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Reference Tracking Controller Applied to the Operation of Hydrothermal Systems ⋆

Luiz Henrique B. Bertolucci * **Eduardo F. Costa** *

* *Department of Applied Mathematics and Statistics/ICMC/USP, São Carlos, Brazil (e-mail: bertolucci, efcosta@icmc.usp.br).*

Abstract: this paper considers the long term operation planning of hydrothermal systems modelled by a Markov jump system, using reference tracking controller. The reference is obtained via optimization of a certain cost functional in a deterministic setup, using deterministic dynamic programming. The control is designed employing a linear Markov jump model and quadratic costs. For selecting the quadratic cost weighting matrices, we propose a scheme based on Monte Carlo simulation of the system with the deterministic dynamic programming solution. We present some preliminary case studies suggesting that the reference tracking controller represent an interesting alternative to the deterministic or stochastic dynamic programming solutions.

1. INTRODUCTION

The hydroelectric operation planning of a multi-plant system means to define rules for water release of each reservoir based, mainly, on the amount of water stored in the system, the water inflow behavior and the considered cost functional. This is a highly complex task since it involves many constraints on the reservoirs capacities and releases, stochastic variables (mainly considering the random water inflow) and nonlinear equations. During the last fifty years, many methods have been proposed to deal with this problem, among which the linear programming, nonlinear programming and dynamic programming are highlighted. An interesting survey on the main developments in this area can be found in Yakowitz [1982], Yeh [1985], Wurbs [1996], Labadie [2004], Castelletti et al. [2008].

Although the Linear Quadratic Control (LQC) is commonly used in many applications, there are few works employing it in the hydroelectric operation planning, e.g. Wasimi and Kitanidis [1983], McLaughlin and Velasco [1990], Özelkan et al. [1997]. In this paper we consider a LQC based on a Markov jump parameters model, featuring easy to find analytical solution (do Val and Basar [1999]) and allowing for a more complex, realistic model for the water flow. The use of LQC in this application has been questioned due to the linear model that is required (Castelletti et al. [2008]), which may yield solutions that do not satisfy the constraints, however, some simple heuristics may circumvent this difficulty with little reflex on the cost. Another drawback of the LQC is that there is no definitive, meaningful rule to select the weighting matrices of the quadratic cost.

In this paper we address the long term planning of hydrothermal systems Soares and Carneiro [1991] using linear quadratic reference tracking (LQRT) control. The reference is obtained via optimization of the energy complementation cost Barros et al. [2003] in a deterministic setup, using deterministic dynamic programming (DDP) (Bertsekas [1995]). The LQRT control is then designed employing a linear Markov jump model for the hydroelectric system and quadratic costs. We also propose a scheme for selecting the weighting matrices based on Monte Carlo simulation of the system with the deterministic dynamic programming solution, see Section 2.2 for the details. The performance of the LQRT control is compared (via case studies) to the ones of the constrained deterministic and stochastic dynamic programming (SDP) solutions in terms of average energy complementation and of generated power, suggesting that the LQRT control may be a viable alternative solution to the long term planning problem.

2. PRELIMINARIES RESULTS

2.1 The hydrothermal planning problem

We consider the following model for the hydroelectric system with N plants,

$$x_i(k+1) = x_i(k) - r_i(k) + \sum_{j\in\Omega_i} r_j(k) + v_i(k) - l_i(k) \quad (1)$$

with $x_i(0) = x_{0,i}$, where $x_i(k)$ represents the stored water volume at reservoir i at the beginning of k-interval, $r_i(k)$ stands for the released volume during the k-interval, $v_i(k)$ is the incremental natural water inflow and $l_i(k)$ is the evaporation loss. We assume that the i-reservoir is supplied by reservoirs $j \in \Omega_i$. The evaporation loss is modelled by

$$l_i(k) = h_i(k)\, a_i(x_i(k)), \quad (2)$$

where $h_i(k)$ is the net mean evaporation rate and $a_i(x)$ describes the surface area of i-th reservoir when x is the stored water volume. We assume the use of spillage to force $x_i(k) \le x_i^{max}(k)$, which leads to

$$r_i(k) = max\Big(u_i(k)\,,\, x_i(k) + \sum_{j\in\Omega_i} r_j(k) + v_i(k) - l_i(k) - x_i^{max}(k)\Big), \quad (3)$$

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where $u_i(k)$ stands for water turbine volume inflow.

