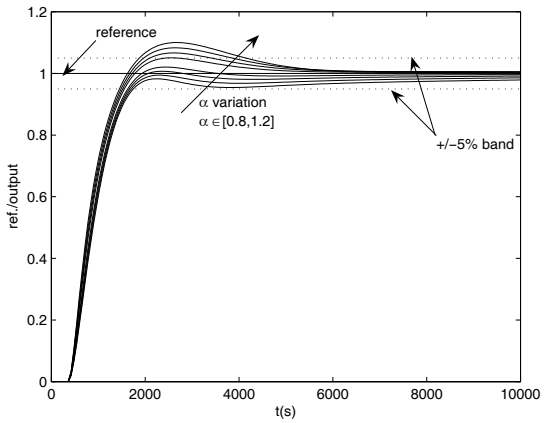

Fig. 6. PI^{α} combined with Smith predictor time response ($\alpha\in[0.8, 1.2]$).

Figure 7 represents the dependence of settling time and overshot in function of α value.

The results shows that the PI^{α} time response strongly depends of the value of α though all these controllers achieve the same two frequency specifications (ω_c and ϕ_m).

In our case of study, only de *PI* controller fulfils the time domain specifications (3).

3.5 Time Domain Tuning

In a previous work (Castillo et al, 2010) a new time domain tuning method was proposed. This method consists on modifying the original frequency specifications for the PI^{α} in function of the value of α. Figure 8 shows the schematic process of this new time domain tuning method and the conventional frequency method.

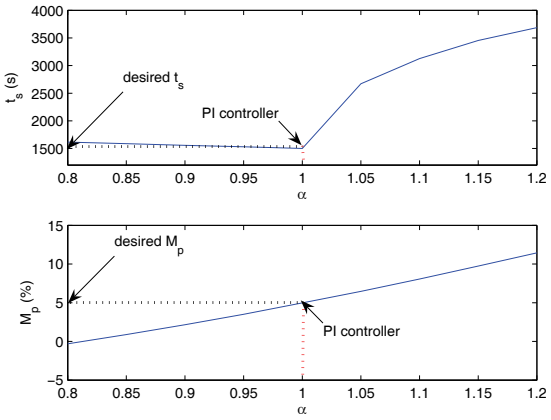

Fig. 7. Dependence on α of the time response characteristic parameters t_s and M_p (frequency domain tuning).

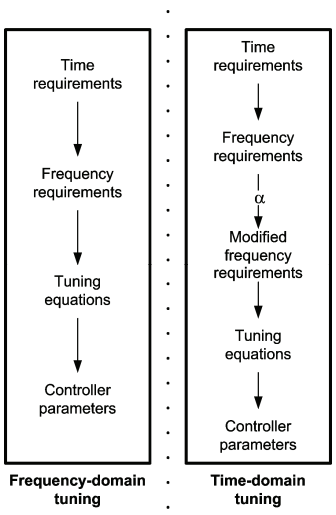

Fig. 8. Frequency-domain tuning method vs. new time-domain tuning method.

The method presented at (Castillo et al, 2010) consists on selecting a new couple of frequency specifications, ω_c* and ϕ_m*, which guarantees the fulfilment of the time response requirements, t_s y M_p. The selection process is carried out by means of the minimization of a time dependant functional.

The particular frequency modification law obtained in order to fulfil (3) in this case is:

$$\begin{aligned} \omega_c^* &= 0.02328\alpha^3 - 0.06429\alpha^2 + 0.05879\alpha - 0.01618 \\ \phi_m^* &= 2.63268\alpha^3 - 6.09549\alpha^2 + 5.15484\alpha - 0.465373 \end{aligned}, \quad (15)$$

$\forall\alpha\in[0.8, 1.2]$. Out of this range no solution that fulfils the time domain specification has been found.

Figure 9 represents the time response that the PI^α controller combined with Smith predictor provides for different values of α, $\alpha\in[0.8,1.2]$, using (15).

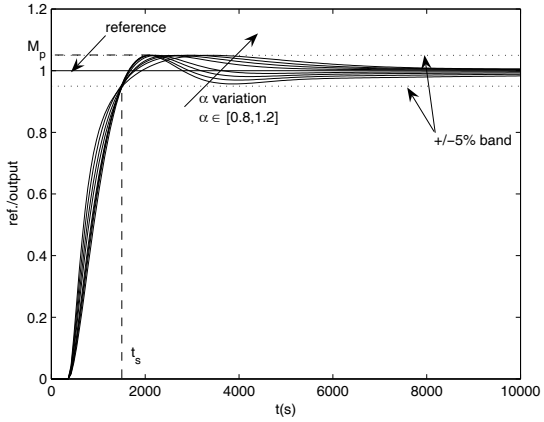

Fig. 9. PI^α combined with Smith predictor time response by time domain tuning ($\alpha\in[0.8, 1.2]$).

