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MISSILE AUTOPILOT DESIGN VIA A MULTI-CHANNEL LFT/LPV CONTROL METHOD

Paulo C. Pellanda * **Pierre Apkarian** ** **Hoang Duong Tuan** ***
Daniel Alazard **

* *IME-DE/3, RJ Brazil - pellanda@aquarius.ime.eb.br*
** *ONERA, Toulouse FRANCE - apkarian(alazard)@cert.fr*
*** *TOYOTA, Nagoya JAPAN - tuan@toyota-ti.ac.jp*

Abstract: The missile pitch-axis autopilot design is revisited using a new and recently available LPV control technique. The missile plant model is characterized by an LFT representation. The synthesis task is conducted by exploiting new capabilities of the LPV method: a set of H_2/H_∞ criteria is considered and different Lyapunov and scaling variables are used for each channel/specification. The method is shown to provide additional flexibility to tradeoff conflicting and demanding performance and robustness specifications for the missile while preserving the practical advantage of previous single-objective LPV methods.

Keywords: missile autopilots, LPV synthesis, LFT, mixed H_2/H_∞, multi-channel control, gain scheduling.

1. INTRODUCTION

Gain-scheduling techniques and Linear Parameter-Varying (LPV) control theory have been used extensively for the synthesis of non-linear controllers and especially in designing missile autopilots (Reichert, 1992; Shamma and Cloutier, 1993; Wu *et al.*, 1995*a*; Schumacher and Khargonekar, 1998; Tan *et al.*, 2000). Despite this popularity, missile autopilot design remains a challenging control problem since it operates over a wide range of flight conditions and tight design specifications are generally prescribed.

The LPV framework provides elegant and solidly founded methodologies to meet design specifications over wide operating ranges. Control problems are formulated as Linear Matrix Inequalities (LMI) optimization problems (Boyd *et al.*, 1994; Gahinet *et al.*, 1994), which are then solved very efficiently using currently available semi-definite programming codes. In Reference (Apkarian *et al.*, 2000) a technique for solving mixed H_2/H_∞ multi-channel Linear Fractional Transformation (LFT)/LPV control problem in discrete-time has been derived. It can be viewed as an extension of LFT/LPV single-objective results in References (Packard, 1994; Apkarian and Gahinet, 1995) and of nominal multi-objective techniques in References (Scherer *et al.*, 1997; de Oliveira *et al.*, 1999). A practically interesting capability of this method is to offer additional flexibility to tradeoff various performance and robustness specifications. Similarly to the LTI case, different Lyapunov and scaling variables are used for each channel/specification to reduce conservatism as compared to earlier methods. In this paper, we discuss its applicability to a realistic LPV missile autopilot design. We show how the method can be used for discrete- or continuous-time LPV systems. We paid a special attention to the controller construction and implementation which are of prior importance in missile problems.

2. MULTI-OBJECTIVE LFT/LPV RESULT

In this section, we state the multi-objective LPV control problem and give a brief overview of the synthesis method in (Apkarian *et al.*, 2000).

Consider a discrete-time LPV plant P with LFT structure

$$\begin{bmatrix} x(k+1) \\ z_\Delta(k) \\ z(k) \\ y(k) \end{bmatrix} = \begin{bmatrix} A & B_\Delta & B_1 & B_2 \\ C_\Delta & D_{\Delta\Delta} & D_{\Delta 1} & D_{\Delta 2} \\ C_1 & D_{1\Delta} & D_{11} & D_{12} \\ C_2 & D_{2\Delta} & D_{21} & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ w_\Delta(k) \\ w(k) \\ u(k) \end{bmatrix} \qquad (1)$$
$$w_\Delta(k) \quad = \quad \Delta(k)\, z_\Delta(k)$$

where $A \in \mathbf{R}^{n\times n}$, $\Delta(k) \in \mathbf{R}^{N\times N}$, $D_{12} \in \mathbf{R}^{p_1\times m_2}$ and $D_{21} \in \mathbf{R}^{p_2\times m_1}$ define the problem dimension. The notation for signals is standard: x for the state vector, w

