Distributed Opportunistic Scheduling in Power Systems - An Optimal Stopping Approach

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Abstract: Demand response plays a key role in saving energy expenditure for future smart grid systems. For a system with multiple users which have different energy consumption and real-time requirements, the scheduling problem is quite challenging. In this work, we propose a novel distributed opportunistic scheduling scheme based on the optimal stopping rule approach. Our objective is to minimize the total energy cost while satisfying the energy and timing requirements of each user. We show that the optimal scheduling scheme is a pure threshold policy, i.e., each user need to be turned on when the electricity price is lower than a certain value; otherwise, remains idle. Our simulation results show that the proposed low-complexity distributed scheduling scheme can dramatically reduce the cost. A heuristic peak avoid approach is also proposed to bypass the peak peak price, which is a often a priori. Our result can also be implemented to the problem of maximizing profit with a fixed amount of energy, which is, in fact, the dual of the energy minimization problem.

1. INTRODUCTION

As an intelligent power generation, distribution and control system, the smart grid provides a more robust, efficient and flexible electrical power system Far [2006], Doe [2008]. A broad class of services can be facilitated by the smart grid, e.g., wide-scale integration of renewable energy sources, demand response programs involving residential and commercial customers, rapid outage detection and granular system health measurement. Millions of smart metering devices are being deployed around the country to enable utilities to interact with consumers. These smart meters can monitor energy consumption in real-time, provide customers with real-time power pricing information, and perform automatic control on smart appliances to conserve energy Bru [2009], Fan [2010], Str [2010].

According to report from Department of Energy in US, 74% of all electrical energy was consumed by buildings Doe [2009]. Therefore, the building loads need to be managed in order to reduce the energy consumption and reduce the peak demand Mag [2004], Chu [2009], HaD [2006], NIST [2009]. In this paper, we focus on the peak demand reduction aspect. To reduce the peak demand, we need to schedule the loads within buildings over time, for example,

some high-load appliances can be scheduled to operate during the off-peak hours.

Most existing load scheduling schemes are centralized and assume future electricity is known or predictable. In Kur [1996], authors use linear programming model for reducing system peak to determine both long and short term control scheduling strategies. Cohen et.al propose to use dynamic programming approach to reduce peak load by minimizing the mean square load in Coh [1987]. The forecasting of load reduction due to load control and energy payback characteristics of various controllable appliances are discussed in Mcr [1985]. Since centralized approaches require global information and usually incur high complexity, distributed approaches are more desirable. A distributed incentive-based load scheduling is proposed in Moh [2010]. A game-theoretic approach based on a simple pricing mechanism is used to provide the users with the incentive to shift the loads over time. Since customers are provided with the real-time power price by smart metering devices, why not use this real price as the mechanism for distributed load control?

The real-time price is an indicator of the system load. In general, the price is high (low) when the load demand is high (low). Some level of peak demand reduction may be automatically achieved by rational customers who aim to minimize the electricity cost. Naturally the customers will

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choose to operate the flexible loads when the real-time price reaches the minimum. In this way, those flexible loads are shifted to the low demand time period, and consequently the peak demand is reduced. However, the real-time price can be quite volatile as shown in Fig. 1, so it is not easy to decide if the minimum is reached. Due to inaccurate price prediction, current existing load scheduling scheme suffer from performance deration. This motivates us to adopt optimal stopping rule as scheduling algorithm which can model electricity price as random price signal. Another problem is that the peak demand may increase rather than decrease if every customer moves the loads to the lowest price time period. The aggregation of enough significant loads may lead to higher spike in the load demand. Therefore, using the real-time price as the mechanism for peak load reduction, we need to be able to not only move the loads to low demand time period but also spread the loads.

Fig. 1. Typical electricity price (collected by Ameren.com from 07/14/2010 to 07/23/2010).

