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Generalized Super-Twisting Observer for Nonlinear Systems

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Abstract: In this paper it is proposed a novel Lyapunov based design of a generalized Super-Twisting Observer for a class of 2-dimensional nonlinear system. The observer can deal with systems whose states are composed of bounded nonlineaer functions.This is the main difference with the classical Super-Twisting observer, in which the second state is only the derivative of the first state. Working with a Strong Lyapunov Function it can be shown sufficient conditions to properly choose the observer gains to ensure finite time convergence to the real states. The obsrver is tested in a mathematical model regarding to the reduced Glucose-Insulin process. The numerical results have shown a better performance of the observer with lineal compensators in comparison to the classical Super-Twisting Observer. The gains for the observer are designed in order to compensate a more general class of perturbations that appear in the suggested glucose-insuline nonlinear model.

Keywords: Sliding Modes, Super-Twisting Observer, Nonlinear systems

1. INTRODUCTION

Sliding modes are well-known for their robustness against perturbations and uncertainties in the mathematical description of several phisycal systems. In most cases, sliding modes are obtained by mean of the injection of a nonlinear discontinuous term. In general this discontinuous term is depending on the output error. This framework may be used to construct robust controlling or observing algorithms. Discontinuous injection must be designed in such a way that system trajectories are enforced to remain in a submanifold contained in the estimation error space (the so-called sliding surface). For both, the control and the observation problem, the resulting motion is referred to as the sliding mode (Utkin (1992)). One additional positve characteristic using this discontinuous term regards to the rejection of external matched disturbances (Tan and Edwards (2001)).

Classical Sliding-Mode Observers (Utkin (1992), Walcott and Zak (1987)) estimate robustly the state when the perturbations/measurement map when the sliding surface has relative degree (RD) one with respect to the system input. However, the disturbance cannot be reconstructed exactly. The observer and controller design based on the second-order sliding modes (SOSM) approach has been considered as an interesting topic by many researchers within the last decade (see Sthessel et al. (2003), Sira-Ramirez (2004), Punta (2006) and the references therein). Some attractive features of SOSM compared to the classical first-order sliding modes are widely recognized: higher accuracy motions, chattering reduction, finite-time convergence for systems with relative degree two (Levant (2005), Boiko et al. (2007), etc.).

In order to perform this disturbance reconstruction task, a Second Order Sliding Mode algorithm, the so-called Super-Twisting Algorithm (STA), has been proposed recently (Davila et al. (2005)) for second-order (mechanical) nonlinear systems. The STA robustly reconstructs, in finite-time, the states, if the perturbation is of relative degree two (RD=2), or reconstructs the perturbation itself, when it is of relative degree one (RD=1).

Besides, the sliding mode observers are widely used due to the finite-time convergence, robustness with respect to uncertainties and the possibility of uncertainty estimation (see, for example, the bibliography in the recent tutorials Barbot et al. (2002), Edwards and Spurgeon (1998), and Poznyak (2001)). In particular, asymptotic observers (Shtessel and Shkolnikov (2003)) and the asymptotic observer for systems with Coulomb friction (Alvarez et al. (2000) and Orlov et al. (2003)) were designed based on the second-order sliding-mode. These observers requires the assumption on the so-called separation principle due to the asymptotic convergence of the estimated values to the real ones.

In general, the convergence of all these algorithms (high order sliding modes) was proved using very complex geometrical conditions Levant (2005). Just a couple of years ago, the Lyapunov methodology was succesfully applied to show how and why these algorithms converge in finite time.

In (Moreno and Osorio (2008)) a strong Lyapunov function is proposed to ensure finite time convergence for the STA. The Lyapunov analysis define how the gains should be selected in order to obtain an acceptable performance produced by the observer (finite time convergence with a predefined convergence time). Additionally, in the same paper,it has been introduced a non classical STA including a linear term proportional to the error trayectories. This observer has terms related to the second order sliding mode theory and another one like the Luenberger observer. This new observer was called Second Order Sliding Mode plus linear observer (SOSML).

This class of observers has been applied to solve the problem of exact differentiation. In this way the main contribution provided in this paper is the convergence analysis for the observer based on the super-twisting algorithm. This algorithm deals with the class of perturbations that are able to grow up with the states. In the same way, the nonlinear model is composed by nonlinear functions in the first and second states.

