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Heuristic for the Resolution of the Cyclic Hoist Scheduling Problem with Multi-Items

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Abstract: The aim of this paper is to present a new heuristic for the resolution of the Cyclic Hoist Scheduling Problem (CHSP). It consists in determining a repetitive sequence of hoist moves that minimizes the duration time (called the cycle time). The obtained cyclic scheduling respects material as well as resource handling constraints. In this approach, time windows are maintained for all soaking operations and overlapping cycles are allowed in order to be as close as possible to optimality. Computational results, including comparison with existing algorithms are presented to show the efficiency of the proposed heuristic. To reduce the cycle time, we integrate in the general heuristic an algorithm with a set of Minimum Part Set (MPS) configurations'. This one allows us to find the best order in which jobs should be introduced into the line.

Keywords: Cyclic Schedule, heuristic, time-windows, electroplating facilities, Minimum Part Set (MPS)

1. INTRODUCTION

This paper tackles on a scheduling problem which is often encountered in electroplating facilities, and is known in the literature as the Cyclic Hoist Scheduling Problem (CHSP). It focuses on minimizing the cycle time, which is the duration of a repetitive sequence of part's moves.

Since the 1970s, and more specifically since the first model given by Philips and Unger (1976), automated electroplating systems have been widely studied (Manier and Bloch 2003). Such systems are composed of tanks containing chemical or electrolytic baths in which parts to be processed are soaked. Transportation of products between tanks is performed by one (or more) hoist(s).

Lei and Wang (1989) proved that the cycle hoist scheduling problem is NP-hard in strong sense, even with identical parts.

A large number of mathematical models (Baptiste et al. 1996, Spacek et al. 1999, Zhou and Ling 2003) and heuristic algorithms (Song et al. 1995) have been developed. Nevertheless, few works have been interested in multi-parts jobs, as mentioned bellow:

Ptuskin (1995) studied a multi-parts problem. In this problem parts are processed according the same sequence, with various processing times, where the sequence alternates periodically jobs of different parts and is known in advance. Moreover, the date of each part has to be computed. This problem is decomposed in several mono-product sub-problems and the solution corresponds to a common period.

Varnier and Jeunehomme (2000) used a branch and bound method for a similar multi-parts problem. A difference with Ptuskin (1995) is that the entry sequence of jobs (configuration) is not known in advance. Each level of the search tree consists in adding a possible state at the beginning of the cycle, called configuration. The possible schedules from this configuration constitute the branches of the search tree and then a linear program is achieved to check if the problem has a solution. If not, backtrack is allowed to consider the other schedules. However, Varnier and Jeunehomme mainly study the transition part of the schedule.

A branch and bound method has also been used by Mateo (2006) to study the CHSP for a two different part jobs problem. The branch and bound procedure builds the sequence of movements progressively. Each level of the search tree consists in adding one tank and thus, the stages to be done on it. A linear program is then solved at each node to check the consistency of the constraint system. Nevertheless, this method can not be easily extended to *r* part-job (where *r*>2).

An analogy between the Hoist Scheduling Problem and the Robotic Flow-shop Problem have been made by Mangione (2003) and he tried to apply the Gilmore and Gomory algorithm to solve the problem for small lines: with three soaking times and two part-jobs.

Many other studies deal with scheduling of the other production lines or even with the CHSP with relaxed constraints: mainly bounded processing times and no-wait constraints; which are problems near the CHSP (Che et al.

2009, El Amraoui et al. 2008, Fleury et al. 2001, Kats et al. 2006, Liu et al. 2002, Song et al. 1995, Deroussi et al. 2008).

Section 2, presents the problem to solve. A mathematical analyse is described in section 3. In section 4, heuristic algorithm is given and applied to 2-cyclic examples. In Section 5, the general heuristic algorithm integrating Minimum Part Set (MPS) configurations' is presented and illustrated by computational results. And section 6 concludes the paper.

