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Robust control of a submarine snorkel system

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Abstract: This paper illustrates a control strategy for a submarine snorkel able to assure that the water surface profile can be tracked by the extremity of the device, called "fluctuating head". The proposed solution is based on adaptive output regulation theory. More precisely, it is assumed that wave profile can be approximated as a linear combination of a number of harmonics of unknown amplitude, phase and frequency and we look for an internal model-based adaptive regulator able to enforce the top of the mast to track the reference profile. The effectiveness of the proposed adaptive scheme is shown through simulations.

Keywords: Robust control (nonlinear case), Robust control applications, Tracking

1. INTRODUCTION

This paper illustrates a control strategy for a submarine snorkel, a device that allows a submarine to operate submerged while still taking in air from above the surface. This device can be usually found on diesel-electric propulsion submarines in order to maintain the concealment also during the battery rechargement phase. The snorkel is, in fact, a pipe that can reach the water surface while the submarine is at its periscope deep to assure a proper air influx for the diesel engines and for room ventilation. The first successful example for this kind of device dates back to November 1926 when Capt. Pericle Ferretti of the technical corps of the Italian Navy ran tests with a ventilation pipe gear installed on board the ex-US H-3 submarine received by the Regia Marina during the 1st World War. This idea was then developed by the Dutch Navy and further improved to appear on the German U-Boot during the 2nd World War to counteract the Alleys' air surveillance. In modern submarines, the snorkel is one of the "mast" that can be raised or lowered depending on the particular operative condition. Normally, the device is inside the submarine, while during its functioning it must emerge for at least 30 cm over the water surface.

The most advanced snorkels for submarines are equipped by a control system able to assure that the water surface profile can be tracked by the extremity of the device, called "fluctuating head". In this way, the system will be above the water surface just for allowing a proper ventilation thus limiting the possibility of being discovered by the enemies' radars (see Fig. 2). It is clear, then, that tracking the wave profile is a key feature of the control system. Calzoni S.r.l. is an important company located in the surroundings of Bologna and founded in 1834 that is one of the worldwide leaders in the Aerospace & Defence market and provides solutions for Marine Handling & Lighting Solutions. Calzoni is the producer of a family of advanced submarine snorkel, the main reference for this work. An example is reported in Fig. 1.

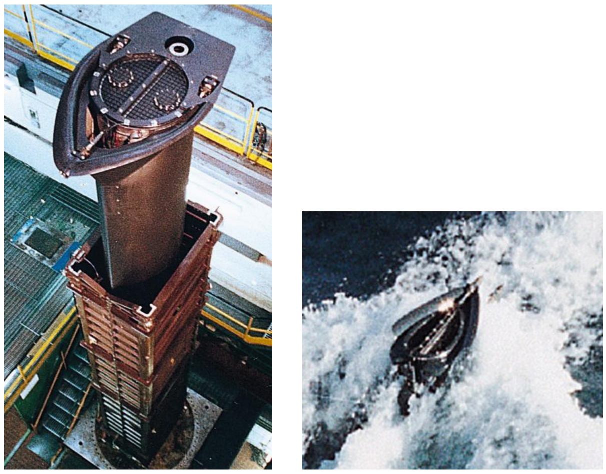

Fig. 1. View of one submarine snorkel developed by Calzoni.

In this paper, a solution to the snorkel tracking problem based on adaptive output regulation theory is proposed. Specifically, we assume that the wave profile can be approximated as a linear combination of a number of harmonics of unknown amplitude, phase and frequency and we look for an internal model-based adaptive regulator able to enforce the top of the mast to track the reference profile.

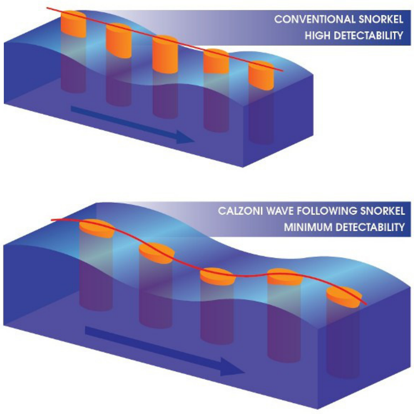

Fig. 2. Comparison between standard submarine snorkel and snorkel equipped with the "fluctuating head".