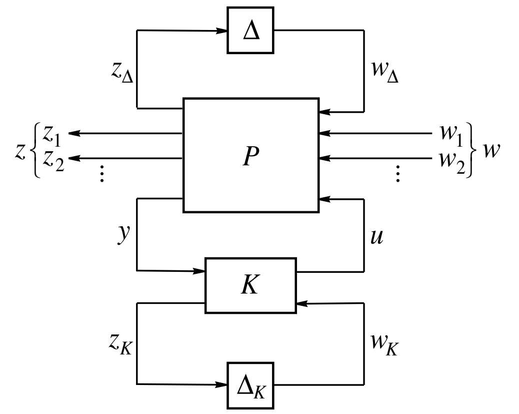

Fig. 1. Mixed H_2/H_∞ multi-channel LPV interconnection

for exogenous inputs, z for controlled or performance variables, u for the control signal, and y for the measurement signal. $\Delta(k)$ is a time-varying matrix-valued parameter evolving in a polytopic set $\mathscr{P}_\Delta$, with

$$\mathscr{P}_\Delta := \mathrm{co}\left\{\Delta_1,\ldots,\Delta_i,\ldots,\Delta_L\right\} \ni 0\,, \qquad (2)$$

where co stands for the convex hull and the Δ_i's denote the vertices of $\mathscr{P}_\Delta$. That is,

$$\Delta := \sum_{i=1}^{L}\alpha_i\Delta_i, \qquad \sum_{i=1}^{L}\alpha_i = 1, \qquad (3)$$

where $\alpha_i \geq 0$ are the polytopic coordinates of Δ. Polytopic coordinates are computed in real time as functions of the scheduling variables (Section 4) and can be exploited by the controller. According to our definitions, the pair (w_Δ, z_Δ) is the gain-scheduling channel.

For the LPV plant (1) the gain-scheduling control problem consists in seeking an LPV controller K with LFT structure

$$\begin{bmatrix} x_K(k+1) \\ u(k) \\ z_K(k) \end{bmatrix} = \begin{bmatrix} A_K & B_{K1} & B_{K\Delta} \\ C_{K1} & D_{K11} & D_{K1\Delta} \\ C_{K\Delta} & D_{K\Delta 1} & D_{K\Delta\Delta} \end{bmatrix} \begin{bmatrix} x_K(k) \\ y(k) \\ w_K(k) \end{bmatrix} \qquad (4)$$
$$w_K(k) \; = \Delta_K(k) z_K(k)\,,$$

$A_K \in \mathbf{R}^{n\times n}$ and $\Delta_K \in \mathbf{R}^{N\times N}$, such that H_2 and/or H_∞ specifications are achieved for a family of channels (w_j, z_j), $j = 1,2,\cdots$, where the w_j and z_j are subvectors of w and z, respectively (Figure 1). In other words, bounds ν_j on the variance of the outputs z_j and/or bounds γ_j on the L_2-induced gain of the operator mapping w_j into z_j are guaranteed for all parameter trajectories $\Delta(k) \in \mathscr{P}_\Delta$. The notation Δ_K is used for the controller gain-scheduling function which is a function of the plant's parameter Δ, that is, $\Delta_K := \Delta_K(\Delta)$.