2. THE PROBLEM TO SOLVE

In this study, we consider the single hoist, multi-parts jobs problem. Parts are to be treated in equal quantities, starting by being loaded from the loading station, and then being successively soaked in *m* tanks, before being unloaded in the unloaded station. The processing time of each part is confined within a minimum and a maximum duration. And any delay in this time window can make the job defective. The transport of jobs between stages is ensured by a transport resource, called hoist.

One of the main specificities of such a system is that the transport durations cannot be ignored as their values are similar to the processing times. Moreover, hoist is the most critical resource of such lines. Besides, there is no buffer between stages.

Fig. 1 shows an example of electroplating line with a single hoist.

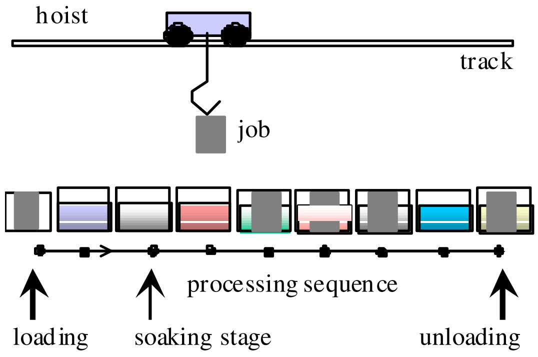

Fig. 1. Example of an electroplating line.

This problem can be considered as a flow shop scheduling problem with very specific constraints. The constraints we consider here are the following ones:

(C1) A hoist can move only one part at a time.

(C2) Between two successive moves in the sequence, hoist must have enough time to travel empty.

(C3) Each tank can receive at most one part job, at the same time.

(C4) In each tank, parts must remain at least a minimal time and at most a maximal duration, that vary from one tank to another due to chemical constraints.

3. MATHEMATICAL FORMULATION

We define the following notation.

Parameters:

np	the total number of jobs to be soaked.
nt^k	the number of processing stages for the k^{th} job, k=1, ..., np.
O_i^k	the i^{th} hoist move operation of job k, k=1, ..., np.
a_i^k	the minimum processing time in soaking i for job k, i = 0,.., nt^k, k=1, ..., np.
b_i^k	the maximum processing time in soaking i for job k, i = 0,.., nt^k, k=1, ..., np.
d_i^k	the time needed for a hoist to move a job k from the tank of the i^{th} soaking stage to the following one on its processing sequence. i = 0,.., nt^k, k=1, ..., np.
$c_{i,j}^{k,p}$	the time needed for a hoist to move empty from the tank of the i^{th} soaking stage of job k to the tank of the j^{th} soaking stage of job p.

Variables:

SH_i^k	start time of the hoist move from in load from the i^{th} soaking stage of job k, i = 0,.., nt^k, k=1, ..., np.
EH_i^k	end time of the hoist move from in load from the i^{th} soaking stage of job k, i = 0,.., nt^k, k=1, ..., np.
SS_i^k	start time of the soaking operation of the job k on its i^{th} soaking stage, i = 0,.., nt^k, k=1, ..., np.
ES_i^k	end time of the soaking operation of the job k on its i^{th} soaking stage, i = 0,.., nt^k, k=1, ..., np.
o_i^k	soaking duration of the job k on its i^{th} soaking stage, i = 0,.., nt^k, k=1, ..., np.
T	the cycle time for the multi-products problem.

One of the main particularities of such a system is that the hoist is not allowed to wait in load and there is no buffer between soaking stages. This means that the end of a hoist move in load is matching the start of the soaking operation and the end of a soaking operation is matching the start of a hoist loaded move (to move a job from one soaking tank to the following one in its processing sequence) (as seen in Fig.2). That is to say scheduling the soaking operations is the same as scheduling the hoist moves in load. And this can be traduced by the following equalities.

$$SH_i^k = ES_i^k. \tag{1}$$

$$ES_i^k = SS_i^k + o_i^k. \tag{2}$$

$$EH_i^k = SS_{i+1}^k. \tag{3}$$

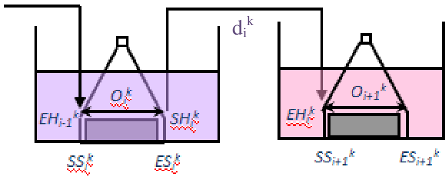

Fig. 2. Example of a hoist move in load.

