Experimenting with the Modified Filtered Smith Predictors for FOPDT Plants

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Abstract: This paper deals with experimental verification of new modified Filtered Smith Predictor with primary 2DOF P-controller that simplifies treatment of the control constraints. Testing the traditional and the new solutions with different disturbance filters by real time experiments on a stable plant fully confirms superiority of the new controllers in constrained control. It also shows that despite to the formal input-to-output equivalence of the new and the traditional solutions, due to different implementation schemes and the plant-model mismatch they give responses to the setpoint and disturbance steps that are slightly different in terms of the speed, monotonicity, as well as noise attenuation. As a by-product of this paper it is shown that both the new and the traditional solutions with the 2nd order disturbance filter are much more sensitive than those with the 1st order one, what may be crucial for their practical use.

1. INTRODUCTION

The Smith Predictor (SP) (Smith, 1957) represents one of the oldest structures of the Dead-Time Compensators (DTCs). Despite the vast amount of available literature (see e.g. Åström and Hägglund, 2005; Guzmán et al., 2008; Normey-Rico and Camacho, 2007; 2008; 2009; Normey-Rico et al., 2009; Tan et al., 2010; Zhang, Rieber and Gu, 2008), from which the related problem may seem to be fully mastered, experimental work with DTCs may still bring several implementation problems.

The first possible problem is related to control saturation that in combination with the PI control used in the primary loop lead to the windup effect (Zhang and Jiang, 2008). So, this complex controller with complicated tuning has yet to be complicated by anti-windup. But, is the windup problem really imperative? Despite that just few of the known DTCs are interpreted as disturbance observer (DO) based structures, in fact, all of them may be shown to include DO for reconstruction of either input or output disturbances that are then compensated by modifying output or input of the primary controller. Due to this the primary controller need not to include the integral action producing windup. The Filtered Smith Predictor (Normey-Rico and Camacho, 2007; 2008; 2009; Normey-Rico et al., 2009) using the parallel plant model (PPM) is based on reconstruction of the output disturbance. Here, it will be shown that it may be simplified by replacing the primary PI control by a 2DOF P-controller, what reasonably simplifies its use in constrained control without decreasing its capability in the disturbance rejection. Simultaneously, the paper is exploring properties achieved by two different DO filters decoupling the loop dynamics.

The paper is structured as follows: Section 2 gives a brief overview of the FSP design for stable First Order Plus Dead Time (FOPDT) plants. New DTC with simplified primary loop enabling constrained control is described in Section 3. Comparison of presented solutions by real time experiments for the relatively short and long dead times is reported in Sections 4 and 5. Contributions of the paper and potential for further development are summarized in Conclusions.

2. THE FSP FOR FOPDT PLANTS

The FSP may be interpreted as the dynamical feedforward control with reference plant model (Åström and Hägglund, 2005; Visioli, 2006), or as the 2DOF IMC structure.

ACRONYMS	
2DOF	Two Degree of Freedom
DO	Disturbance Observer
DTC	Dead-Time Compensator
FOPDT	First Order Plus Dead Time
PI-F1SP, PI-F2SP	Filtered Smith Predictors with PI-controller and the 1st, or the 2nd order disturbance filters
P-F1SP, P-F2SP	Filtered Smith Predictors with P-controller and the 1st, or the 2nd order disturbance filters
IAE	Integrated Absolute Error
IMC	Internal Model Control
PPM	Parallel Plant Model
SP	Smith Predictor
TV	Total Variance

The unified approach to designing FSPs for the FOPDT plants considers compensation of an output disturbance by correction of the reference value, whereby the disturbance is reconstructed by using the PPM. For stable plants $P(s)$, $P_0(s)$ is denoting its "fast" delay-free nominal dynamics and $P_n(s)$ its nominal model with particular set of parameters considered in controller tuning

$$P(s)=\frac{Ke^{-\Theta s}}{Ts+1}\ ;\ P_0(s)=\frac{K_0}{T_0 s+1};\ P_n(s)=\frac{K_0 e^{-\Theta_0 s}}{T_0 s+1} \tag{1}$$