In the past the problem of output regulation has been extensively studied for linear systems starting with the work of Francis and Wonham [1976] and Francis [1977] and for nonlinear systems beginning with Isidori and Byrnes [1990]. A noteworthy step forward in the theory has been achieved when adaptive output regulation has been addressed, namely the exosystem, and not only the controlled plant, has been regarded as an uncertain system. This, indeed, is the case when the exogenous signal to be tracked/rejected is given as a sum of harmonics with uncertain frequency. In this respect a number of works, starting from the original formulation and solution provided in Serrani et al. [2001], have been presented in literature. Among others it is worth recalling Delli Priscoli et al. [2006b], where the theory of adaptive observers for nonlinear systems was shown to be effective in designing nonlinear internal models in a semi-global minimum-phase setting, Marino and Tomei [2003] and Zing [2003] where global adaptive tools was applied to solve the problem for linear and nonlinear systems in a global setting, and Ye and Huang [2003] where the case of "large-scale systems" has been dealt with.

The problem of adaptive output regulation can be also considered as a particular case of a more general nonlinear output regulation framework presented in Marconi et al. [2007], Marconi and Praly [2008]. In the present paper, as preliminary attempt, we adapt the original approach presented in Serrani et al. [2001] to the problem at hand. The goal of the work is to show the effectiveness, through simulation results, of the adaptive scheme proposed in Serrani et al. [2001].

2. MODEL OF THE SUBMARINE SNORKEL SYSTEM

From a functional point of view, the submarine snorkel system is an hydraulic piston whose motion is perpendicular with respect to the submarine and, then, to the water surface. As schematically reported in Fig. 3, the motion is the result of the action of three forces, the input force F_u, the normal force F_n and the hydro-dynamics force F_h.

The input force F_u acts along the direction of motion of the piston and it is generated by the pressure of the oil in the hydraulic circuit. This force is proportional to the

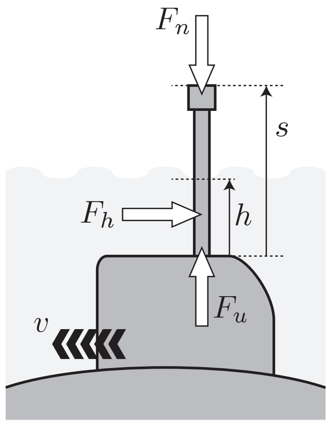

Fig. 3. Schematic view of the submarine snorkel.

area of the contact surface oil-piston A_p and to the fluid pressure P_f, i.e.:

$$F_u = A_p P_f \tag{1}$$

Also the normal force F_n acts along the direction of motion of the piston, but in the opposite direction of the input force. This force takes into account the combined effect of gravity and buoyancy. The latter follows from the Archimedes' principle, i.e. "Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object." It is clear, then, that the buoyancy force depends on the volume of the snorkel that is immersed in water. If s denotes the extension of the snorkel, being $s = 0$ the case in which the snorkel is completely inside the submarine, and h the distance of the water surface from the exit point of the snorkel itself, it is easy to see that:

$$F_n = \begin{cases} mg - \rho g A_s s & \text{if } 0 \le s < h \\ mg - \rho g A_s h & \text{if } h \le s \end{cases} \tag{2}$$

Here, m is the mass of the piston, ρ the water density, A_s the stem area, while g is the gravity acceleration.

Differently from F_u and F_n, the hydro-dynamics force F_h acts along the direction of motion of the submarine and is due to the viscous friction between snorkel and water, which is proportional, among others, to the area of the snorkel in water. If v denotes the velocity of the submarine with respect to the water, this force is given by

$$F_h = \begin{cases} \frac{1}{2} C_R v^2 l s & \text{if } 0 \le s < h \\ \frac{1}{2} C_R v^2 l h & \text{if } h \le s \end{cases} \tag{3}$$

where C_R is the aerodynamics coefficient and l is the snorkel width, so that $l \cdot s$ or $l \cdot h$ are equal to the area of the snorkel profile in water.