In this problem there are a total of N variables to find. These variables include the cycle time duration and the hoist move sequence as well as the job soaking sequence.

$$N = 1 + \sum_{k=1}^{np} 2.nt^k. \tag{4}$$

Nevertheless, finding the hoist move sequence means in other terms finding the job soaking sequence, as analyzed above. And thus, the total number of variables can be reduced to N':

$$N' = 1 + \sum_{k=1}^{np} nt^k. \tag{5}$$

To our knowledge, many studies on cyclic hoist scheduling problems with time windows constraints deal with exact solving method. But most of them are dedicated to identical part problems and used to solve small size instances (lines with less than 10 tanks). In the following section, we introduce a new heuristic approach that handles the problem with distinct multi-parts jobs.

4. METHODOLOGY DESCRIPTION

4.1 Cyclic Multi Parts Earliest Starting Time heuristic

Song et al. (1995) proposed a heuristic approach for the mono-product CHSP. This heuristic is an adaptation for the Yih (1994) heuristic to the cyclic problem. In order to complete a job as early as possible, authors start each soaking task as early as possible. They call this iterative principle as Earliest Starting Time (EST). In this heuristic, authors schedule the hoist operations for the first job then the following ones one by one. But if it turns out that a no-wait condition has been violated, the procedure re-schedule the entire job.

The procedure guarantees the generation of a feasible schedule but, flexible processing times are not taken into account by "EST". And to simplify the model, authors fixed the processing times as equal to the minimum times required. Which can prevent them to reach the optimal solution.

In order to take advantage of flexible processing time for the cyclic multi-items hoist scheduling problem and to reduce the gap between the time duration, provided by the "EST" heuristic and the optimal duration we propose a new heuristic labelled Cyclic Multi-Parts Earliest Starting Time (CMPEST).

The originality of this new heuristic is that the cycle degree is not a parameter but a variable of the model. This last defines the number of jobs to be introduced and extracted from the electroplating line during each cycle. For example, in a two-cyclic scheduling, exactly two jobs enter the line and two jobs leave it. The main idea of this scheduling heuristic is to set a cyclic list for the job to be introduced into the line (i.e., A-B/A-B/…/A-B). This list is closely related to the Minimum Part Set (MPS) configuration and the quotient of each job in this considered MPS. To find the best cyclic list minimizing the means optimal cycle time is not so easy and the procedure of how to get it will be presented in section 5.

We used the Earliest Starting Time (EST) as the iterative principle. When all of these jobs in the cyclic list are scheduled, the permanent smallest pattern of hoist moves in load will define the cycle degree.

The principle of the algorithm we propose can be resumed as follow: in each step a list of options is proposed according to an insertion procedure and each one is checked; then only one is picked to be scheduled.

- *The general algorithm for the "CMPEST" heuristic*

Select the first hoist move of job 1 from tank 0 to tank 1as the first hoist move to be scheduled.

```
While (all the hoist moves in load are not yet scheduled)
    Procedure to move to the iteration $k+1$.
    Procedure to insert new options to the list.
    Sort the options in the list $L^{k+1}$ by the starting hoist move time from lower to higher.
    Select the first option from the sorted list.
    If (the tank destination of the option is free)
        If (time window constraint is respected)
        Then the first option is picked for scheduling
              Else
                While (no solution found)
                Backtrack to the following list $L^k$ and select the following option on the list.
                End While
                Else
                Select the following option on the list $L^{k+1}$.
        End If
    End If
End While
```