Usually, the primary PI-controller

$$C_0(s)=\frac{U(s)}{E(s)}=K_c\frac{1+T_is}{T_is} \quad ; T_i=T_0 \; ; \; K_c=\frac{T_0}{T_rK_0} \tag{2}$$

is used, whereby T_r is the time constant of the (fast) primary loop described by the transfer function

$$C(s)=\frac{U(s)}{E(s)}=\frac{C_0}{1+C_0P_0}=\frac{1}{K_0}\frac{1+T_0s}{1+T_rs} \tag{3}$$

The nominal setpoint-to-output transfer function considering $P=P_n$ is

$$H_{rn}(s)=\frac{Y(s)}{R(s)}=C(s)P(s)=\frac{e^{-\Theta s}}{1+T_rs} \tag{4}$$

When extending the DO by the 1st and 2nd order filters

$$F_{r11}(s)=\frac{1+\beta_{11}s}{1+T_fs} \; ; \; F_{r22}(s)=\frac{(1+T_rs)(1+\beta_{22}s)}{(1+T_fs)^2} \tag{5}$$

where T_f represents their time constant, the loop may be simplified by introducing equivalent controller

$$C_e(s)=C/(1-CPF_r) \tag{6}$$

The loop transfer functions corresponding to the output disturbance d_o, to the input disturbance d_i and to the measurement noise n become

$$\begin{aligned} H_o(s)&=\frac{Y(s)}{D_o(s)}=\frac{1}{1+C_ePF_r}=1-\frac{F_r(s)e^{-\Theta s}}{1+T_rs} \\ H_i(s)&=\frac{Y(s)}{D_i(s)}=\frac{P}{1+C_ePF_r}=P(s)H_o(s) \\ H_n(s)&=\frac{U(s)}{N(s)}=\frac{C_eF_r}{1+C_ePF_r}=C(s)F_r(s) \end{aligned} \tag{7}$$

In order to get rejection of piecewise stable input and output disturbances at least in steady states and in order to eliminate the possibly slow plant dynamics from the input disturbance response, following requirements should hold

$$H_o(0)=0 \;;\; H_i(0)=0 \text{ and } H_i(-1/T)=0 \tag{8}$$

The first-order filter (5) gives the PI-F1SP controller and

$$\beta_{11}=T\left[1-(1-T_r/T)(1-T_f/T)e^{-\Theta/T}\right] \tag{9}$$

The second-order filter gives the PI-F2SP controller and

$$\beta_{22}=T\left[1-(1-T_f/T)^2e^{-\Theta/T}\right] \tag{10}$$

One of the questions to be dealt in this paper is how the type of the filter (5) influences dynamics and robustness of the corresponding controller.

3 NEW P-F1SP AND P-F2SP CONTROLLERS

Next, we will firstly show that an equivalent solution may also be achieved by considering the primary 2DOF P control instead of (2). By considering particular filters (5), new DTCs denoted as the P-F1SP and P-F2SP will be introduced.

3.1 Primary 2DOF P-controller

Simpler P controllers instead of PI ones were recommended in a slightly modified setting e.g. in Liu, Zhang and Gu (2005) and Lu et al. (2005). The 2DOF controller will be expressed as the P controller with the gain K_p extended by the static feedforward control u_0

$$u=K_Pe+u_0 \;;\quad u_0=r/K_0\,;\, K_P=(T_0/T_r-1)/K_0; \tag{11}$$

Thereby, the fast model parameters T_0, K_0 correspond to the estimates of the plant parameters T and K, T_r represents the time constant of the fast primary loop. In controlling the nominal delay-free P_0 by (11) the setpoint-to-output transfer function $H_{rn}(s)=Y(s)/R(s)$ equals to (4) and also $C(s)=U(s)/E(s)$ representing the filtered inversion of the fast plant dynamics will be equally given by (3). Despite to this formal equivalence, both controllers show different properties in constrained mode.