In this application, we consider the special situation in which Δ has a block-diagonal structure determined by a vector of parameters $\theta := (\theta_1,\cdots,\theta_r)^T$ with

$$\Delta = \mathrm{diag}(\theta_1 I_{s_1},\cdots,\theta_r I_{s_r}) \qquad (5)$$

Also, we assume that θ evolves in a box defined as

$$\theta_l \in [\underline{\theta}_l, \bar{\theta}_l], \quad \underline{\theta}_l < \bar{\theta}_l, \quad \forall t \geq 0 \qquad (6)$$

The assumptions (5) and (6) mean that:

- the time-varying parameter θ is valued in a hyper-rectangle $\mathscr{P}_\Theta$ of $\mathbf{R}^r$, with

$$\mathscr{P}_\Theta := \mathrm{co}\left\{\Theta_1,\ldots,\Theta_L\right\}\,, \qquad (7)$$

  where the Θ_i are the vertices of $\mathscr{P}_\Theta$;
- Δ and θ have the same polytopic coordinates $\{\alpha_i\}$;
- $L = 2^r$ and $N = \sum_{l=1}^{r} s_l$.

Hereafter, $i\ (=1,\cdots,L)$ indexes the vertices Θ_i and Δ_i, $j\ (=1,2,\cdots)$ indexes the channels and specifications, and $l\ (=1,\cdots,r)$ indexes the parameters.

It is shown in Reference (Apkarian *et al.*, 2000) that sufficient conditions for the existence of a solution to the multi-objective LPV control problem can be written as an LMI program. The general synthesis scheme is described below.

Algorithm 2.1. Controller synthesis

Step 1: Define the following general non symmetric decision variables which are common to all specifications and channels (Table 1):
- the set S_v of general slack variables; the set K_v of transformed controller variables, whose dimensions must be defined in according to the controller dimensions; and the set Δ_{Kv} of scheduling function coefficients.

Step 2: For each H_2-channel, define the set H_{2v} of the following symmetric decision variables:
- Lyapunov variables (X_{2j} and Z_j); scaling variables (Q_{1j}, Q_{2j}, R_{1j} and R_{2j}); and a performance variable (ν_j).

Step 3: For each H_∞-channel, define the set $H_{\infty v}$ of the following symmetric decision variables:
- a Lyapunov variable ($X_{\infty j}$); scaling variables ($Q_{\infty j}$ and $R_{\infty j}$); and a performance variable (γ_j).

Step 4: For each channel/specification, construct the LMI constraint system derived in Appendix A of (Apkarian *et al.*, 2000) and represented here by the simple notations below:
- H_2 performance:

$$\mathscr{L}_{H_2}(S_v, K_v, \Delta_{Kv}, H_{2v}, \Delta_i, P_j) < 0 \qquad (8)$$

- H_∞ performance:

$$\mathscr{L}_{H_\infty}(S_v, K_v, \Delta_{Kv}, H_{\infty v}, \Delta_i, P_j) < 0 \qquad (9)$$

  where P_j is the set of state-space matrices representing the LPV plant (1) with only the channel/specification (w_j, z_j) under consideration.

Step 5: (LMI optimization problem) - Minimize a specific performance variable γ_j or ν_j subject to the LMI constraints (8)-(9), fixing the remaining performance variables at some adequate set of values; or simply compute a feasible solution to the LMI constraints (8)-(9).

Step 6: As described in (Apkarian *et al.*, 2000), compute the LPV controller data (4) as functions of the decision variables (Table 1) obtained in Step 5. Note that the set K_v (bold notation) does not represent the set of controller data. The controller gain-scheduling function is determined by

$$\Delta_K(\Delta) := \sum_{i=1}^{L} \alpha_i \Phi_i, \tag{10}$$

where the Φ_i can be computed off line as functions of the decision variables.

3. DISCRETIZATION

While genuine extensions of the foregoing method to the continuous-time case remain challenging, it can be indirectly applied to continuous plants with the help of a formal bilinear transformation.