In this paper, we propose to apply the Optimal stopping rule Fer [2006], Shi [2008] to determine when to operate the flexible loads given the volatile real-time price. Optimal stopping rule is proved to perform excellent in communication and network area Zhe [2009]. In this paper we extend the application of optimal stopping rule to power grid area. We take into the consideration the time requirement of the load. If a user does not have time requirement, it will always choose to operate at the time when the electricity price is the lowest to minimize the electricity cost. However, many users, such as washing machine, is sensitive to the waiting time. So the spent time (includes waiting time and service time) must be taken into consideration. Therefore, the cost is modeled as the wait cost plus the electricity cost. Our objective is to minimize the total cost by choosing the best operating time. We show that the optimal scheduling scheme is a pure threshold policy, i.e., each user need to turn on the load when the electricity price is lower than a certain value; otherwise, remains idle. Our simulation results show that the proposed low-complexity distributed scheduling scheme can dramatically reduce the cost. In other words, the loads are effectively shifted to the low demand time period. Furthermore, we extend our scheme to a dynamic threshold policy by peak avoidance function which can efficiently reduce waiting time and reduce the peak load at the same time. Our result can also be implemented to the problem of maximizing profit with a fixed amount of energy, which is, in fact, the dual of the energy minimization problem.

The reminder of this paper is organized as follows. In Section 2, we briefly introduce the optimal stopping theory and formally formulate the opportunistic scheduling in power systems. The main results are presented in Section 3. We conduct simulation to verify our results in Section 4. Section 5 concludes this work.

2. BACKGROUND, PROBLEM STATEMENT AND PRELIMINARIES

2.1 Preliminary of Optimal Stopping Theory

The optimal stopping theory is concerned with the problem of choosing a time to take an action based on sequentially observed random variables in order to the maximize an expected payoff or to minimize an expected cost. Specifically, let $(X_1, X_2, \dots)$ denote a random process whose joint distribution is assumed known, and $(y_0, y_1(x_1), y_2(x_1, x_2), \dots, y_\infty(x_1, x_2, \dots))$ denote a sequence of real-valued reward functions. We need choose a stopping time N , which satisfies $\{N = n\} \in F_n$ where F_n is the σ -algebra generated by $(X_j, j \leq n)$, to maximize or minimize the expected return $\mathbb{E}[Y_N]$ Zhe [2009]. In our scenario, a action means an electricity user starts to run, the observation is the electricity price and the objective is to minimize cost or maximize profit. More details about the optimal stopping theory can be found in Fer [2006] Shi [2008].

2.2 Problem Statement

We consider a power systems with M users. Time is divided to slots with length τ . The electricity price process is modeled by a random process $\{P_n\}$. We assume the electricity price is constant during a slot but may vary over different slots. Our model of electricity price, in the wireless communication scenario, is analogous to the well-known block fading channel model Wang [1995].

Though the interference between users is of more interests, in this initial work, we assume users do not interfere with each other and propose the analysis of user interference for future work. This is valid in large power systems since a small amount of users have negligible affect on electricity price. Thus we can consider an arbitrary user. We also assume that once a user is turned on, the electricity price is constant during its service time. This assumption can be validated when the change speed of electricity price is slow compared with the service time of users (e.g., electricity price may change hourly, while we can complete washing clothes within 40 minutes).

Consider an arbitrary user who has a load to sun, let S denote the service time of the load, K denote the amount of electivity(in KWH) during the service time, R denote the amount of reward that can be made from running the load. We assume the user starts at time N . In many circumstances, users have time requirement. Thus there are two sources of costs that a user perceives: 1) cost due to purchasing electricity, denoted by C_p , and 2) cost due to the spent time, denoted by C_t . It is easy to see

$$C_p = K \times P_N. \quad (1)$$

The total spent time is the summation of waiting time and service time:

$$T_N = N \times \tau + S. \quad (2)$$

In this work, we assume C_t is a linear function of the total spent time, i.e.,

$$C_t = \mu \times T_N, \quad (3)$$

where μ is a positive constant for a specified user and is referred as the time factor in this work.

The energy management problem under study can be formulated as either a Min-Cost problem or a Max-Profit problem.

The Min-Cost Problem If we are interested in minimizing the total cost in a running time, then we need to find an optimal stopping rule N which solving the following minimization problem.

$$\min_{N \in \mathcal{C}} \mathbb{E}(K \times P_N + \mu \times T_N), \quad (4)$$

where $\mathcal{C}$ is the class of stopping rules,

$$\mathcal{C} = \{N : N \geq 1, \mathbb{E}[T_N] < \infty\}. \quad (5)$$

The Max-Profit Problem If we are interested in maximizing the rate of return, then we need to find an optimal stopping rule N which solving the following maximization problem.

$$\max_{N \in \mathcal{C}} \frac{\mathbb{E}(R - K \times P_N)}{\mathbb{E}[T_N]}. \quad (6)$$

The following lemma shows that the above two optimization problems are actually equivalent.