3.2 Respecting the control signal constraints

Control of real plants is always subject to constraints expressed e.g. in the form of the saturation function

$$u_r=sat(u)=\begin{cases} U_{\max}; & u>U_{\max} \\ u; & U_{\min}\le u\le U_{\max} \\ U_{\min} & u<U_{\min} \end{cases} \tag{12}$$

In controlling stable plant P_0 (1) with built in constraints (12) by the P-controller (11) the loop remains stable without taking any additional measures for any r satisfying

$$\begin{aligned} r\in(Y_{\min},Y_{\max})\,;\, Y_{\min}&=K(U_{\min}+d_i)+d_o; \\ Y_{\max}&=K(U_{\max}+d_i)+d_o \end{aligned} \tag{13}$$

This may be shown e.g. by choosing appropriate Ljapunov function, by the circle criterion, by the Popov criterion, or by the passivity approach (Föllinger, 1993; Glattfelder and Schaufelberger, 2003; Hsu and Meyer, 1968).

In the much more complicated FSP, outputs of the primary controller (11) yields inversion of the fast dynamics. In order to respect constraints imposed on the real plant input, outputs of the primary loop can not be simply generated from the setpoint signal by the transfer function, but the primary loop must be implemented by including at least so strong constraints as those at the plant input. Furthermore, in order to guarantee relevance of information used in the disturbance reconstruction, also the DO must be supplied with the constrained control signal. The corresponding predicted signal $\hat{x}_p$ (in Fig. 1, $\hat{x}_p$ plays role of the output predicted with respect to the delayed output x_d), what again requires to work with constrained primary loop.

3.3 Modified P-FSP Controllers

Since both nominal primary controllers are given by (3), also the nominal closed loop transfer functions (7) are equal, i.e. both structures are fully equivalent. Also the requirements (8) on filters (5) remain unchanged that finally leads to tuning (9-10). The equivalence holds, however, just for the nominal tuning ($P=P_n$) and without control constraints. In both F1SP alternatives, the disturbance response is formally not fully decoupled from the setpoint response, i.e. T_r influences both the setpoint as well as the disturbance response, but still it is possible to tune these responses separately.

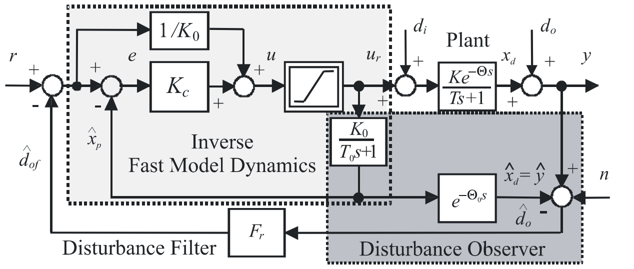

Fig. 1 Modified P-FSP with the primary loop using 2DOF P-controller (11) with the disturbance filters (5)

4. COMPARING PI-FSPs WITH P-FSPs: THE LAG DOMINANT PLANT

The thermo-optical laboratory plant offers control of 8 measured process variables (Huba and Šimunek, 2007) by 3 manipulated (voltage) variables (0–5 V) influencing the bulb (main heat & light source), the light-diode (disturbance light source) and the fan (system cooling). Within schemes in Matlab/Simulink or Scilab/Scicos the plant is represented as a single block and so limiting needs on costly and complicated software packages for real time control. The optical channel used in following experiments consists of the light intensity produced by bulb measured by a photodiode and filtered by an analogue low pass filter with the time constant about 20s.

Dependence of the output y on the control signal u and on the disturbance signal d_i was approximated by the FOPDT model

$$Y(s)=\frac{Ke^{-\theta s}}{Ts+1}\left[U(s)+K_iD_i(s)\right] \tag{14}$$

by identifying series of input steps with incrementally increasing initial value. Thereby, the gain K_0 chosen with respect to the best approximation of the steady-state input-to-output characteristic in Fig. 2 does not allow more precise matching of the gains identified from the step responses that show a clearly increasing trend (Fig. 3).