The procedures used by the general algorithm are described as follows:

- *Procedure to move to the iteration k+1*

- the option list L^k is paste in L^{k+1}.
- the O_i^j , the i^{th} hoist move of job j, is moved from the L^k list.
- the starting time SH_k^p for each option O_k^p in L^k is to be updated by: $\max\{SH_k{}^p; EH_i{}^j + c_{i+1,k}{}^{j,p}\}$. (6)

- *Procedure to insert new options to the list*
  (depends on the O_i^j option selected in the following iteration)

- if the hoist move operation O_i^j is not the last operation for job j, then insert the operation $O_{i+1}{}^j$ in the list L^{k+1} and its starting time is computed by: $SH_{i+1}{}^j = EH_i{}^j + a_{i+1}{}^j$. (7)
- if the hoist move operation O_i^j is the first operation for job j (i=0), then insert, in the list L^{k+1}, the $O_{i+1}{}^j$ and the first operation of the following job in the cyclic list ($O_0{}^{j+1}$). Then starting times are computed as follows:

$$SH_{i+1}{}^j = EH_i{}^j + a_{i+1}{}^j. \tag{8}$$

$$SH_{i+1}{}^j = EH_i{}^j + c_{i+1,0}{}^{j,j+1}. \tag{9}$$

- *Procedure to find the best cyclic sequence*

\- When all the hoist moves in load are scheduled, then a cyclic sequence, defined by a parameter *n* (called degree), is to find.

The best cyclic sequence to find is the one which minimizes the parameter *n*, and where during each one cycle, exactly *n* parts enter the line and *n* parts leave it. We define the mean cycle time as the cycle time divided by *n*.

4.2 Computational results

In order to illustrate the heuristic performance, we use instances found in literature and proposed by Mateo. In fact, few benchmarks are available related to Hoist Scheduling Problems and mainly for cyclic problems with single part jobs. In Mateo (2001), one can find tables providing the data associated with each studied benchmarks (the time unit is the second (s)). Those ones correspond to facilities composed of 5, 6, 7, 8, 9 and 10 tanks (without the load and unload stations), for 2-heterogenous part jobs.

Table 1 presents some results obtained for the linear model, the "EST" heuristic and the "CMPEST" heuristic. These experiments show how we have improved the mean value of the cycle duration for cyclic sequence with degrees upper than two.

Table 1. Results: linear model, "EST" heuristic and "CMPEST" heuristic

Example Reference	Optimal Solution			"EST" Solution			"CMPEST" Solution		
(Mateo 01)	*n*	*T*	*T/n*	*n*	*T*	*T/n*	*n*	*T*	*T/n*
117071706	2	362	181	2	746	373	4	700	175
125043007	2	653	326,5	2	1115	557,5	6	1868	311
137073308	2	583	291,5	2	1157	578,5	10	2858	285
140043108	2	585	292,5	2	1077	538,5	6	1694	282
256085510	2	756	378	2	1410	705	6	2128	354

The simulations presented as follows are the results of more than 180 benchmarks and with various soaking time windows and hoist speed. Table 2 shows the gap (%) between the solutions found by the "EST" heuristic (respectively the "CMPEST" heuristic) and the mean optimal cycle time. We spend less than one second to solve the problems instances with 5 to 10 tanks, using C++ software, on a Pentium 4 with 3 Ghz frequency processor. About 14 minutes are necessary to solve our linear model to optimality, using Cplex software.

Table 2. Results : gap (%) between "EST" heuristic, "CMPEST" heuristic and the means optimal cycle time (C.P.U.<1s)

Gap (%)	Tanks					
	5	6	7	8	9	10
“EST“ vs means optimal cycle time	92,6	86,5	97,6	85,2	75,9	77,4
“CMPEST“ vs optimal cycle time	8,7	9,1	8,3	7,1	6,4	5,4

These results seem to be interesting in terms of performance. Moreover, we can notice two important effects. Firstly, the Gap (%) between the solutions found by the "EST" heuristic and the means optimal cycle time is not proportional to the number of tanks. Secondly, the mean gap for the six test series is about 86% for the "EST" heuristic while it is less than 8% for the "CMPEST" heuristic.