Since the snorkel motion is along the vertical direction, it seems that the hydro-dynamics force is not playing any role. However, this force is responsible for a friction force on the piston. Several friction models can be adopted but, for this particular application, the static friction could be neglected and just the kinematic friction has been taken into account. Such friction force is, then, proportional to F_h and opposite to the direction of motion of the snorkel. Consequently, taking into account (1), (2) and (3), the dynamics of the snorkel is given by the following nonlinear ODE:

$$m\ddot{s} = F_u + F_n - \mu F_h \operatorname{sign}\dot{s} \tag{4}$$

where μ is the friction coefficient. On the other hand, the usual operative condition of the snorkel, i.e. when the piston head has to track the water surface, corresponds to the case in which $h \leq s$. From (4), the following differential equation can be obtained:

$$m\ddot{s} = A_p P_f - mg + \rho g A_s h - \frac{1}{2}\mu C_R v^2 l h \operatorname{sign}\dot{s} \tag{5}$$

3. FORMULATION OF THE REGULATION PROBLEM

In this section we reformulate the problem at hand as a problem of adaptive output regulation. It is worth mentioning that at present stage the regulator has been developed under the hypothesis that the complete error signal is available. On the other hand, the only variable available from measurements is the snorkel extension s, which is not sufficient for the development of a robust regulator in the sense of Francis and Wonham [1976], as pointed out at the end of the section. The idea will to combine the available measurement with digital information provided by further sensors that are mounted (or that can be installed) on the snorkel head, but this topic is still under investigation. To this end we assume that h, the distance between of the water surface and the top of the snorkel, is given by

$$h = Qw \qquad Q \in \mathbb{R}^{1\times(2r+1)}$$

in which w is governed by the "exosystem"

$$\begin{cases} \dot{\omega} = 0 \\ \dot{w} = S(\omega)w \end{cases} \qquad \omega \in \mathbb{R}^r,\; w \in \mathbb{R}^{2r+1} \tag{6}$$

where $\omega = (\omega_1, \ldots, \omega_r)^{\mathrm{T}}$ and

$$S(\omega) = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & S_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_r \end{pmatrix} \qquad S_i = \begin{pmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{pmatrix}$$

In other words it is assumed that h is a linear composition of a constant bias and r harmonics with constant frequencies ω_i and amplitudes and phases depending on the initial condition $w(0)$ of (6). In the following we assume that the initial state $(\omega(0), w(0))$ is unknown but ranging in a known compact set $\Omega \times W \subset \mathbb{R}^r \times \mathbb{R}^{2r+1}$. This means that the signal h is composed by r harmonics whose amplitudes, phases and frequencies are unknown but with known upper bounds.

We change coordinates by defining the error variables

$$e_1 = s - Qw \qquad\qquad e_2 = \dot{s} - QS(\omega)w \tag{7}$$

so that the system composed by (5) and (6) reads as

$$\begin{cases} \dot{\omega} = 0 \\ \dot{w} = S(\omega)w \\ \dot{e}_1 = e_2 \\ m\dot{e}_2 = u - \Gamma w - \frac{1}{2}\mu C_R v^2 l Q w \operatorname{sign}(e_2 + QS(\omega)w) \end{cases} \tag{8}$$

where $u = A_p P_f - mg$ and

$$\Gamma = mQS^2(\omega) - \rho g A_s Q$$

Note that Γ is an unknown vector and that the gravity force has been balanced out. This system has regulation error $e = e_1$ and available measurement $y = e_1 + Qw$. Our problem is to design an output feedback regulator of the form

$$\begin{cases} \dot{\xi} = \varphi(\xi, y) \\ u = \gamma(\xi, e) \end{cases} \qquad \xi \in \mathbb{R}^m$$

such that for any initial condition $(\omega, w, e_1, e_2, \xi) \in \Omega \times W \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}^m$ the closed loop system has bounded trajectories and $\lim_{t\to\infty} e(t) = 0$. For control purposes we design the regulator by neglecting the non linear discontinuous term in (8), or equivalently in (5), that, however, will be considered in the simulative analysis presented in Sect. 6. The error dynamics is, then, described by the following linear system:

$$\dot{e} = Ae + Bu - \begin{pmatrix} 0 \\ \Gamma \end{pmatrix} w \tag{9}$$

in which $e = (e_1, e_2)^{\mathrm{T}}$ and

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad\qquad B = \begin{pmatrix} 0 \\ 1/m \end{pmatrix}$$