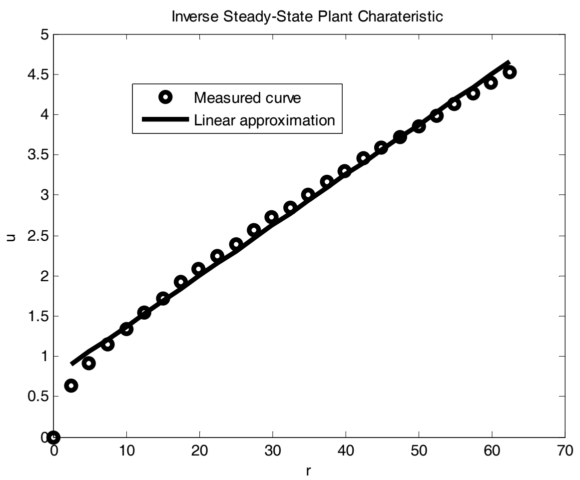

Fig. 2 The steady-state controller output versus the reference signal and its approximation for the feedforward control

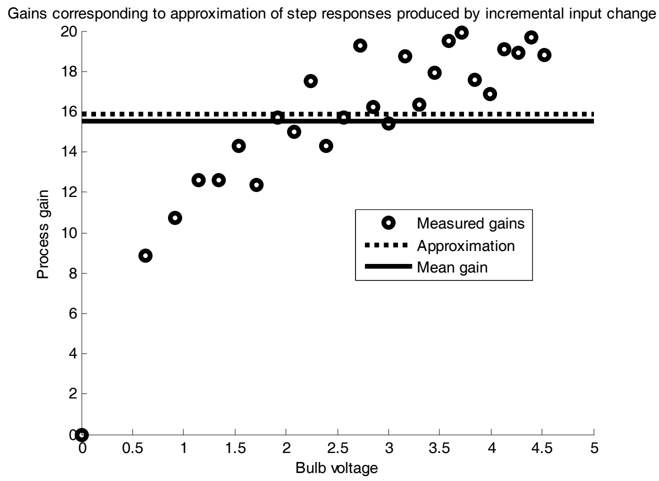

Fig. 3 Plant gains identified by series of step responses with incremental input increase versus the initial input value

So, the plant was approximated by uncertain model with

$$\begin{aligned}&K\in\langle 3.9706,\quad 13.8093\rangle\ K_i\in\langle 4.31, 4.58\rangle;\\&\theta\in\langle 0,0.5\rangle;\ T\in\langle 17.0096,\ 25.8687\rangle\end{aligned} \tag{15}$$

Obviously, it is lag dominant. The controllers used in following experiments correspond to the operating point

$$K=15.8730;\theta=0.34;\ \ T=22.5521 \tag{16}$$

and to the quasi-continuous control with the sampling period $T_{samp}=0.1\text{sec}$. The testing sequences (as shown in Fig. 4) were evaluated for the disturbance filter time constants

$$T_f=T_r/c\ ;\ c=\{1,2,8,16\}\Rightarrow T_f\in\langle 0.6551,10.4822\rangle \tag{17}$$

In evaluating transients achieved with particular controllers, speed and duration of the plant output responses were characterized by the IAE performance index defined as

$$IAE=\int_0^\infty |e(t)|dt;\ e(t)=r(t)-y(t) \tag{18}$$

Besides of this, monotonicity of the output responses has been evaluated. The control effort was characterized by the TV criterion (Skogestad and Postlethwaite, 1996) defined as

$$TV=\int_0^\infty\left|\frac{du}{dt}\right|dt\approx\sum_i|u_{i+1}-u_i| \tag{19}$$