This scheme was tested on a biological nonlineal system (a Minimal Model for Glucose - Insulin Kinetics). The system is affected by external perturbation in the output and in the dynamics description. Using the results developed by (Moreno and Osorio (2008)), a strong Lyapunov function is proposed to prove practical stability and to obtain the boundary layer where the estimation error converges. In the following section, it is presented the classical Super-Twisting observer and the class of perturbations that the observer can deal with it. In section III the Generalized Super-Twisting observer (GSTO) is presented and the class of bounded perturbations are introduced. The problem statement and the main result are discussed in section IV. In Section V, a numerical example regarding to a glucose-insulin model is given. Finally in section VI the conclusions are given.

2. CONVENTIONAL SUPER-TWISTING OBSERVER

The super-twisting algorithm has been used for second order systems, especially those that accept the mechanical-like form. The class of nonlinear perturbed second order system considered to apply this so-called conventional super-twisting observer is governed by the two following differential equations:

$$\begin{aligned} \dot{x}_{1,t} &= x_{2,t} \\ \dot{x}_{2,t} &= f(x_t, u_t) + \xi_t \\ y_t &= x_{1,t} \end{aligned} \tag{1}$$

Here, $x_t = [x_{1,t}, x_{2,t}]^\intercal \in \Re^2$ is the state vector and $u_t \in \Re^m$ is the control action applied to the system. The signal ξ_t represents internal disturbances in the nonlinear structure of the system. The second equation can include discontinuous parts as dry-friction. Therefore, the solution of the unperturbed differential equation for the nonlinear system is understood in Filippov sense (Filippov (1998)). It means, that the second equation in (5) (where $f(x_t, u_t)$ appears) is replaced by the equivalent differential inclusion $\dot{x}_{2,t} \in \bar{F}(x_t, u_t) + \xi_t$. In view of the continuity almost everywhere of f, the set-valued $\bar{F}(x_0, u_0) = \left[\bar{F}_i(\cdot)\right]$ is the convex closure of f. This is the set of all limits of $F(x_t, u_t)$ as $[x_{a,t}, u_{a,t}] \to [x_{0,t}, u_{0,t}]$ where $[x_{0,t}, u_{0,t}]$ is the set of all continuity points of f for any $x_{a,t} \in X \subset \Re^{2n}$ and $u_{a,t} \in U^{adm}$. This is, in fact, a direct consequence of the belonging of $x_{\alpha,t} \in X \ \forall t$. The set of all admissible nonlinear controllers U^{adm} is defined by

$$U^{adm} := \left\{ u : \|u_t\|^2 \le v_0 + v_1 \|x_t\|^2_{\Lambda_u} < \infty \right\} \tag{2}$$

In previous papers, it was supposed that the following assumption was fulfilled: The disturbance is weighted (with weight Λ_ξ) quadratically bounded, that is

$$\|\xi_t\|^2_{\Lambda_\xi} \le \Upsilon, \ \Lambda_\xi = \Lambda_\xi^\top > 0 \tag{3}$$

This is a very important restriction regarding the class of second order nonlinear systems that may be analyzed. The super-twsting algorithm used for this class of systems has the following structure:

$$\begin{aligned} \frac{d}{dt}\hat{x}_{1,t} &= \hat{x}_2 - k_1 |\tilde{x}_{1,t}|^{1/2} \operatorname{sign}(\tilde{x}_{1,t}) \\ \frac{d}{dt}\hat{x}_{2,t} &:= f(x_t, u_t) - k_2 \operatorname{sign}(\tilde{x}_{1,t}) \\ \tilde{x}_{1,t} &:= x_{1,t} - \hat{x}_{1,t} \end{aligned}$$

In this structure, the gains $k_i \ i = \overline{1,2}$ must be selected in such a way to ensure finite time convergence. Recently, this observer was modified to include a larger class of perturbations. This new observer which has been called the linear super-twisting algorithm. This observer includes two additional linear terms:

$$\begin{aligned} \frac{d}{dt}\hat{x}_{1,t} &= \hat{x}_2 - k_1 |\tilde{x}_{1,t}|^{1/2} \operatorname{sign}(\tilde{x}_{1,t}) + k_3 \tilde{x}_{1,t} \\ \frac{d}{dt}\hat{x}_{2,t} &:= f(x_t, u_t) - k_3 \operatorname{sign}(\tilde{x}_{1,t}) + k_4 \tilde{x}_{1,t} \end{aligned} \tag{4}$$

The class of perturbations considered in this situation may be not absolutely bounded. Nevertheless, such perturbations must disappear when the surface is reached. This is an unnatural assumption even for mechanical systems. This paper generalizes (for a bigger class of perturbations). the observer structure considering the option where more general perturbations appears.