4. DESIGN OF THE ADAPTIVE INTERNAL MODEL

In this part we specialize the theory of adaptive output regulation, originally proposed in Serrani et al. [2001], to the problem at hand. As a first preliminarily design stage, we construct a *state-feedback* regulator, namely a regulator processing the full state (e_1, e_2). The knowledge of (e_1, e_2), not known in the considered application, will be then substituted by an appropriate estimation obtained by elaborating the measurement y.

By following the main lines of Serrani et al. [2001] we focus on the regulator of the form

$$\begin{cases} \dot{\xi} = F\xi + Gu + N\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \\ u = \hat{\Psi}\xi + v_{st} \end{cases} \qquad \xi \in \mathbb{R}^{2r+1} \tag{10}$$

in which (F, G) is a controllable pair with F Hurwitz, N is a $(2r+1)\times 2$ matrix to be chosen, and v_{st} is a residual control input which will be designed later. The vector $\hat{\Psi}$, of dimension $1 \times (2r+1)$, contains further regulator state variables which have to be adapted so that the proposed regulator structure has desired asymptotic properties. The regulator structure is thus completed by an adaptation law for $\hat{\Psi}$ of the form

$$\dot{\hat{\Psi}} = v_{ad}$$

where v_{ad} will be chosen later.

Ideally, the vector $\hat{\Psi}$ should be chosen equal to the vector $\Psi = \Gamma T^{-1}$, where $T \in \mathbb{R}^{2r+1} \times \mathbb{R}^{2r+1}$ is the nonsingular solution of the Sylvester equation

$$TS(\omega) - FT = G\Gamma \tag{11}$$

which always exists as the spectrum of F and $S(\omega)$ are disjoint (see Serrani et al. [2001]). As a matter of fact, it is easy to realize that the matrices $S(\omega)$ and $F + G\Psi$ are similar under T and the regulator (10), if $\hat{\Psi}$ were chosen equal to Ψ and it were initialized at $\xi(0) = Tw(0)$, is able to reproduce the ideal steady state control input $\Gamma w(t)$ needed to achieve the regulation objective. This is what, in the terminology proposed in Byrnes and Isidori [2003], has been referred to as *internal model property*. Unfortunately the ideal choice

$$\hat{\Psi} = \Psi$$

cannot be implemented as Ψ depends, via T, on the uncertain frequency ω. Hence an adaptive law for $\hat{\Psi}$ will be sought.

In the forthcoming analysis we choose the pair $(F,\, G)$ in the canonical form

$$F = \begin{pmatrix} 0 | I_{2r} \\ \hline f \end{pmatrix} \qquad\qquad G = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{12}$$

in which I_{2r} denotes the identity matrix of dimension $2r$ and $f = (f_1,\, \dots,\, f_{2r+1})^{\mathrm{T}}$ contains the coefficients of an Hurwitz polynomial. Following Serrani et al. [2001], consider the preliminary change of variables

$$\xi \;\mapsto\; \tilde{\xi} = \xi - Tw$$

and note that the regulator dynamics (10) in the new coordinates read as

$$\dot{\tilde{\xi}} = (F + G\Psi)\tilde{\xi} + Gv_{st} + G\tilde{\Psi}\xi + N(e_1,\, e_2)^{\mathrm{T}}$$

where

$$\tilde{\Psi} = \hat{\Psi} - \Psi$$

By bearing in mind that $\Psi T = \Gamma$, the system dynamics transform as

$$\begin{cases} \dot{e}_1 = e_2 \\ m\dot{e}_2 = \Psi\tilde{\xi} + v_{st} + \tilde{\Psi}\xi \end{cases}$$