4.1 Setpoint steps

From the setpoint-to-output transfer functions (4) that are equivalent in the nominal case, one could expect - at least for the relatively small setpoint step changes, when the control signal saturation is not active – also equal transients (Fig. 4, smaller steps and larger steps upwards). The plant-model mismatch that is visible especially for large setpoint steps. The chosen approximation of the static controller characteristics in Fig. 2 gives good static feed forward performance, but it does not sufficiently characterize the dynamical changes and so the disturbance observer generates internal disturbance during transient responses leading e.g. to undershooting of the downwards steps (Fig. 5 above). For short dead time these negative effects can be suppressed by speeding up the DO loop. For the relatively large T_f (for $c=1$ and $c=2$ in (17)) all setpoint responses corresponding to the relatively large downward steps show undershooting (Fig. 5 above). When speeding up corrective disturbance rejection by $T_f \le T_r/8$, both the P-F1SP and the P-F2SP controllers (immune against windup) yield already monotonic output responses that are clearly superior over the PI-FSP ones (influenced by the windup, Fig.5). For large setpoint steps the dependence of the achieved dynamics on the ratio $c = T_r/T_f$ is negligible, what confirms expectation of nominal independence of the setpoint dynamics from the DO.

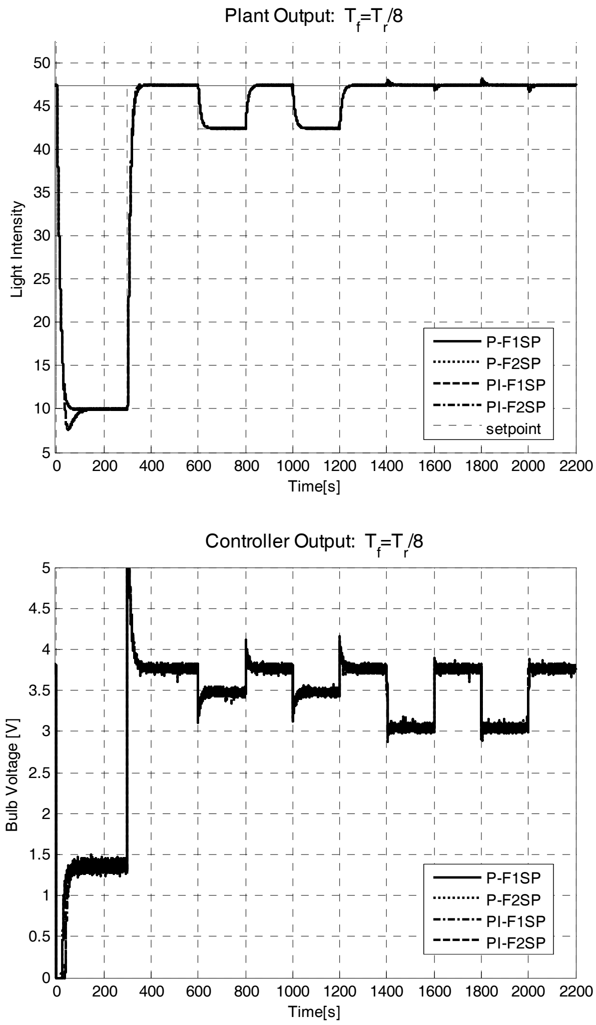

Fig. 4 Testing sequence: large (0-600s) and small (600-1400s) setpoint steps and disturbance steps $\Delta d = \pm 3V$ (1400-2200s) produced by LED equivalent to $\Delta d_i = \pm 0.7V$

However, the IAE and TV values for small setpoint steps (Fig.6) that enable already a finer analysis of the dependence on $c = T_r/T_f$ (not superimposed by the windup and plant nonlinearity) show that in all cases decreasing of the filter time constant T_f leads to increased figures both in IAE as well as TV. Thereby, lower values correspond to the P- and PI-F1SP controllers with F_{1r} and both P-FSP controllers are at least equivalent to the PI-FSP ones.