Consider a continuous-time system

$$\begin{aligned} \dot{x}(t) &= \tilde{A}x(t) + \tilde{B}\xi(t) \\ \psi(t) &= \tilde{C}x(t) + \tilde{D}\xi(t) \end{aligned} \tag{11}$$

A corresponding discrete-time state-space realization is obtained as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \mathscr{F}_u\left(\begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix}, \begin{bmatrix} I & \sqrt{2}I \\ \sqrt{2}I & I \end{bmatrix} \right) \tag{12}$$

where $\mathscr{F}_u$ is the customary notation for upper LFT. A transformation from the discrete-time domain to the continuous-time domain can be obtained similarly:

$$\begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix} = \mathscr{F}_u\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}, \begin{bmatrix} -I & \sqrt{2}I \\ \sqrt{2}I & -I \end{bmatrix} \right) \tag{13}$$

Supposing that $\xi(t) := [w_\Delta^T(t),\, w^T(t),\, u^T(t)]^T$ and $\psi(t) := [z_\Delta^T(t),\, z^T(t),\, y^T(t)]^T$ in Equation (11), a corresponding discrete-time system in the form (1) is readily obtained by applying the bilinear transformation (12). Since H_2 problems are properly posed in continuous time, the methodology described in the previous section can be applied without restrictions to the transformed system. For the H_2 performance index ν_j to be well defined in continuous time, the state-space data must be such that the closed-loop feedthrough term of the channel/specification j is zero. Without imposing restrictions to the controller, this is achieved with $\tilde{D}_{11j} = 0$ and either $\tilde{D}_{1\Delta j} = 0$ and $\tilde{D}_{12j} = 0$ or $\tilde{D}_{\Delta 1j} = 0$ and $\tilde{D}_{21j} = 0$.

Once the LFT discrete-time controller has been computed, one can use the transformation (13) to recover the corresponding continuous-time controller. It is worth mentioning that only the LTI components of the LFT plant and of the LFT controller are modified by bilinear transformations, whereas the Δ- and Δ_K-blocks remain unchanged.

4. POLYTOPIC COORDINATES

Modern flight control systems undergo highly maneuverable trajectories which requires a fast controller update. Controllers designed through general LPV/gridding techniques (Wu *et al.*, 1995*b*; Apkarian and Adams, 1998) show little conservatism but require more complex on-line computations at the gain-scheduling level. Contrarily, LFT/LPV controllers are often more conservative but their favorable LFT structure offers obvious advantages in this respect. In comparison with the single-objective H_∞ LFT/LPV control methods (Packard, 1994; Apkarian and Gahinet, 1995), the foregoing mixed H_2/H_∞ multi-objective approach allows to consider a richer class of scheduling functions (10), instead of replicating the parameter block of the plant ($\Delta_K := \Delta$). This is another factor which reduces conservatism and that is immediately penalized by an increase in complexity of on-line computations. Fast algorithms for the calculation of polytopic coordinates should therefore be utilized in order to overcome this difficulty.

For a parameter evolving in a hyper-rectangle, barycentric coordinates can be directly and quickly computed by ratio of hyper-volumes. The following algorithm describes a procedure for the computation of polytopic coordinates to general hyper-rectangles (6) with vertices in Equation (7):

Algorithm 4.1. Computation of polytopic coordinates

Step 1: Given a parameter $\theta := (\theta_1, ..., \theta_r)^T$, compute its normalized coordinates

$$\vartheta_l := \frac{(\bar{\theta}_l - \theta_l)}{(\bar{\theta}_l - \underline{\theta}_l)}, \quad l = 1, \cdots, r\,.$$

Step 2: For each vertex Θ_i, $i = 1, ..., L$, compute the corresponding polytopic coordinates

$$\alpha_i = \prod_{l=1}^{r} \tilde{\vartheta}_l\,, \quad \text{where}$$

$$\tilde{\vartheta}_l = \begin{cases} \vartheta_l & \text{if} \quad \underline{\theta}_l \quad \text{is a coordinate of} \quad \Theta_i \\ 1 - \vartheta_l & \text{if} \quad \bar{\theta}_l \quad \text{is a coordinate of} \quad \Theta_i \end{cases}$$

Then, computing polytopic coordinates from measured rectangular coordinates is not a costly procedure. It can be readily performed on line through simple operations basically consisting in (r) scalar normalizations and $(Lr - L)$ scalar multiplications.