3. GENERALIZED SUPER-TWISTING OBSERVER

3.1 Class of Nonlinear System

The class of nonlinear perturbed second order system considered throughout this paper is governed by the following two differential equations:

$$\begin{aligned} \dot{x}_{1,t} &= g(x_t, t) + \xi_{1,t} \\ \dot{x}_{2,t} &= f(x_t, u_t) + \eta(x_t, t) + \xi_{2,t} \\ y_t &= x_{1,t} \end{aligned} \tag{5}$$

Here, $x_t = [x_{1,t}, x_{2,t}]^\intercal \in \Re^2$ is the state vector and $u_t \in \Re^m$ is the control action applied to the system. The signals $\xi_{1,t}$ and $\xi_{2,t}$ represent internal disturbances in the nonlinear structure of the system. $g(\cdot,\cdot) : \Re^2 \to \Re$ and $f(\cdot,\cdot) : \Re^2 \to \Re$ are nonlinear bounded functions. This class of systems is more general compared to those described above. Indeed, the class of uncertainties is also larger and characterizes many systems.

3.2 Generalized Super-Twisting Observer

The observation scheme is based on the classical Super-Twisting Observer. The scheme is composed by the plant's reproduction (5) and a set of corrective terms (mixing continuous and discontinuous elements) using the available information for the uncertain system. The state estimator uses the approach developed in (Davila et al. (2006)) with a new linear correcting term in the structure (Moreno and Osorio (2008)). The generalized Super-Twisting Observer (GSTO) has the following structure

$$\frac{d}{dt}\hat{x}_t = \begin{bmatrix} \hat{g}\left(x_{1,t}, \hat{x}_{2,t}, t\right) \\ +\beta_1 \lambda\left(\tilde{x}_{1,t}\right) \text{sign}\left(\tilde{x}_{1,t}\right) + \beta_3 \tilde{x}_{1,t} \\ \\ \hat{f}\left(x_{1,t}, \hat{x}_{2,t}, u_t\right) + \eta\left(x_{1,t}, \hat{x}_{2,t}, t\right) \\ +\beta_2 \text{sign}\left(\tilde{x}_{1,t}\right) + \beta_4 \tilde{x}_{1,t} \end{bmatrix} \quad (6)$$

$$\lambda(z) := |z|^{1/2}$$

The estimation error defined as $\tilde{x}_t$ is build up by the component $\tilde{x}_{1,t} := x_{1,t} - \hat{x}_{1,t}$. The change of $\tilde{x}_{1,t}$ along the time is represented by the next differential equation

$$\begin{gathered} \frac{d}{dt}\tilde{x}_{1,t} = g(x,t) - \hat{g}\left(x_1, \hat{x}_2, t\right) - \\ \beta_1 \lambda\left(\tilde{x}_{1,t}\right) sign\left(\tilde{x}_{1,t}\right) - \beta_3 \tilde{x}_{1,t} \\ \frac{d}{dt}\tilde{x}_{2,t} = f\left(x_t, u_t\right) - f\left(\hat{x}_t, u_t\right) + \eta\left(x_t, t\right) - \eta\left(\hat{x}_t, t\right) \\ +\xi_{2,t} - \beta_2 sign\left(\tilde{x}_{1,t}\right) - \beta_4 \tilde{x}_{1,t} \end{gathered} \quad (7)$$

In this paper, it is supposed that following assumptions are fulfilled:

A1. The set of all admissible nonlinear controllers U^{adm} is defined by

$$U^{adm} := \{|u| : |u| \leq u_0 + u_1 \|\hat{x}\| \leq u_0 + u_1 \left(\|\tilde{x}_t\| + \|x\|\right)\} \quad (8)$$

A2. The nonlinear functions that describe the nonlinear dynamics for the first and second state are bounded

$$\begin{gathered} |f(x,u) - f(\hat{x},u)| \leq f_0 + f_1 \|\tilde{x}_t\| \\ |g(x,t) - \hat{g}\left(\bar{x}_1, \hat{x}_2, t\right)| \leq g_0 + g_1 \|\tilde{x}_t\| \\ |\eta(x,t)| \leq \eta_0 + \eta_1 \|x\| \end{gathered} \quad (9)$$

A3.The disturbances are absolutelly bounded

$$\left|\xi_{i,t}\right| \leq \xi_i^+ \qquad i = \overline{1:2} \quad (10)$$

A4. Lets consider a class of systems with bounded dynamics:

$$\sup_t \|x(t)\| = d^+ \quad (11)$$

4. PROBLEM STATEMENT AND MAIN RESULT

The main problem to deal with in this paper regards to design the high-order variable structure observer for uncertain 2-dimensional nonlinear system based on the Lyapunov second method. The observer was proposed using the structure developed in (Davila et al. (2005)) and (Moreno and Osorio (2008)). This novel observer is developed under the assumption of the presence of nonlinear functions in the first and second states describing the nonlinear system's dynamics. Therefore, it must be proved the robustness of the observer suggested in this paper.