Furthermore, if we let

$$\tilde{e}_2 = e_2 + e_1$$

and we chose

$$v_{st} = m\,(e_1 - \tilde{e}_2) + v'_{st}$$

the resulting system turns out to be

$$\begin{cases} \dot{e}_1 = -e_1 + \tilde{e}_2 \\ m\dot{\tilde{e}}_2 = \Psi\tilde{\xi} + v'_{st} + \tilde{\Psi}\xi \end{cases}$$

We consider now the additional change of variable, meant to eliminate the control variable v_{st} from the $\tilde{\xi}$ dynamics, given by

$$\tilde{\xi} \;\mapsto\; \chi = \tilde{\xi} - mG\tilde{e}_2$$

which, by choosing the degree-of-freedom N in the regulator (10) such that

$$\begin{aligned} N\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} &= -mG\,(e_1 - \tilde{e}_2) - mFG\tilde{e}_2 \\ &= -mGe_2 - MFG\,(e_2 - e_1) \end{aligned} \tag{13}$$

transforms the overall dynamics as

$$\begin{cases} \dot{\chi} = F\chi \\ \dot{e}_1 = -e_1 + \tilde{e}_2 \\ m\dot{\tilde{e}}_2 = m\Psi G\tilde{e}_2 + \Psi\chi + v'_{st} + \tilde{\Psi}\xi \\ \dot{\tilde{\Psi}} = \dot{\hat{\Psi}} = v_{ad} \end{cases}$$

This system has a friendly cascade structure with the driving system $\dot{\chi} = F\chi$ which is Hurwitz. The regulator design can be then completed as follows. At first, let us chose

$$v'_{st} = -k\tilde{e}_2 \tag{14}$$

being $k > m\Psi G$. Moreover, by considering the candidate Lyapunov function

$$V(\chi, \tilde{e}_2, \tilde{\Psi}) = \tilde{e}_2^2 + \kappa\chi^{\mathrm{T}}P_F\chi + \frac{1}{\gamma}\tilde{\Psi}\tilde{\Psi}^{\mathrm{T}}$$

where $P_F = P_F^{\mathrm{T}} > 0$ is such that $P_F F + F^{\mathrm{T}}P_F = -I$, and $\kappa > 0$ is a sufficiently large positive number, standard completion of squares arguments and the choice

$$v_{ad} = -\,\mathrm{dzn}_\ell\,\hat{\Psi} - \gamma\xi^{\mathrm{T}}\tilde{e}_2 \tag{15}$$

lead to the following bound on $\dot{V}$:

$$\dot{V} \le -\frac{1}{2}\tilde{e}_2^2 - q\chi^{\mathrm{T}}\chi$$

where q is a positive number. In (15), γ is an arbitrary positive and $\mathrm{dzn}_\ell(\cdot)$ is a "dead-zone" vector function [1] defined as $(\mathrm{dzn}\, s)_i = s_i - \ell\,\mathrm{sign}\, s_i$ if $|s_i| \ge \ell$ and $(\mathrm{dzn}\, s)_i = 0$ otherwise, were ℓ is any positive number such that $\ell \ge \max_{\omega\in\Omega}\Psi_i$. Note that, since Ψ depend on ω, the tuning of ℓ requires the knowledge of an upper bound on the uncertain frequency. Then, by invoking standard LaSalle arguments, it is concluded that the overall closed-loop trajectories are bounded and attracted by the set

$$\Big\{(\chi,\, e_1,\, \tilde{e}_2,\, \tilde{\Psi}) \in \mathbb{R}^{2r+1} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}^{2r+1} \;: \;:\; e_1 = 0,\, \tilde{e}_2 = 0,\, \chi = 0\Big\}$$

The previous analysis proves that the regulator (10), (13), (14) and (15) solves that problem at hand for the simplified linear system for any possible *constant* frequency $\omega \in \Omega$. According to the general theory presented in Serrani et al. [2001], the analysis above is not conclusive as far as the possible convergence of $\tilde{\Psi}$ to 0 is concerned. Additional persistence of excitation conditions should be assumed to formally prove asymptotic properties of $\tilde{\Psi}$.