5. MISSILE CONTROL PROBLEM

In this section, we apply the technique presented in Sections 2-4 to a realistic missile gain-scheduling autopilot problem. This problem consists in controlling a missile to track commanded normal acceleration $\eta_c(t)$, by generating a commanded tail fin deflection $\delta_c(t)$.

The nonlinear pitch-axis missile model and actuator dynamics are available in References (Reichert, 1992; Nichols *et al.*, 1993). They involve the angle of attack $\alpha(t)$, the pitch-rate $q(t)$ and the tail deflection angle $\delta(t)$ and its derivative $\dot{\delta}(t)$. Normal acceleration $\eta(t)$

Table 1. Decision variables

Set	Variables	Dimension	Number of scalar variables
S_v	U, V_{11}, W_{11}	$n \times n$	$3n^2$
	$M, N, E_{11}, F_{11}, G_{11}, H_{11}$	$N \times N$	$6N^2$
K_v	$A_K, B_{K1}, B_{K\Delta}, C_{K1}, C_{K\Delta}, D_{K11}, D_{K1\Delta}, D_{K\Delta 1}, D_{K\Delta\Delta}$	Appropriate	$n^2+N^2+2nN+(n+N)(m_2+p_2)+m_2p_2$
Δ_{Kv}	$\Delta_{K,i}, i=1,\cdots,L$	$N \times N$	LN^2
H_{2v}	X_{2j}	$2n \times 2n$	$n(2n+1)$
	Z_j	$p_{1j} \times p_{1j}$	$p_{1j}(p_{1j}+1)/2$
	$Q_{1j}, Q_{2j}, R_{1j}, R_{2j}$	$2N \times 2N$	$4N(2N+1)$
	ν_j	Scalar	1
$H_{\infty v}$	$X_{\infty j}$	$2n \times 2n$	$n(2n+1)$
	$Q_{\infty j}, R_{\infty j}$	$2N \times 2N$	$2N(2N+1)$
	γ_j	Scalar	1

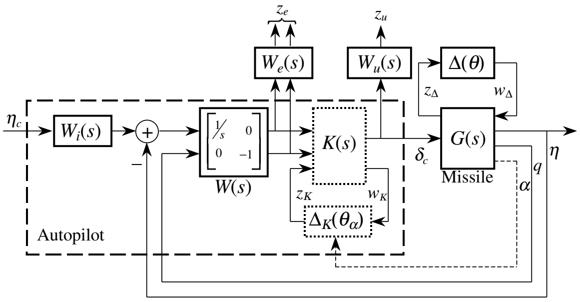

Fig. 2. Control and synthesis structure

and pitch-rate are measured outputs. The plant dynamics can be parameterized by $\theta(t) = [\alpha(t),\ M(t)]^T$, where the Mach number $M(t)$ is an exogenous variable which is treated here as an uncertainty. We consider that only the state variable $\alpha(t)$ is available for scheduling purposes. In fact, the parameter channel can be split into two channels by defining $z_\Delta := [z_\alpha^T,\ z_M^T]^T$ and $w_\Delta := [w_\alpha^T,\ w_M^T]^T$. Due to the missile symmetry about $\alpha = 0$, controllers are designed for $\alpha \geq 0$ and scheduled on $|\alpha|$.

5.1 *Performance objectives and control structure*

The performance and robustness specifications for the closed-loop system are similar to those detailed in References (Wu *et al.*, 1995*a*; Nichols *et al.*, 1993). Our aim is to maintain robust stability over the entire operating range, $\alpha \in [-30, 30]$ degrees and $M \in [2, 4]$, and to track step commands in η_c with time constant no more than 0.35 s, maximum overshoot of 10%, steady-state error less than 1% and an adequate high-frequency roll-off for noise attenuation and withstand neglected high frequency dynamics and flexible modes. In order to avoid saturation of the actuator, the maximum tail deflection rate for 1g step command in η_c should not exceed 25 °/s.