The problem considered here can be formulated as follows:

To select an adequate combination of gains β_1, β_2, β_3 and β_4 in such a way, *under the assumptions considered in this paper for the nonlinear second order system (5), for any admissible control injection* $u_t \in U^{adm}$, and the assumptions presented in (8), (9), (10) and (11) the trajectories of the state estimator given in (6) converge exponentially to a small ball $B_\delta := \{\tilde{x}_t : \|x_t - \hat{x}_t\| \leq \delta\}$ surrounding the real trajectories of the system under analysis.

The main result developed in this paper is presented in the following theorem

Theorem. Consider the nonlinear second order system described in (5). Now, lets use the state modified Super-Twisting observer defined in (6). If the observer gains are selected in such a way the following matrices Q_0 and Q_1 are positive definite and negative definite respectively

$$\begin{gathered} Q_0 := \left[q_{i,j}\right], \quad i,j = \overline{1:3} \\ q_{11} = -\epsilon_0^{-1} p_{11}^2 + p_{11}\beta_1 + p_{13}\beta_2 \\ q_{22} = p_{12}\left(g_1 \text{sign}\left(\Delta_{1,t}\right) - \beta_3 - p_{12}\epsilon_2^{-1}\right) \\ q_{33} = p_{31}\left(-p_{31}\epsilon_1^{-1} + g_1 \text{sign}\left(\Delta_{2,t}\right)\right) \\ q_{21} = q_{12} = \frac{1}{2}\left(-p_{11} g_1 \text{sign}\left(\Delta_1\right) + p_{11}\beta_3 - p_{12}\beta_1 + p_{23}\beta_2\right) \\ q_{32} = q_{23} = \frac{1}{2}\left(p_{12} g_1 \text{sign}\left(\Delta_{2,t}\right) - p_{31}\beta_3 + p_{31} g_1 \text{sign}\left(\Delta_{1,t}\right)\right) \\ q_{13} = q_{31} = \frac{1}{2}\left(-p_{11} g_1 \text{sign}\left(\Delta_{2,t}\right) - p_{31}\beta_1\right) \end{gathered} \quad (12)$$

and

$$\begin{gathered} Q_1 := \left[a_{i,j}\right], \quad i,j = \overline{1:4} \\ a_{11} = p_{12}^2 \epsilon_3^{-1} + p_{12}\beta_1 + p_{13}^2 \epsilon_6^{-1} \\ a_{22} = p_{22}^2 \epsilon_4^{-1} + \text{sign}\left(\tilde{x}_{1,t}\right) p_{22} g_1 + p_{23}^2 \epsilon_7^{-1} \\ -p_{23} f_1 \text{sign}\left(\tilde{x}_{1,t}\right) + p_{23}\beta_4 \\ a_{33} = -p_{22}\beta_3 + p_{23}^2 \epsilon_5^{-1} - p_{23} \text{sign}\left(\tilde{x}_{2,t}\right) \\ + p_{33}^2 \epsilon_8^{-1} + p_{33} f_1 \text{sign}\left(\tilde{x}_{2,t}\right) \\ a_{21} = a_{12} = -p_{12} g_1 \text{sign}\left(\tilde{x}_{1,t}\right) + p_{12}\beta_3 \\ -p_{22}\beta_1 - p_{13} f_1 \text{sign}\left(\tilde{x}_{1,t}\right) + p_{13}\beta_4 \\ a_{32} = a_{23} = p_{22} g_1 \text{sign}\left(\tilde{x}_{2,t}\right) - p_{23} g_1 \text{sign}\left(\tilde{x}_{1,t}\right) + p_{23}\beta_3 \\ -p_{23} f_1 \text{sign}\left(\tilde{x}_{2,t}\right) + p_{33} f_1 \text{sign}\left(\tilde{x}_{1,t}\right) - p_{33}\beta_4 \\ a_{13} = a_{31} = -p_{12} g_1 \text{sign}\left(\tilde{x}_{2,t}\right) + p_{23}\beta_1 \\ -p_{13} f_1 \text{sign}\left(\tilde{x}_{2,t}\right) - p_{33}\beta_2 \end{gathered} \quad (13)$$