Figure 10 represents the dependence of settling time and overshot in function of α value.

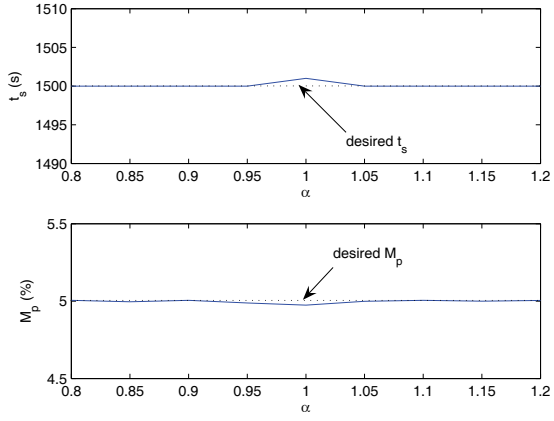

Fig. 10. Dependence on α of the time response characteristic parameters t_s and M_p (time domain tuning).

The results demonstrate that the frequency modification law (15) guarantees the fulfilment of the desired time domain specification.

Once the PI^α ensures the fulfilment of the same time response under nominal conditions as the *PI* controller, their robustness can be compared.

4. ROBUSTNESS IMPROVEMENT APPLICATION

Assuming parameters variations in (1), the maximum allowed values before instability can be obtained by means of simulation for both controllers.

Table 2 shows the maximum allowed values of K, T_1, T_2 and L for both controllers keeping the other parameters in nominal values.

Table 2. Maximum allowed values of *K*, T_1, T_2 *and L*

	K^{max}	T_1^{max}	T_2^{max}	L^{max}
α=0.80	0.1245	>10e5	>10e4	1289
α=0.85	0.1284	>10e5	>10e4	1289
α=0.90	0.1325	>10e5	>10e4	1289
α=0.95	0.1359	>10e5	2510	1289
PI	0.1205	>10e5	2325	1289
α=1.05	0.1356	>10e5	2190	1290
α=1.10	0.1308	42200	2170	1349
α=1.15	0.1242	24100	2260	1349
α=1.20	0.1174	18500	2750	1349

Thus, the *PI* controller combined with Smith predictor can not fulfil simultaneously the nominal time response requirements (3) and the robustness range (2) and only the PI^α controllers with $\alpha\in[0.85, 0.95]$ can guarantee both requirement sets.

The best PI^α controller that fulfils (2) and (3) is:

$$R^f(s) = 22.0641 + \frac{0.1069}{s^{0.90}}, \quad (16)$$

which keeps the same robustness to L and T_1 changes than *PI* controller, increases a 10% the robustness to K changes, and has no limit to changes in T_2.

In order to illustrate the robustness performance of both controllers, Fig. 11 shows the time response of both controllers in each parameter limit value (2).

5. CONCLUSIONS

In this paper a *PI* and PI^α controller combined with a Smith predictor have been compared.

A method, previously developed in Castillo et al. 2010, that guarantees the fulfilment of the time specifications in the case of the standard closed-loop control for first order plants, has been applied to tune PI^{α} controllers combined with a Smith predictor structure in the case of first order plus delay plants.

A family of PI^{α} controllers that provides the same time response than the *PI* controller under nominal condition has been found.

The additional parameter α has been chosen in order to maximize the robustness to plant parameters changes.

The comparative results show that the *PI* controller combined with a Smith predictor can not fulfil simultaneously the nominal time response requirements (3) and the robustness range (2).

The best PI^{α} controller has been chosen in order to maximize the robustness range.

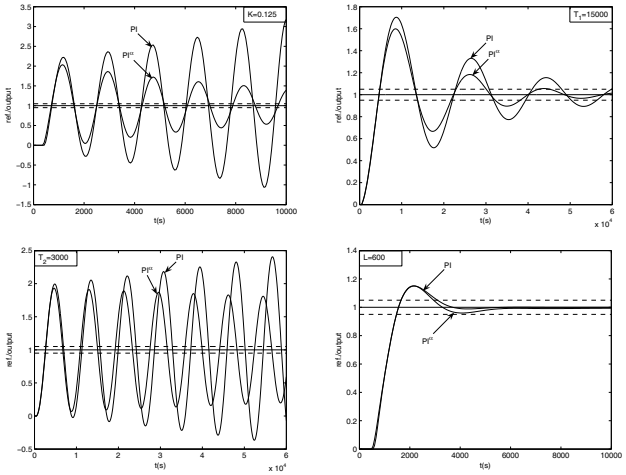

Fig. 11. *PI* and PI^{α} controllers combined with Smith predictor: time response in the limit allowed limit of K, T_1, T_2 and L.

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