We adopt the closed-loop control structure depicted in Figure 2. The LFT missile model $F_u(G(s), \Delta(\theta))$ is derived in the full version of this paper (Pellanda *et al.*, 2001). In order to utilize the approach discussed in this paper, we express the performance objectives by choosing appropriate weighting functions. The precompensator $W_i(s)$ is used to achieve the command shaping. The weighting functions $W(s)$ and $W_e(s) := \text{diag}(W_e'(s), 0.01)$ penalize the tracking error and $W_u(s)$ incorporates bounds on the norm of unmodeled dynamics and also reflects magnitude restriction on the control signal.

Hence, the specifications above can be met by a controller $K(s)$ together with its scheduling function $\Delta_K(\theta_\alpha)$, $|\theta_\alpha(t)| = \left|\frac{|\alpha(t)|-15}{15}\right| \leq 1$, which:

- minimize the L_2-induced gain γ_M of the operator mapping z_M into w_M,
- maintain the variance of z_e due to the disturbance η_c below an appropriate bound ν_e,
- guarantee an upper bound γ_u on the L_2-induced gain of the operator mapping η_c into z_u,

for all trajectories $\alpha(t) \in [-30, 30]$ degrees.

Then, this problem can be solved by running Algorithm 2.1 and consists in finding an adequate compromise between three conflicting objectives over the entire operating range: one H_2 and two H_∞ specifications. Note that such a problem cannot be solved by earlier LPV methodologies for plants described by LFT representations.

The discrete-time synthesis plant $P(z)$ and the final continuous-time controller $K(s)$ are computed through bilinear transformations, respectively from $P(s)$ and the designed $K(z)$, as indicated in Section 3. The continuous-time synthesis plant $P(s)$ is readily obtained from the connections in Figure 2 and incorporates the missile model $G(s)$ and the weighting functions, $W_i(s)$, $W(s)$, $W_e(s)$, and $W_u(s)$. These frequency-dependent weights have been tuned by performing a few trials-and-errors of synthesis and simulations for the nominal plant. That is, an LTI plant model obtained from the linearization about the point $\theta = [0,\ 0]^T$, ($\alpha = 15$, $M = 3$), and an appropriate compromise between ν_e and γ_u have guided our weight selection. This has been carried out by using the same synthesis methodology described in Section 2 with $\Delta = 0$. The adopted frequency shapes for the filters are fairly standard and their numerical data are available in the Reference (Pellanda *et al.*, 2001).

5.2 *Results and simulations*

In order to put in light the potentials of our multi-channel LPV synthesis method and to allow comparisons, we have considered two designs. The first LPV controller, $K_1(s)$ and $\Delta_{K_1}(\theta_\alpha)$, has been synthesized considering M as a constant $(= 3)$; the second one, $K_2(s)$ and $\Delta_{K_2}(\theta_\alpha)$, considers M as a bounded uncertain parameter ($M \in [2,4]$). In short, we have used the following strategy to compute these controllers:

- $K_1(s)$ and $\Delta_{K_1}(\theta_\alpha)$:
  - Starting with a small value ν_e, synthesize controllers which minimize the H_∞ performance γ_u subject to a H_2 constraint $\sqrt{\nu_e}$.
  - Through successive relaxations in ν_e, find a reasonable compromise between these objectives. To check out when a good balance has been achieved, perform non-stationary $(\alpha(t))$ and nonlinear simulations for $M = 3$ and evaluate the closed-loop performance in the time domain.
- $K_2(s)$ and $\Delta_{K_2}(\theta_\alpha)$:
  - As mentioned in the previous subsection and analogously to $K_1(s)$, minimize γ_M subject to the constraints γ_u and ν_e.
  - Starting with the final values γ_u and ν_e obtained in designing $K_1(s)$, relax them alternately in order to find an adequate balance between the three objectives.