Then the observation error provided by the suggested sliding mode observer (6) converges to a small ball B_δ whit radius given as follows

$$\delta \leq \max\left(\left(\frac{\rho_1}{\alpha_1}\right)^2, \frac{\rho_2}{\alpha_2}\right)$$

$$\alpha_1 := \frac{\lambda^{1/2}\min(P)\,\lambda\min(Q_0)}{\lambda\max(P)} \tag{14}$$

$$\alpha_2 := \lambda\min(P),$$

$$\rho_1 := (\epsilon_0+\epsilon_1+\epsilon_2+\epsilon_3+\epsilon_4+\epsilon_5)\left(g_0+\xi_1^+\right)^2$$

$$\rho_2 := (\epsilon_6+\epsilon_7+\epsilon_8)\left(f_0+\eta_0+\xi_2^+ + \eta_1 d^+\right)^2$$

The proof of the main theorem is developed in the Appendix.

Comment. The assumption about the possitiveness of Q_0 and negativeness of Q_1 could seem to be very restrictive. Nevertheless, using the S-procedure, one can simplify this condition. Indeed, if one try to solve such problem as is, it is natural to obtain more equations than variables. But, using the S-procedure, one will obtain the same number of variables than equations leading to a well defined solution. Therefore, the theorem statement may be changed to:

There are real numbers $\tau_1 \geq 0$ and $\tau_2 \geq 0$ $(\tau_1+\tau_2 > 0)$ such that

$$Q := \tau_1 Q_1 + \tau_2 Q_2 < 0 \tag{15}$$

This transformation changes the problem of the state observation defined in the theorem to find the adequate values of

$$\beta_i, i = \overline{1,4} \text{ and } \tau_j, j = \overline{1,2} \tag{16}$$

such that $Q < 0$.

Several numerical procedures may be proposed to obtain the solution to the problem recently introduced. In this paper, the interior point method was used to obtain the solution for the linear matrix inequality stated in (15). Nevertheless, a non-complex analysis may lead to find some analytical solutions for (16). Similar analysis were developed in (Poznyak (2008)).

Remark 1. LMI's defined by Q_0 and Q_1 are actually depending on $\text{sign}(\tilde{x}_{1,t})$ and $\text{sign}(\tilde{x}_{2,t})$. Therefore, one can see that, for example the term $a_{22} = p_{22}^2\epsilon_4^{-1} + \text{sign}(\tilde{x}_{1,t})\, p_{22} g_1 + p_{23}^2\epsilon_7^{-1} - p_{23} f_1 \text{sign}(\tilde{x}_{1,t}) + p_{23}\beta_4$ will have two values. Therefore, $Q_0 := Q_0(\text{sign}(\tilde{x}_{1,t}), \text{sign}(\tilde{x}_{2,t}))$ and $Q_1 := Q_1(\text{sign}(\tilde{x}_{1,t}), \text{sign}(\tilde{x}_{2,t}))$ represents a number of 4 different matrixes when

$$\begin{array}{l} (Q_{01},Q_{11}) := \{(Q_0,Q_1) \mid \tilde{x}_{1,t} > 0, \tilde{x}_{2,t} > 0\} \\ (Q_{02},Q_{12}) := \{(Q_0,Q_1) \mid \tilde{x}_{1,t} > 0, \tilde{x}_{2,t} < 0\} \\ (Q_{03},Q_{13}) := \{(Q_0,Q_1) \mid \tilde{x}_{1,t} < 0, \tilde{x}_{2,t} > 0\} \\ (Q_{04},Q_{14}) := \{(Q_0,Q_1) \mid \tilde{x}_{1,t} < 0, \tilde{x}_{2,t} < 0\} \end{array} \tag{17}$$

The case where $\tilde{x}_{1,t} = \tilde{x}_{2,t} = 0$ is not considered by the solution offered by the Lyapunov analysis. This point of the trajectory may be excluded from the analysis considered the assumptions and results founded in the theorem 1.