The problem of adaptive output regulation by using the available output y is indeed more challenging and under study by the authors. In this respect it is worth noting that, by using the terminology in Proposition 1 in Francis and Wonham [1976], the regulation error e_1 is not *readable* from the output y. Namely, there does not exist a vector ℓ such that $e_1 = \ell y$. As shown in Francis and Wonham [1976], readability of the error from the available output is a necessary condition for a *robust* (in the sense of Francis and Wonham [1976]) linear regulator to exists. This condition has been extended to a nonlinear setting in Delli Priscoli et al. [2006a], where it has been shown that a necessary condition for a robust nonlinear regulator to exist is that the available output vanishes in steady state.

All these facts suggest that in the addressed case, characterized by the non-zero steady state output y, the search of a robust regulator (where the notion of robustness must be interpreted in the general sense of the quoted works) is hopeless. Nevertheless, possible robust solutions, under study by the authors, are in principle possible by "relaxing" the requirement of robustness underlying the problem. Furthermore, the solution can take advantage from available information coming from "on-off" sensors mounted on the submarine snorkel. With reference to the standard solution provided by Calzoni, for example we are thinking about some digital sensors mounted on the fluctuating head of the snorkel and able to determine if the head is over or under the water surface. Adaptive observers and extended Kalman filters are some of the observer theories that are explored.

[1] The presence of the dead-zone function does not play any role in the forthcoming stability analysis and only considered for practical numerical reasons in the simulations.

m	$650\,\mathrm{kg}$	ρ	$1\,\mathrm{kg/m^3}$
g	$9,81\,\mathrm{m/sec^2}$	A_s	$0.125\,\mathrm{m}^2$
C_R	$1\,\mathrm{kg/m^3}$	v	$6,168\,\mathrm{m/sec}$
l	$0,4\,\mathrm{m}$	μ	$0,8$

Table 1. Parameters of the submarine snorkel.

5. DESIGN OF A REDUCED-ORDER ADAPTIVE INTERNAL MODEL

The regulator designed in the previous section turns out to be redundant as it comes from an off-the-shelf application of the general theory in Serrani et al. [2001]. It turns out that the order of the regulator, previously equal to $2(2r+1)$ plus the dimension of the Kalman observer, can be reduced by a mild adaptation of the analysis presented in the previous section. For illustrative reason we develop the case in which $r = 3$. Furthermore, we assume that the frequencies of the three harmonics are multiple of a fundamental unknown frequency ω_f, namely $\omega_3 = \omega_f$, $\omega_2 = n_2\omega_f$ and $\omega_1 = n_1\omega_f$, with n_1, n_2 known real numbers. In this case note that the characteristic polynomial of the matrix $S(\omega)$ is given by

$$\det\left(sI - S(\omega)\right) = s^7 + c_1\omega_f^2 s^5 + c_2\omega_f^4 s^2 + c_3\omega_f^6$$

with $c_1 = n_1 + n_2 + 1$, $c_2 = n_1 + n_2 + n_1 n_2$, $c_3 = n_1 n_2$ known numbers. Hence, given an arbitrary pair $(F,\, G)$ of the form (11), it turns out that the ω-dependent vector Ψ having the property that $(F + G\Psi)$ and $S(\omega)$ are similar is necessarily of the form

$$\Psi = \Psi_3(\omega)L - f$$

in which

$$\Psi_3(\omega) = \begin{pmatrix} -\omega_f^6 & -\omega_f^4 & -\omega_f^2 \end{pmatrix}$$

while L is a $\mathbb{R}^3 \times \mathbb{R}^7$ matrix with all zeros except the elements (1, 2), (2, 4) and (3, 6) which are set equal to c_1, c_2 and c_3 repsectively. According to the previous facts, all the arguments used in the previous section can be repeated with mild modifications by estimating not the entire $2r+1$-order vector Ψ but rather the 3-order vector Ψ_3. In particular the proposed regulator is of the form