We consider the energy complementation cost (Barros et al. [2003]), for a time horizon $T < \infty$, given by

$$\mathcal{W}_u^T(x_0) = \sum_{k=0}^{T} \left[D(k) - \sum_{i=1}^{N} P_i(k) \right]^2 , \qquad (4)$$

where $D(k)$ is the mean power demand to be supplied in the period k and $P_i(k)$ is the mean power generated by hydroelectric plant i,

$$P_i(k) = K_i \Big(\varphi_i(x_i(k)) - \mu_i(r_i(k)) \Big) u_i(k) \qquad (5)$$

where K_i is an efficiency factor for the $i-th$ plant, and φ_i and μ_i are polynomials that convert volumes (of the reservoir and tail, respectively) to heights. The long term planning problem can be formulated as

$$\begin{aligned} W = &\inf_u E\{W_u^T(x_0)\} \\ s.t.: \quad & (1,3) \\ & u_i^{min}(k) \le u_i(k) \le u_i^{max}(k) \\ & x_i(k) \ge x_i^{min}(k), \quad i = 1, ..., N \end{aligned} \qquad (6)$$

where $E\{\,.\,\}$ is the expected value and $u_i^{min}(k)$, $u_i^{max}(k)$ and $x_i^{min}(k)$ are scalar constraints. When we consider (6) with a deterministic model for $v_i(k)$, we denote the optimization problem by $\mathbf{P}_d$, its solution by $u_i^d(k)$ and the associated state trajectory by $x_i^d(k)$. Solving $\mathbf{P}_d$ using DDP we obtain a feedback control law in the form $u^d\big(k, x_1(k), \ldots, x_N(k)\big)$. If a stochastic model is taken into account for v (we have used a Markov jump model), the problem is denoted by $\mathbf{P}_s$. Following the SDP approach to solve $\mathbf{P}_s$ and adding the water inflow into the state vector we get a feedback solution in the form $u^s\big(k, x_1(k), \ldots, x_N(k), v_1(k-1), \ldots, v_N(k-1)\big)$.

2.2 The linear quadratic reference tracking controller

The LQRT control aims at minimizing the following quantity,

$$\begin{aligned} z(k) = &\Big(x(k) - x^d(k)\Big)' Q(k) \Big(x(k) - x^d(k)\Big) \\ &+ \Big(u(k) - u^d(k)\Big)' R(k) \Big(u(k) - u^d(k)\Big) \end{aligned}$$

where $Q(k) \in R^{N\times N}$ and $R(k) \in R^{N\times N}$ are the weighting matrices, and the target $x^d(k) = [x_1^d(k), \ldots, x_N^d(k)]' \in R^N$ and $u^d(k) = [u_1^d(k), \ldots, u_N^d(k)]' \in R^N$ are given. $x(k) \in R^N$ represents the stored water in the reservoirs and $u(k) \in R^N$ stands for the water turbine inflows; employing the Markov chain model for $v_i(k)$ proposed by Karamouz and Vasiliadis [1992], considering $a_i(\cdot)$ as a linear function, and assuming $r_i(k) = u_i(k)$, $k \ge 0$, $i = 1, \ldots, N$, we obtain

$$x(k+1) = A(k)x(k) + B(k)u(k) + e_{\theta(k)}(k) \qquad (7)$$

where $\theta(k)$ is the state of a periodic Markov chain and $e_{\theta(k)}(k) \in R^N$ is related to the water inflow and evaporation. The LQRT controller is defined, for a time horizon $H < \infty$, as

$$\begin{aligned} u^\star(k) = &\arg\inf_u E\left\{ \sum_{k=0}^{H} z(k) \right\} \\ s.t.: \quad & (7). \end{aligned} \qquad (8)$$

In this paper, $x^d(k)$ and $u^d(k)$, $0 \le k \le T$, are obtained by solving the problem $\mathbf{P}_d$; in accordance, we set $H \le T$. In fact, by taking $T > H$ in $\mathbf{P}_d$ can be regarded as a "financial valued" manner of including the stored water volume at the end of the planning period in the objective function; to be more precise, by Bellman's Principle of Optimality,

$$W = \inf_u E\{W_u^T(x_0)\} = \inf_u E\{W_u^H(x_0) + V(x(H+1))\}$$

where $V(x) = \inf_u E\{W_u^{T-H}(x))$, the optimal cost for the next $T-H$ stages, plays the role of a terminal penalization.

An analytic solution for $u^\star(k)$ can be found in [do Val and Basar, 1999, theorem 14], assuming $\theta(k)$ is observed. In this application, $\theta(k-1)$ is available at time instant k and we consider the control $u(k) = E\{u^\star(k)|\theta(k-1)\}$.