5. GENERAL ALGORITHM FOR MULTI-ITEMS CHSP

During the process of scheduling, whenever a first operation of a job is scheduled, the question arises as to which kind of job first operation to consider next as a new option. Finding an optimal cycle time for the multi-products problem involves two types of sequencing: one is hoist move sequencing and the other is job sequencing.

The first one is ensured by the heuristic steps while the second one is ensured by a new technique as detailed in this part. In each operation cycle, one Minimum Part Set (MPS) is at least produced. An example of such a MPS is {A, B, C}, which consists of one job of type A, one job of type B and one job of type C. The MPS is supported to be repeated on the line. The considered procedure consists in scheduling all the possible mixing of products. We define a configuration $C_{x1, \ldots, xk}$ as a possible state of how to start introducing part on the line during the cycle process. In our example, one configuration could be $C_{A,B,C}$ or $C_{A,C,B}$. When an MPS is considered, the best solution of the multi-products cyclic HSP can be obtained by generating a set of MPS configurations' (the cyclic list of jobs to be introduced into the line); and then a schedule is computed for each considered configuration by applying the heuristic steps. The general algorithm steps are reported in fig. 3.

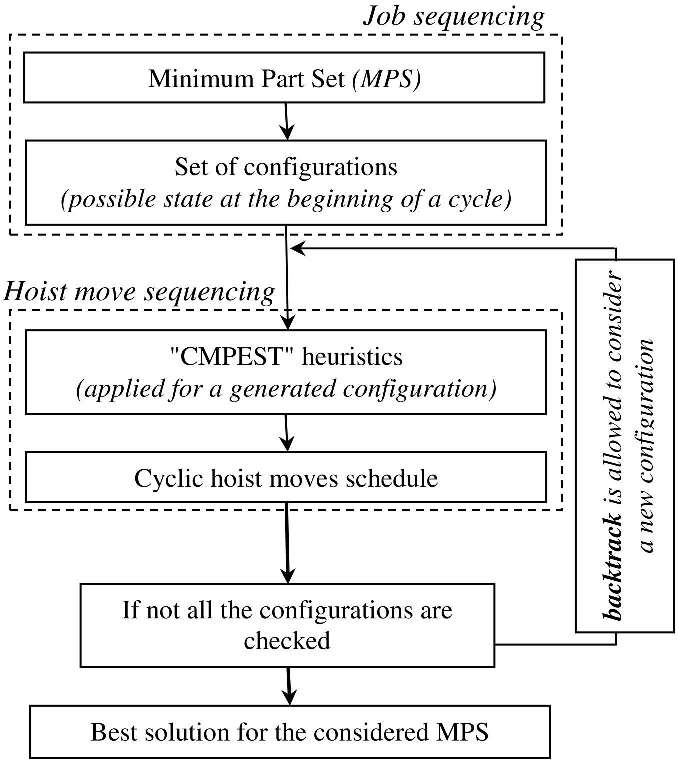

Fig. 3. The elaborated scheduling procedure.

To show the performances of the procedure, we use instances found in literature and proposed by Mateo (2001). These instances are adjusted to an MPS with three different part jobs and one job for each kind.

Table 3 presents the results (cycle degree: *n* and cycle time durations: $TC_{A,B,C}$ and $TC_{A,C,B}$) for six line instances composed of 5, 6, 7, 8, 9 and 10 tanks (without the load and unload stations) for two different configurations $C_{A,B,C}$ and $C_{A,C,B}$. For more details, instances are reported in appendix A.