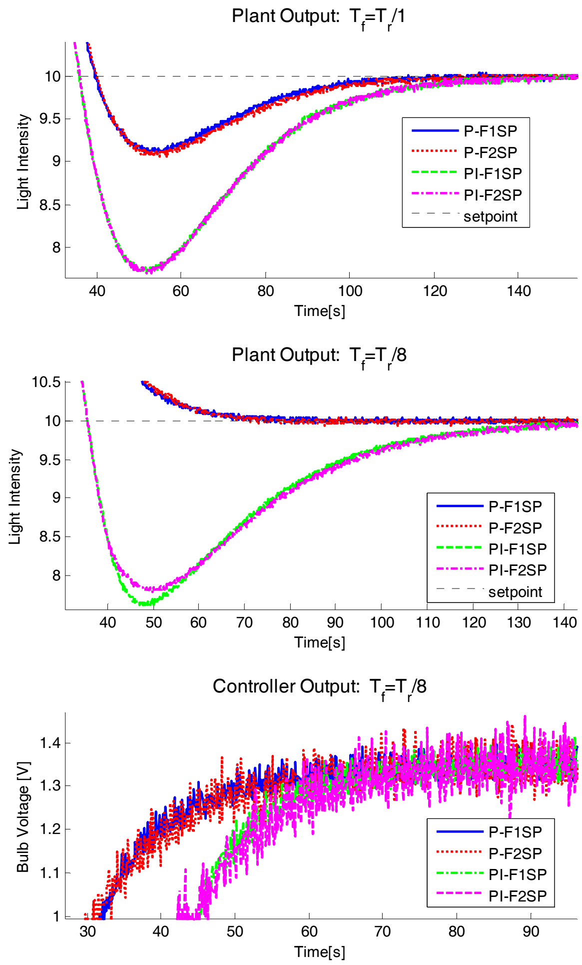

Fig. 5 Details of the large downward steps with typical windup of both PI-FSPs remaining also for shorter T_f

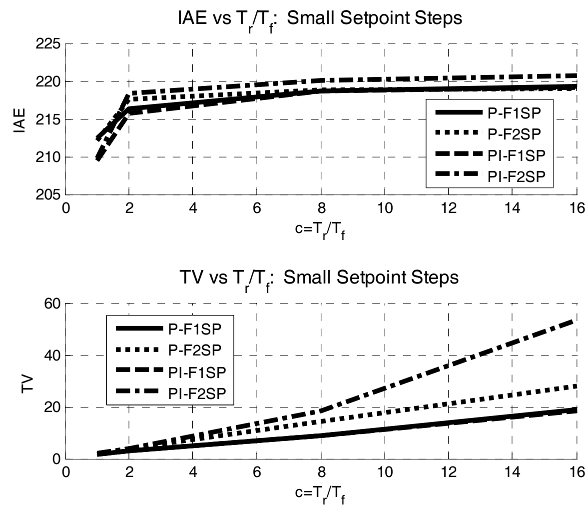

Fig. 6 IAE and TV values corresponding to $T_r = T/2$ and T_f (17) for small setpoint steps

The existing differences may be explained by imperfections (uncertainty, nonlinearity) of the plant model. Due to this the DO is influencing the loop behaviour also in situations with zero external disturbances, whereby the reconstructed disturbances correspond to the plant-model mismatch.

These experiments show that both P-FSP controllers yield performance similar, or even superior to the PI-FSP ones, whereby they do not exhibit windup and are easier to explain.

4.2 Disturbance steps

With respect to the formally equal disturbance responses of the PI- and P-F2SP, or PI- and P-F1SP one could again expect confirmation of this fact by the real time experiments. Example of measured disturbance responses is in Fig. 7. As it is also evident from characteristics in Fig. 8, by decreasing T_f the IAE values decrease, but the required control effort expressed by the TV values increases. The fastest responses correspond to the PI-F2SP, but on the cost of the largest TV values. The P- and PI-F1SP controllers yield practically the same dynamics that is less depending on T_f than for F_{2r}.