Lemma 1. (i) If $\min_{N \in \mathcal{C}} \mathbb{E}(K \times P_N + \mu \times T_N) = R$, then $\max_{N \in \mathcal{C}} \mathbb{E}(R - K \times P_N)/\mathbb{E}[T_N] = \mu$. Moreover, if $\mathbb{E}(K \times P_N + \mu \times T_N) = R$ is minimized at $N^* \in \mathcal{C}$, then N^* is also the solution for maximizing $\mathbb{E}(R - K \times P_N)/\mathbb{E}[T_N]$.

(ii) Conversely, if $\max_{N \in \mathcal{C}} \mathbb{E}(R - K \times P_N)/\mathbb{E}[T_N] = \mu$ and the if the maximization is attained at $N^* \in \mathcal{C}$, then N^* also minimizes $\mathbb{E}(K \times P_N + \mu \times T_N)$.

The proof of Lemma 1 is similar to Theorem 1 in Chapter 6 of Fer [2006], thus we ignore here.

Remark 1. Lemma 1 tells us that by choosing appropriate μ and R , solutions of the Min-Cost problem and the Max-Profit problem are equivalent. Thus, we only need to consider one of these two optimization problems. In the next section, we only study the Min-Cost Problem.

3. MAIN RESULTS

In Section 2, we have not made any assumption about the electricity price process $\{P_n\}$. An easy choice is modeling the electricity price as an independent identically distributed (i.i.d.) random process. However, the i.i.d. model is a little oversimplified since the electricity process has memory in general, i.e., the electricity price does not change too much over two consecutive slots. Thus a more realistic candidate is the finite-state Markov chain (FSMC) model. Moreover, the electricity price has an explicit trend in general, e.g., it may have a peak at a fixed time of every day. In this section, we begin with solving the optimization problem (4) based on the i.i.d. electricity price model with the optimal stopping theory Fer [2006], Shi [2008]. For the

generally price process case, it is very difficult to obtain the optimal scheduling policy. Alternatively, we propose a peak avoidance approach for electricity price processes with explicit trends. More realistic electricity price process models, e.g., Makov model, are proposed for future work.

3.1 Optimal Scheduling for i.i.d. Electricity Price Processes

Now we assume $\{P_n\}$ is an i.i.d. random process. Without the loss of generality, we assume that the first two moments of P_n exist and both are limited. Then we have the following result.

Theorem 2. Let $\{P_n\}$ be independent identically distributed, then the optimal stopping rule, which solves the Min-Cost problem, exists, and is given by

$$N^* = \min \{n \geq 1 : P_n \leq \lambda^*\} \quad (7)$$

where λ^* is the unique solution of

$$\mathbb{E}(\lambda - P_n)^+ = \frac{\mu\tau}{K}. \quad (8)$$

Proof. The proof of Theorem 2 is similar to the solution to the problem of selling an asset without recall in Chapter 4 of Fer [2006]. Thus we only indicate the difference here.

Let $X_n = KP_n + \mu S$, $c = \mu\tau$, then the total cost can be expressed by $C(n) = X_n + nc$. Then from Chapter 4 of Fer [2006], we see that the optimal stopping rule exists are given by

$$N^* = \min \{n : -X_n \geq V^*\}, \quad (9)$$

where V^* is the unique solution of

$$\mathbb{E}(-X_n - V^*)^+ = c. \quad (10)$$

By simply changing variables, we can get (7).

Remark 2. The above theorem reveals that the optimal scheduling scheme N^* is a pure threshold policy, i.e., the user should be turn on once the electricity price is less than a fixed value.

Note that the distribution of P_n is required for solving (8). Many probabilistic models can be chosen for modeling the electricity price. Two natural choices are Gaussian distribution and uniform distribution. Now we give the solution of (8) under the this two models.

If P_n is uniformly distribution on $[a, b]$, it is easy to obtain that

$$\lambda^* = \sqrt{2(b-a)\mu\tau/K} + a. \quad (11)$$

If P_n is Gaussian distributed, there is no closed-form solution for (8). Assuming $P_n \sim N(\bar{P}, \sigma^2)$, it is straightforward to show that (8) equivalent to

$$\sqrt{\frac{\sigma^2}{2\pi}} \left(1 + \text{sign}(\lambda - \bar{P}) \left(1 - e^{-\frac{(\lambda - \bar{P})^2}{2\sigma^2}} \right) \right) + \frac{\lambda - \bar{P}}{2} \left(1 + \text{erf} \left(\frac{\lambda - \bar{P}}{2\sigma} \right) \right) = \frac{\mu\tau}{K} \quad (12)$$

which is can be solved numerically.