$$\begin{cases} \dot{\xi} = F\xi + Gu + N\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \\ \dot{\hat{\Psi}}_3 = -\gamma\xi^{\mathrm{T}}\left(e_1 + e_2\right) \\ u = \hat{\Psi}_3 L\xi - f\xi + me_2 - k\left(e_1 + e_2\right) \end{cases} \tag{16}$$

with $\xi \in \mathbb{R}^7$ and $\hat{\Psi}_3 \in \mathbb{R}^3$, in which $N(e_1,\, e_2)^{\mathrm{T}}$ and k are chosen as in (13) and (14) respectively. The arguments used in the previous analysis allow one to conclude that the previous controller guarantees that the overall closed-loop trajectories are bounded and that $\lim_{t\to\infty} e_1(t) = 0$ as required by the regulation problem.

6. SIMULATION RESULTS

In this section, the effectiveness of the proposed control solution is show by means of simulation. As far as the physical parameters of the submarine snorkel is concerned, they have been chosen as reported in Table 1. This corresponds to a typical solution developed by Calzoni.

In the first simulation, it is supposed that the error variables (7) are available and the controller given by (10), (13), (14) and (15) is implemented. The water surface is supposed to be the result of a constant offset of $1\,\mathrm{m}$ plus a couple of sinusoids of amplitude $0.5\,\mathrm{m}$ and $0.2\,\mathrm{m}$, and pulse ω_1 and ω_2 respectively. To test the performance of the adaptive scheme, the pulse of each sinusoid is unknown and it has been changed in $t = 30\,\mathrm{sec}$ and in $t = 60\,\mathrm{sec}$. So, initially ω_1 is equal to $0.9425\,\mathrm{rad/sec}$, and then changed into $0.6283\,\mathrm{rad/sec}$ and finally into $1.5708\,\mathrm{rad/sec}$. As far as ω_2 is concerned, it assumes the following values: $3.7322\,\mathrm{rad/sec}$, $1.6965\,\mathrm{rad/sec}$ and $5.4287\,\mathrm{rad/sec}$. Finally, the friction force has been taken into account.

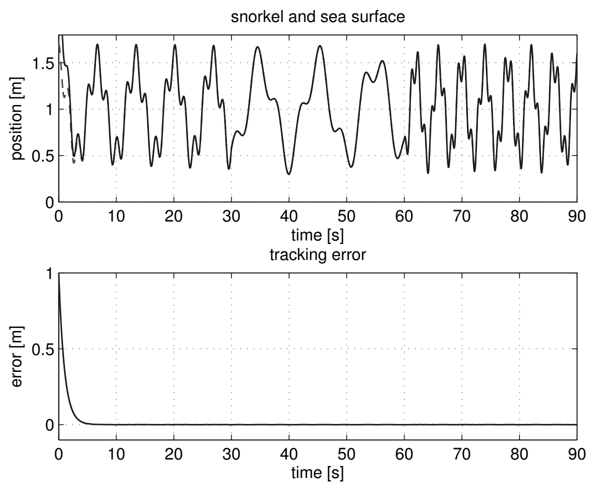

Fig. 4. Tracking performances in case the error (e_1, e_2) is available for measurement.

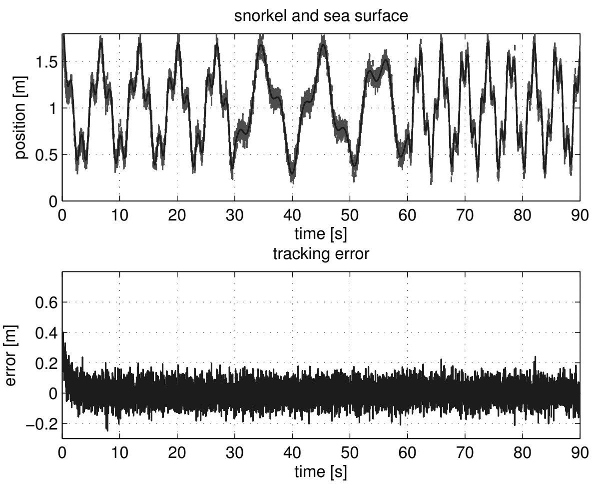

Fig. 5. Tracking performances in presence of noise.