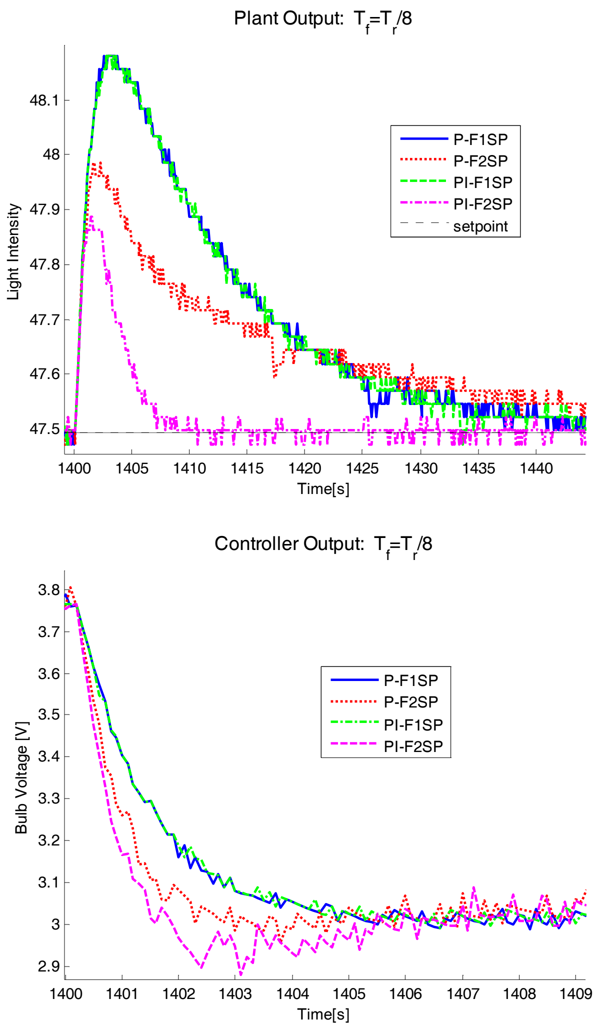

Fig. 7 Responses corresponding to the disturbance step

5. COMPARING PI- AND P-FSP: LONGER DEAD TIME VALUES

In order to evaluate the closed loop performance for longer dead time values and to compare transients with those achieved for the lag-dominated loop, the natural plant delay θ_0 (16) was increased in Simulink by an artificial delay to

$$\theta = \theta_0 + \theta_a \ ; \ \theta_a = 20\,s \tag{20}$$

So the total dead time θ was increased from the relatively short value θ_0 up to the value close to the time constant *T*. This results into decreased controller ability to compensate for the model uncertainty expressed by the reasonably increased closed loop sensitivity leading to increased amount of higher harmonics, what is visible especially in the large downward setpoint steps, when all tested controllers undershoot (Fig.9). The increased sensitivity prevents the P- and PI-F2SP controllers with F_{2r} to work with the time constants $T_f < T_r/2$, since already for $c = 8$ the transients are permanently oscillating at the controller output and not converging at the plant output.

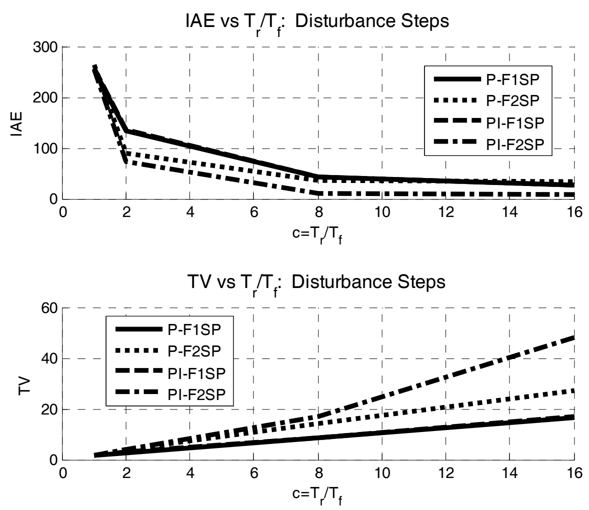

Fig. 8 IAE and TV values corresponding to $T_r = T/2$ and (19) for disturbance steps

Experiments again show that the P-FSP controllers yield performance comparable, or even superior to that of the PI-FSP ones. For longer dead time values, due to the increased sensitivity the use of F_{2r} (5) is not recommended.