3.2 Suboptimal Scheduling for Electricity Price Processes with Trends

As mentioned before, the i.i.d. model is an oversimplified model. Generally, the real electricity price has an explicit trend. By observing the realtime electricity price database,

we find that there is a peak price at about 3:00PM with high probability. This motivates us to improve the results in Section 3.1 by utilizing the trend information. Deriving of optimal stopping rule for electricity price processes is very difficult. Thus, in this work, we propose a heuristic approach, which we call the Peak Avoidance approach, to improve the threshold-based solution in Section 3.1.

Let t denote the time a user enters the system, and t_p denote the electricity price peak time. Since the trend is a priori information, a good stopping rule is expected to have the following properties. If t is closed to t_p , and

- (1) if $t < t_p$, the user should start to run as soon as possible to avoid the coming peak;
- (2) if $t > t_p$, the user should wait a while for lower price.

Thus we can adjust the threshold derived in Theorem 2 based on above observations. We propose a new threshold

$$\lambda_a^* = \lambda^* + \text{sign}(t_p - t)\beta e^{-\alpha(t-t_p)^2} \quad (13)$$

where λ^* is the unique solution of (8), α, β are positive parameters, which can be determined by experiments. It is easy to see that i) if $t < t_p$, the threshold is increased, thus the user tends to start earlier; ii) if $t > t_p$, the threshold is decreased, thus the user tends to start later. When t is far from t_p , $\lambda_a^* \approx \lambda^*$.

4. SIMULATION RESULTS

In this section, we first conduct simulation to study the properties of the optimal stopping rule, then we assess the performance of our proposed algorithm with real electricity price data.

4.1 Properties of the Optimal Stopping Rule

Now we assume the electricity price is uniformly distributed on $[0.01, 0.2]$. Then the threshold λ^* is given by (11). Fig. 2 shows the relation between threshold and time factor, while Fig. 3 shows the relation between threshold and energy consumption.

Fig. 2. Threshold v.s. time factor μ . $P_n \sim U(0.01, 0.2)$.

From these figures, we can see that the threshold is increasing on the time factor but decreasing on the power consumption. This is because if one user requires more energy and is less sensitive to the waiting time, it is more willing to wait for a good electricity price.

Fig. 3. Threshold v.s. power consumption K . $P_n \sim U(0.01, 0.2)$.

4.2 Performance of the Optimal Stopping Rule

In order to illustrate the merit of our proposed scheduling scheme, we use both randomly generated data and real data to test the performance of the optimal stopping rule. We compare the optimal stopping rule with an “Instant Service Scheme”, under which a user starts to run at once when it enters the system (i.e., no waiting time).

Fig. 4 shows the performance of the optimal stopping rule with randomly generated electricity price compared with the instant service scheme. In this simulation, the electricity is generated with uniform distribution on $[0.01, 0.2]$. From this figure, we can see the optimal stopping rule provides dramatic cost reduction compared with the instant service scheme. Moreover, we see that if a user is less sensitive to the waiting time, the optimal stopping rule saves more compared with the instant service scheme. This is because if a user is less sensitive to the waiting time, it tends to wait longer thus is more likely to get a better price. Now we apply the optimal scheduling scheme

Fig. 4. Performance of the optimal stopping rule with randomly generated data.

to real data. We use the realtime electricity price from Ameren.com (<https://www2.ameren.com/RetailEnergy/realtimeprices.aspx>), data collected from July 14th to July

23rd, 2010). Fig. 5 shows the performance of our proposed scheme compared with the instant service scheme. From this figure we can see that although the real price is not uniformly distributed, the optimal stopping rule can still provide remarkable cost reduction, which corroborates our work.

Fig. 5. Performance of the optimal stopping rule with real data.

5. CONCLUSION

In this work, optimal stopping theory is applied to the scheduling problem in power systems. Multiple electricity users which have different power consumption and time requirement are scheduled to minimize the cost. If the electricity price is modeled as a simple i.i.d. random process, we showed that the optimal scheduling scheme is a pure threshold policy. Since the i.i.d. model is a little over simplified, a heuristic peak avoidance approach is proposed to exploit the trend information of electricity price processes. simulation results show that our proposed scheduling policy can dramatically reduce the cost of users.

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