Table 3. Results : "CMPEST" solutions for two different MPS configurations'

Configuration	Tanks					
$C_{A,B,C}$ - $C_{A,C,B}$	5	6	7	8	9	10
n	3	3	12	12	6	3
$TC_{A,B,C}$	616	587	2924	2735	2509	1117
$TC_{A,C,B}$	603	563	2958	2584	2516	1107
Reduction (%)	2.1	4.1	1.1	5.5	0.3	0.9

These results show that by considering different configurations of the MPS, we can relatively improve the cycle time duration and then the throughput rate of the production line.

6. CONCLUSION

In the aim to optimize the cycle time and to reduce the simulation time for a Cyclic Hoist Scheduling Problem, we have proposed an original heuristic approach for the multi-items case. The obtained results show the effectiveness of the proposed approach. In fact, by using the flexibility provided by the non fixed processing time, we show how we can obtain very interesting solutions for a cycle degree upper than 2 with 2-cyclic heterogeneous benchmarks. Moreover, using the non-fixed processing time can help us to reduce considerably the cycle duration. Despite the cycle degree of the schedule is not fixed in advance, the elaborated heuristic approach provides very good results compared with exact one. To minimize the means optimal cycle time, a set of Minimum Part Set (MPS) configurations' are generated and checked in the heuristic general algorithm.

Future work is to improve the performance of the CMPEST algorithm and extend it to multi-hoists cyclic problem.

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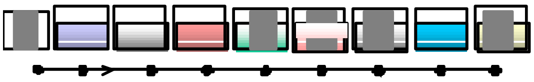
Appendix A



Job	Soaking tanks
Number	Soaking time windows
Hoist move in (load; unload) between 2 consecutive tanks	

Job	tank1	tank2	tank3	tank4	tank5
1	50-114	80-184	60-145	80-191	28-67
2	49-126	71-178	54-152	53-159	56-166
3	77-184	57-142	67-160	45-105	27-65
12; 8					*(5 tanks benchmark)*

Job	tank1	tank2	tank3	tank4	tank5	tank6	tank7	tank8
1	36-87	33-78	35-85	37-91	38-86	41-100	46-107	40-93
2	63-173	49-140	72-203	53-138	46-132	76-199	40-116	24-65
3	60-142	67-147	53-125	32-80	35-86	29-68	45-108	26-64
7; 5								*(8 tanks benchmark)*

Job	tank1	tank2	tank3	tank4	tank5	tank6
1	30-70	48-119	70-155	45-105	24-57	72-170
2	73-212	38-113	69-183	23-59	64-179	49-131
3	30-70	59-144	44-108	45-109	24-57	32-76
9; 6						*(6 tanks benchmark)*

Job	tank1	tank2	tank3	tank4	tank5	tank6	tank7
1	75-173	67-156	80-183	65-152	45-99	71-169	75-184
2	54-150	49-127	76-220	24-71	57-152	67-189	29-75
3	41-409	67-731	34-331	30-150	50-205	34-213	30-329
10; 7							*(7 tanks benchmark)*

Job	tank1	tank2	tank3	tank4	tank5	tank6	tank7	tank8	tank9
1	42-100	66-152	36-87	21-50	29-68	32-74	68-168	60-146	36-85
2	65-184	65-189	76-192	21-53	38-113	56-160	49-128	51-152	22-56
3	40-122	40-122	40-122	40-122	40-122	40-122	40-122	40-122	40-122
13; 9									*(9 tanks benchmark)*

Job	tank1	tank2	tank3	tank4	tank5	tank6	tank7	tank8	tank9	tank10
1	42-98	58-127	36-82	70-170	22-55	46-112	71-158	44-108	72-173	21-47
2	78-214	31-90	49-141	33-88	32-93	74-197	44-111	69-202	37-97	65-174
3	73-167	45-104	40-88	34-75	62-140	33-73	33-76	50-120	41-97	46-110
9; 6										*(10 tanks benchmark)*