6. CONCLUSIONS

New formulations of the FSPs were proposed based on simplified primary loop with the 2DOF P-controller. Two solutions denoted as P-F1SP and P-F2SP corresponding to the 1st and the 2nd order filters were considered and compared with the PI based FSP with equivalent disturbance filters.

The essential advantage of the new solutions is that, due to the memoryless controller in the primary loop and due to the IMC structure they do not generate the windup effect, in contrast to the PI-FSP ones that under constrained control tend to the windup and so they require appropriate anti-windup measures. This reasonably restricts possibility of direct use of the traditional PI-FSP in situations with the relatively short dead time and low uncertainty, where the new solution enables to use much faster tuning leading to saturation during significant phase of the setpoint steps.

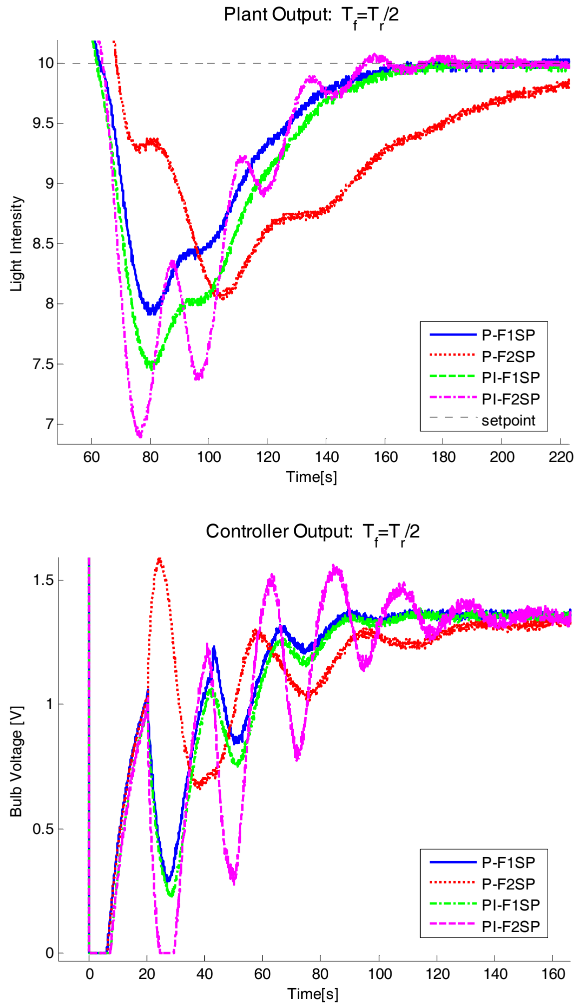

Fig. 9 Details of the downward step

Due to different implementation schemes and the plant-model mismatch, the real time experiments corresponding to particular solutions are different also for relatively small changes. Thereby, the traditional solutions based on the primary PI controller give better performance just for a relatively narrow range of parameters for the disturbance response. Also here the experiments confirmed attractiveness of the new solutions that is not restricted just to the case of constrained control.

Experience achieved in the DTCs tuning shows that there is no principal difference in dealing with the lag dominant and the dead time dominant systems. It is just to remember that the closed loop dead time is limiting speed of the correcting processes within the loop and so increasing demands on precision of the plant model. In the considered case of the optical plant, higher requirements on control quality lead in the case of longer dead time to necessity of considering a more precise nonlinear plant model. Use of the 2nd order DO filter (5) that enables formally fully independent tuning of the setpoint and disturbance responses shows to be much more sensitive to different loop imperfections and therefore it has to be used very carefully.

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