The simulation results have been reported in Fig. 4. As the plot on top clearly shows, after a short transitory the system is able to track the unknown water surface profile. The dashed line is the unknown reference signal (i.e., the water surface), while the solid line represents the position of the top of the snorkel. In the second plot, the time evolution of the error variable e_1 is reported, showing the very good performances of the proposed solution. On the other hand, the robustness property of the whole scheme is shown in Fig. 5 in which noise has been added both on the water surface profile and to the error signal. Even in

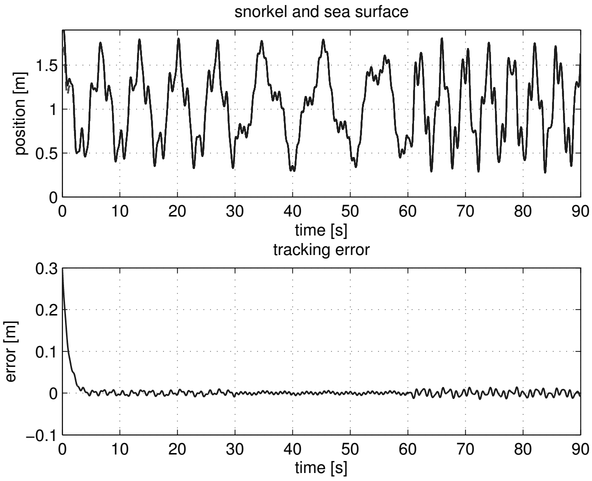

Fig. 6. Tracking performances in case the internal model unit is under-dimensioned (without noise).

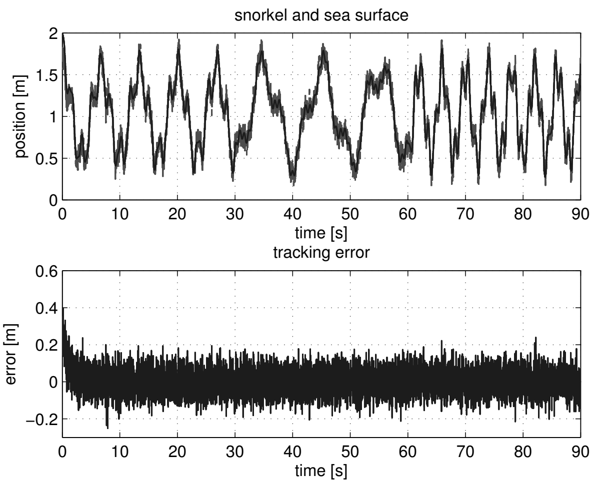

Fig. 7. Tracking performances in case the internal model unit is under-dimensioned (with noise).

this situation, the system is able to track the water profile in a very accurate manner.

The robustness of the proposed solution is also proved in case the internal model unit is under-dimensioned with respect to the number of harmonics that compose the water surface profile. In particular, a third harmonics with amplitude $0.25\,\mathrm{m}$ and pulse $8.2550\,\mathrm{rad}/\mathrm{sec}$ has been added to the same profile used in the previous simulations. The results are reported in Fig. 6 for the situation in which no noise is present, while in Fig. 7 when noise is present. As expected, in both the cases the internal model unit is not able to bring the steady state error to zero. Nevertheless, the water surface profile can be tracked quite accurately and the performances are quite promising for an application of the proposed control scheme in a real scenario.

7. CONCLUSIONS

In this paper, a novel control strategy for a submarine snorkel able to assure that the water surface profile can be tracked by the extremity of the device, called "fluctuating head" is presented. The theoretical background lies within the adaptive output regulation theory under the hypothesis that the wave profile can be approximated as a linear combination of a number of harmonics of unknown amplitude, phase and frequency. Then, an internal model-based adaptive regulator able to enforce the top of the mast to track the reference profile is proposed. At present stage, the regulator has been developed under the hypothesis that the complete error signal is available. On the other hand, the only variable available from measurements is the snorkel extension, which is not sufficient for the development of a robust regulator. The possibility of combining the available measurement with digital information provided by further sensors that are mounted (or that can be installed) on the snorkel head is under investigation.

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