Following Wasimi and Kitanidis [1983], Özelkan et al. [1997], we consider diagonal weighting matrices Q and R, and its elements have been selected as we explain next. Let w be an element of the Markov chain probability space in such a manner that $\theta(0, w), \ldots, \theta(H, w)$ is a realization of the Markov chain. For each realization w we employ the feedback control $u^d(\cdot, \cdot)$ (obtained via DDP) to obtain the (state) trajectory realization $x_i^d(k, w)$. Employing Monte Carlo simulation we obtain M realizations $w_1, \ldots, w_M$ and we set, for $0 \le k \le H$ and $i = 1, \ldots, N$,

$$Q_{ii}(k) = \sum_{j=1}^{M} M^{-1} \left(x_i^d(k, w_j) - x_i^d(k) \right)^{-2}$$

and

$$R_{ii}(k) = \sum_{j=1}^{M} M^{-1} \left(u_i^d(k, x^d(k, w_j)) - u_i^d(k) \right)^{-2} .$$

The basic idea is to associate large weights with variables whose realizations present small variations from the target $x_i^d(k)$ or $u_i^d(k)$.

3. STUDY CASE

The Sao Francisco river is considered one of most important rivers in Brazil. The proposed LQRT controller have been applied to the hydroelectric power plants lying in Sobradinho and Itaparica, along this river. All required data to perform this study can be obtained from the official websites of the company responsible for the reservoirs (www.furnas.com.br), but also in brazilian National Operating Agency (www.ons.org.br). In our model, we consider monthly temporal discretization, e.g. $x(k)$ stands for the water storage at the $k-$th month. We have solved the problem $\mathbf{P}_d$ using DDP with time horizon $T = 108$, and obtained $u^\star(k)$ solution of (8) with time horizon $H = 60$. Figures 1 and 2 show the target $x_1^d(k)$ and the average stored water in one of the hydroelectric plants operated with optimal controls obtained via $\mathbf{P}_s$, $\mathbf{P}_d$ and with the LQRT, denoted respectively by $x_1^{SDP}(k)$, $x_1^{DDP}(k)$ and $x_1^{LQRT}(k)$.

We compare the controller performances in two different scenarios, (I) taking into account the evaporation phenomenon and (II) zero evaporation. The main results are presented in Table 1 and 2, obtained via Monte Carlo simulation with $M = 3000$ realizations. We denote $J_{DDP} = W_{u^d}^T(x_0)$ (cost incurred by the feedback control $u^d(\cdot, \cdot)$,

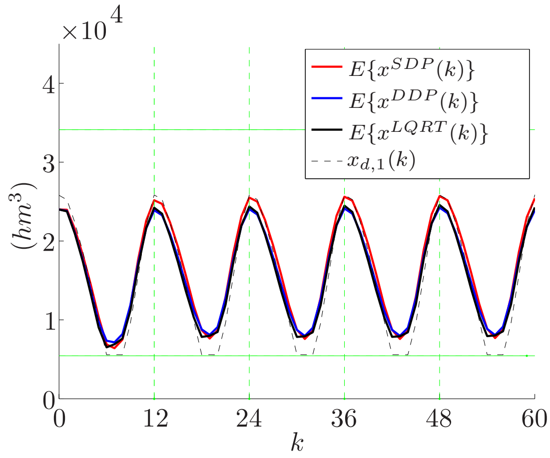

Figure 1. Average storages in Sobradinho - scenario I

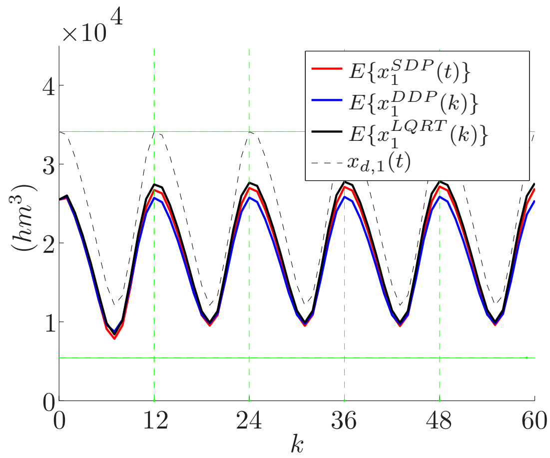

Figure 2. Average storages in Sobradinho - scenario II

a random variable), $J_{SDP} = W_{u^s}^T(x_0)$ (cost incurred by $u^s(\cdot,\cdot)$) $J_{LQRT} = W_u^T(x_0)$ (cost incurred by the LQRT control); similarly, P_{DDP}, P_{SDP} and P_{LQRT} stand for the power generated using the respective controls; $\mathrm{cov}(\cdot)$ stands for the covariance of a random variable.