Based on the representation of four matrices explained in this remark, it is necessary to extend the application of the S-Procedure described in the previous comment. Now, eight different positive scalars $\tau_{1i} > 0$, $\tau_{2i} > 0$ $i = 1..4$. such that

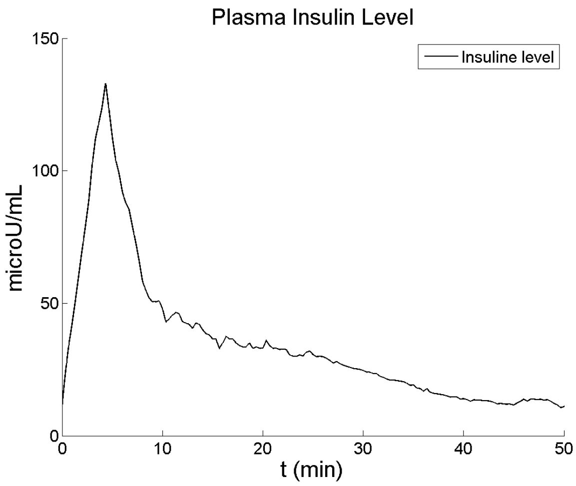


Fig. 1. Plasma Insulin Level as a control input

$$Q := \sum_{i=1}^{4}(\tau_{1i}Q_{0i} + \tau_{2i}Q_{1i}) < 0$$

where Q_{0i} and Q_{1i} are the versions of Q_0 and Q_1 respectively when each combination in (17) is considered. Therefore, the results obtained in the main theorem introduced in this paper is well posed.

5. NUMERICAL RESULTS

As an illustration of the results presented in this paper, it is designed a GSTO for a nonlinear system describing the glucose-insuline interactions in the human body. Consider a second order nonlineal math model given by

$$\begin{array}{c} \frac{d}{dt}G_t = S_G(G_b - G_t) - X_t G_t \\ \frac{d}{dt}X_t = k_3\left[S_I(I_t - I_b) - X_t\right] \\ y = G_t \end{array} \tag{18}$$

In these equations, t is the independent model variable time $[min]$, t_0 is the time of glucose injection, G_t is the plasma glucose concentration $[mg/dL]$, I_t is the plasma insulin level $[\mu U/mL]$ and X_t is the interstitial insulin activity. Looking at the structure of the equation (18), it is clear that X_t does not represent a physiological, measurable quantity, this variable is related to the effective insulin activity. G_b is the basal plasma glucose concentration $[mg/dL]$ and I_b is the basal plasma insulin concentration $[\mu U/mL]$. Basal plasma concentrations of glucose and insulin are typically measured before administration of glucose (or sometimes 180 minutes after). The parameter S_G is defined as the insulin sensitivity whereas S_I represents the glucose effectiveness. For simulation these were chosen as

$$\begin{array}{ll} S_I = 5.0\times 10^{-4}\,[mg/dL] & I_0 = 0\,[\mu U/mL] \\ S_G = 2.6\times 10^{-2}\,\left[min^{-1}\right] & I_b = 11\,[\mu U/mL] \\ G_b = 90\,[mg/dL] & G_0 = 279\,[mg/dL] \end{array}$$

The input of the system, given by I_t is displayed in the figure 1 according to the results presented in (Van Riel (2004)). The figure 2 shows the estimation of the first state, the plasma glucose Concentration, it can be seen how after a short period of time, the GSTO reach the real trayectories. of the system. For the second state,

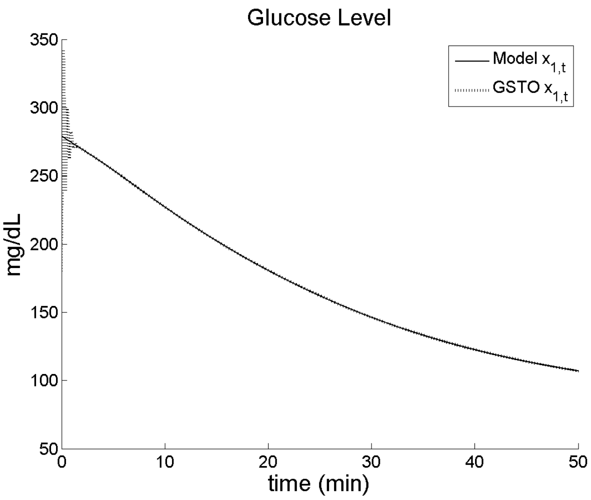
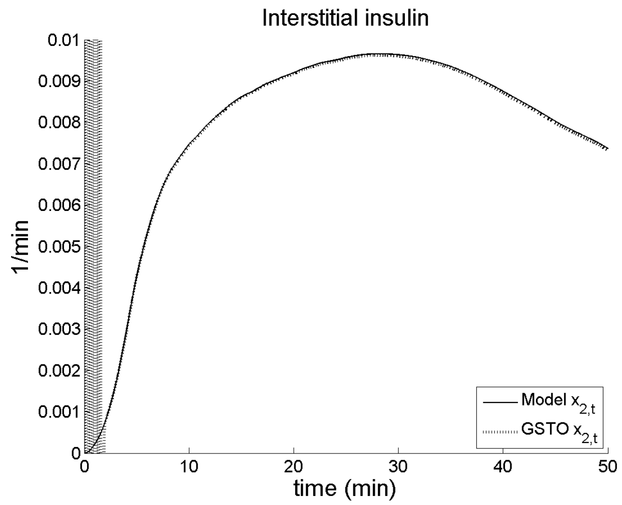