Table 1. Relative performance - scenario (I)

$\frac{E\{J_{LQRT}\}}{E\{J_{SDP}\}} = 1.0001$	$\frac{E\{J_{LQRT}\}}{E\{J_{DDP}\}} = 0.9783$
$\frac{E\{P_{LQRT}\}}{E\{P_{SDP}\}} = 1.0145$	$\frac{E\{P_{LQRT}\}}{E\{P_{DDP}\}} = 1.0163$
$\frac{cov\{J_{LQRT}\}}{cov\{J_{SDP}\}} = 1.1478$	$\frac{cov\{J_{LQRT}\}}{cov\{J_{DDP}\}} = 1.1016$
$\frac{cov\{P_{LQRT}\}}{cov\{P_{SDP}\}} = 1.0562$	$\frac{cov\{P_{LQRT}\}}{cov\{P_{DDP}\}} = 1.1319$

Table 2. Relative performance - scenario (II)

$\frac{E\{J_{LQRT}\}}{E\{J_{SDP}\}} = 1.0069$	$\frac{E\{J_{LQRT}\}}{E\{J_{DDP}\}} = 0.9776$
$\frac{E\{P_{LQRT}\}}{E\{P_{SDP}\}} = 1.0006$	$\frac{E\{P_{LQRT}\}}{E\{P_{DDP}\}} = 1.0019$
$\frac{cov\{J_{LQRT}\}}{cov\{J_{SDP}\}} = 0.9751$	$\frac{cov\{J_{LQRT}\}}{cov\{J_{DDP}\}} = 0.8971$
$\frac{cov\{P_{LQRT}\}}{cov\{P_{SDP}\}} = 0.9456$	$\frac{cov\{P_{LQRT}\}}{cov\{P_{DDP}\}} = 0.9418$

4. CONCLUSION

We have investigated the use of a LQRT control for the operation of hydrothermal systems, based on the DDP solution, and we have proposed a scheme for selecting the weighting matrices of the controller. The performance of the LQRT control was compared to the ones of the DDP and SDP controls via Monte Carlo simulation in a real system, suggesting that the performance is better than the DDP and similar to the SDP, whereas the computational burden is similar to the one of DDP. Future work will look into the sensitivity of the weighting matrices, as well as include massive simulation for different systems and scenarios.

REFERENCES

M. T. L. Barros, Frank T-C. Tsai, S. Yang, J. E. G. Lopes, and W. W-G. Yeh. Optimization of large-scale hydropower system operations. *Journal of Water Resources Planning and Management*, 129(3):178–188, 2003.

D. P. Bertsekas. *Dynamic Programming and Optimal Control*. Athena Scientific, 1995.

A. Castelletti, F. Pianosi, and R. Soncini-Sessa. Water reservoir control under economic, social and environmental constraints. *Automatica*, 44(6):1595 – 1607, 2008.

J. B. R. do Val and T. Basar. Receding horizon control of jump linear systems and a macroeconomic policy problem. *Journal of Economic Dynamics and Control*, 23:1099–1131, 1999.

M. Karamouz and H. V. Vasiliadis. Bayesian stochastic optimization of reservoir operation using uncertain forecasts. *Water Resour. Res.*, 28(5):1221–1232, 1992.

J. W. Labadie. Optimal operation of multireservoir systems: State-of-the-art review. *Journal of Water Resources Planning and Management*, 130(2):93–111, 2004.

D. McLaughlin and H. L. Velasco. Real-time control of a system of large hydropower reservoirs. *Water Resour. Res.*, 26(4):623–635, 1990.

E. C. Özelkan, Á. Galambosi, E. Fernndez-Gaucherand, and L. Duckstein. Linear quadratic dynamic programming for water reservoir management. *Applied Mathematical Modelling*, 21(9):591 – 598, 1997.

S. Soares and A.A.F.M. Carneiro. Optimal Operation of Reservoirs for Electric Generation. *Power Delivery, IEEE Transactions on*, 6(3):1101 –1107, 1991. ISSN 0885-8977. 10.1109/61.85854.

S. A. Wasimi and P. K. Kitanidis. Real-time forecasting and daily operation of a multireservoir system during floods by linear quadratic gaussian control. *Water Resour. Res.*, 19(6):1511–1522, 1983.

R. Wurbs. *Modeling and analysis of reservoir system operations*. Prentice-Hall, 1996.

S. Yakowitz. Dynamic-programming applications in water-resources. *Water Resour. Res.*, 18(4):673–696, 1982.

W. W. G. Yeh. Reservoir management and operations models: A state-of-the-art review. *Water Resources Research*, 21(12):1797–1818, 1985.