Fig. 2. Estimation of the Glucose Plasma Concentration using the GSTO



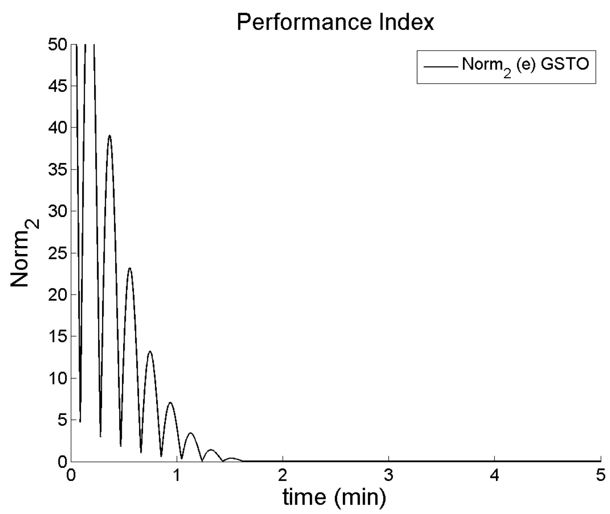


Fig. 3. Performance index for the estimation erro

corresponding to the Interstitial insulin level, the figure 5 depicts the estimation process and the effectiveness of the proposed observer. The performance index for the GSTO, using the Euclidean norm is represented in the figure 3. To

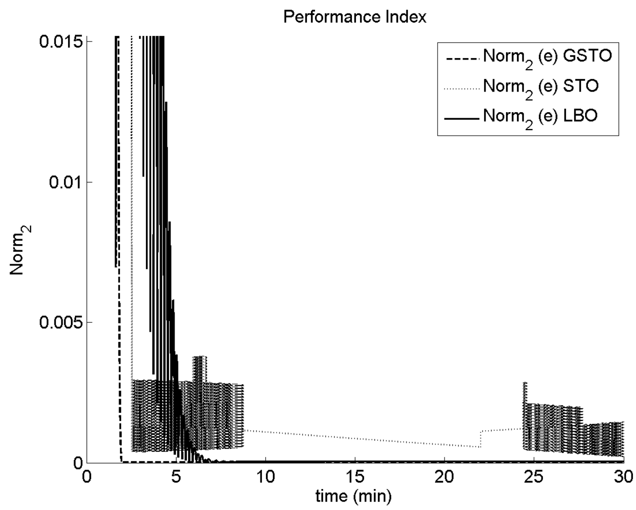


Fig. 4. Comparation between the GSTO, the classical STO and a Luenberger observer (LBO)

demonstrate the difference between the observer working with four correction terms and the previous achieves it is presented in the figure 4, the performance index for the GSTO presented in this paper, the classical STO and a Luenberger Observer. (LBO). It can be seen advantages to use the GSTO, it reach the manifold $\tilde{x} = 0$ in less time than the others algorithms. The *chattering* effect decrease in comparison with the classical Super-Twisting observer, and the class of perturbations is extended.

6. CONCLUSION

It has been presented a novel design for a Generalized Super-Twisting Observer based on Strong Lyapunov Functions, the perturbations in the state grow up together with the nonlinear states of the system. This is the principal difference between this result and previous results using the same technique. The performance of the observer applying in a nonlinear biomedical model, suggest a good behavior under the presence of disturbances and uncertainties in the model.

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Appendix A. PROOF OF THE MAIN THEOREM

Consider the following strong Lyapunov function

$$V(\tilde{x},t) = \varsigma^{\intercal}(\tilde{x}_t) P \varsigma(\tilde{x}_t)$$
$$\varsigma(\tilde{x}_t) := \begin{bmatrix} \lambda(\tilde{x}_{1,t}) \operatorname{sign}(\tilde{x}_{1,t}) \\ \tilde{x}_{1,t} \\ \tilde{x}_{2,t} \end{bmatrix}$$

if $P = P^T > 0$ is selected in the form

$$P = \frac{1}{2} \begin{bmatrix} 4\beta_3+\beta_1^2 & \beta_1\beta_2 & -\beta_1 \\ \beta_1\beta_2 & 2\beta_4+\beta_2^2 & -\beta_2 \\ -\beta_1 & -\beta_2 & 2 \end{bmatrix}$$

Note that $V(\tilde{x},t)$ is continuous but not differentiable at $\tilde{x}_{1,t} = 0$. Moreover, it satisfies

$$\lambda \min(Q) \|\varsigma(\tilde{x}_t)\|_2^2 \leq V(\tilde{x},t) \leq \lambda \max(Q) \|\varsigma(\tilde{x}_t)\|_2^2 \quad \text{(A.1)}$$

Taking the derivative for the Lyapunov functions leads to $\dot{V}(\bar{x}_t, \hat{x}, t) = 2\varsigma^{\intercal} P \dot{\varsigma}$ and $\dot{\varsigma}$ is given by the following equation

$$\dot{\varsigma}(\tilde{x}_t) = \begin{bmatrix} \frac{1}{2}\lambda^{-1}(\bar{x}_1) \frac{d}{dt}\tilde{x}_1 \\ \frac{d}{dt}\tilde{x}_{1,t} \\ \frac{d}{dt}\tilde{x}_{2,t} \end{bmatrix}$$

substituting the trayectories. of the error dynamics given by the equation (7) it is possible to obtain the next equation for the $\dot{\varsigma}(\tilde{x}_t)$, defining $\tilde{g} := g(x,t) - \hat{g}(x_1, \hat{x}_2, t)$, $\tilde{f} := f(x_t, u_t) - f(\hat{x}_t, u_t)$ and $\tilde{\eta} := \eta(x_t, t) - \eta(\hat{x}_t, t)$

$$\dot{\varsigma}(\tilde{x}_t) = \begin{bmatrix} \frac{1}{2}\lambda^{-1}(\tilde{x}_{1,t}) \left(\tilde{g} + \xi_{1,t} - \beta_1 \lambda(\tilde{x}_{1,t}) \operatorname{sign}(\tilde{x}_{1,t}) - \beta_3 \tilde{x}_{1,t}\right) \\ \tilde{g} + \xi_{1,t} - \beta_1 \lambda(\tilde{x}_{1,t}) \operatorname{sign}(\tilde{x}_{1,t}) - \beta_3 \tilde{x}_{1,t} \\ \tilde{f} + \tilde{\eta} + \xi_{2,t} - \beta_2 \operatorname{sign}(\tilde{x}_{1,t}) - \beta_4 \tilde{x}_{1,t} \end{bmatrix}$$

Solving the Lyapunov function $\dot{V}(\tilde{x},t) = 2\varsigma^T(\tilde{x}_t) P \dot{\varsigma}(\tilde{x}_t)$ leads to

$$\dot{V}(\tilde{x},t) = -\left(\varsigma_t^T Q_0 \varsigma_t - \rho_1\right) + \xi_t^{\top} Q_1 \xi_t + \rho_2$$

with Q_0 and Q_1 defined in (12) and (13). If the gains for the observer are selected in such way Q_0 is positive definite and Q_1 is negative definite simultaneously, it follows that $\dot{V} \leq -\lambda^{-1}(\tilde{x}_{1,t}) \lambda \min\{Q_0\} \|\varsigma(\tilde{x}_t)\|_2^2 - \lambda \min\{Q_1\} \|\varsigma(\tilde{x}_t)\|_2^2$
Using equation (A.1) and the fact that

$$\lambda(\tilde{x}_{1,t}) \leq \|\varsigma(\tilde{x}_t)\|_2 \leq \frac{V^{1/2}(\tilde{x},t)}{\lambda^{1/2} \min\{Q_0\}}$$

it follows that

$$\dot{V}_t \leq -|\bar{x}_1|^{-1/2} \left(\alpha_1 \sqrt{V_t} - \beta_1\right) - \alpha_2 V_t + \beta_2$$

if the following conditions are fulfilled (Poznyak et al. (1998))

$$\sqrt{V_t} \geq \frac{\rho_1}{\alpha_1}, \qquad V_t \geq \frac{\rho_2}{\alpha_2}$$

the zone of convergence for the estate estimator (6) is

$$V_t \leq \max\left(\left(\frac{\rho_2}{\alpha_1}\right)^2, \frac{\rho_2}{\alpha_2}\right)$$

and this conclude the proof.