Numerical data for $K_2(s)$ and its scheduling function coefficients Φ_i's are provided in the Reference (Pellanda *et al.*, 2001).

Nonlinear simulation results for fixed values of M are displayed on Figure 3. Figure 4 shows nonlinear simulations for time-varying $M(t)$. The input is a sequence of step commanded acceleration η_c whose amplitudes have been chosen such that the parameter α covers most of the scheduling range, thus inducing significant variations in the aerodynamic coefficients. The Mach number time trajectory has been generated as in References (Nichols *et al.*, 1993; Wu *et al.*, 1995*a*). As theoretically expected, all performance objectives are met for all considered trajectories when (K_2, Δ_{K_2}) is employed for controlling the system. In contrast, the desired closed-loop behavior is satisfied only at the central point ($M = 3$) for (K_1, Δ_{K_1}). We recall that (K_2, Δ_{K_2}) has been computed in order to ensure robustness with respect to variations in the Mach number through an extra H_∞ constraint on the Mach channel M. From the later result, the advantages of using this multi-objective LPV synthesis method became evident.

6. CONCLUSIONS

We have discussed a multi-objective/channel H_2/H_∞ LPV control technique for the design of a missile autopilot over a broad range of operating conditions in both the angle of attack and the Mach number. The proposed method provides additional flexibility to handle various and stringent specifications attached to the missile problem while maintaining the same operational simplicity as earlier single-objective LPV techniques:

- the missile nonlinearities are captured through the use of an LFT representation,
- different channels are defined to translate tracking performance, control limitation and robustness properties,
- balancing the different design requirements is carried out in a very natural way within the proposed design framework and conservatism is kept reasonable thanks to the use of different Lyapunov and scaling variables for each channel/specification,
- besides, we describe simple schemes to construct the controller scheduling function and show how all these manipulations carry over the continuous-time case.

The determination of a full genuine continuous-time methodology remains, however, challenging and future research should be oriented in this direction. Also, we think that other practical reasons might dictate the use of observer-based LPV controllers. This is a delicate and seemingly untouched topic that will be considered in a future study.

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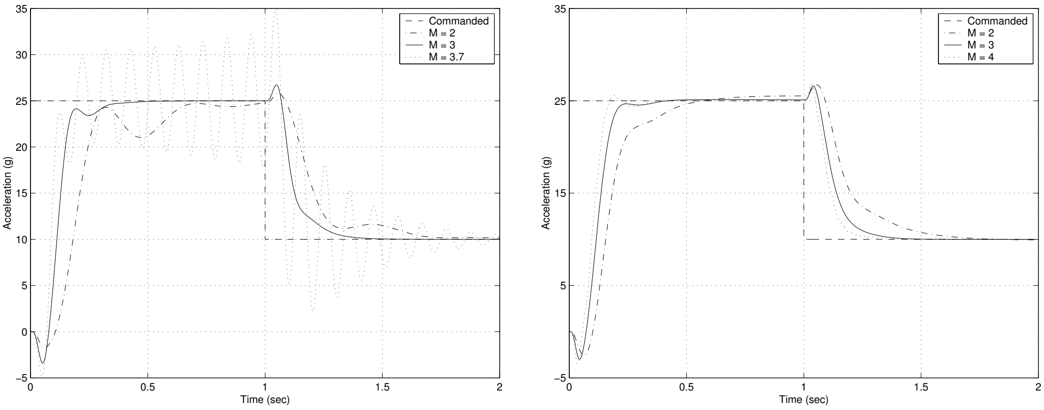

Fig. 3. Nonlinear closed-loop response for fixed M: (K_1, Δ_{K_1}) on the left and (K_2, Δ_{K_2}) on the right

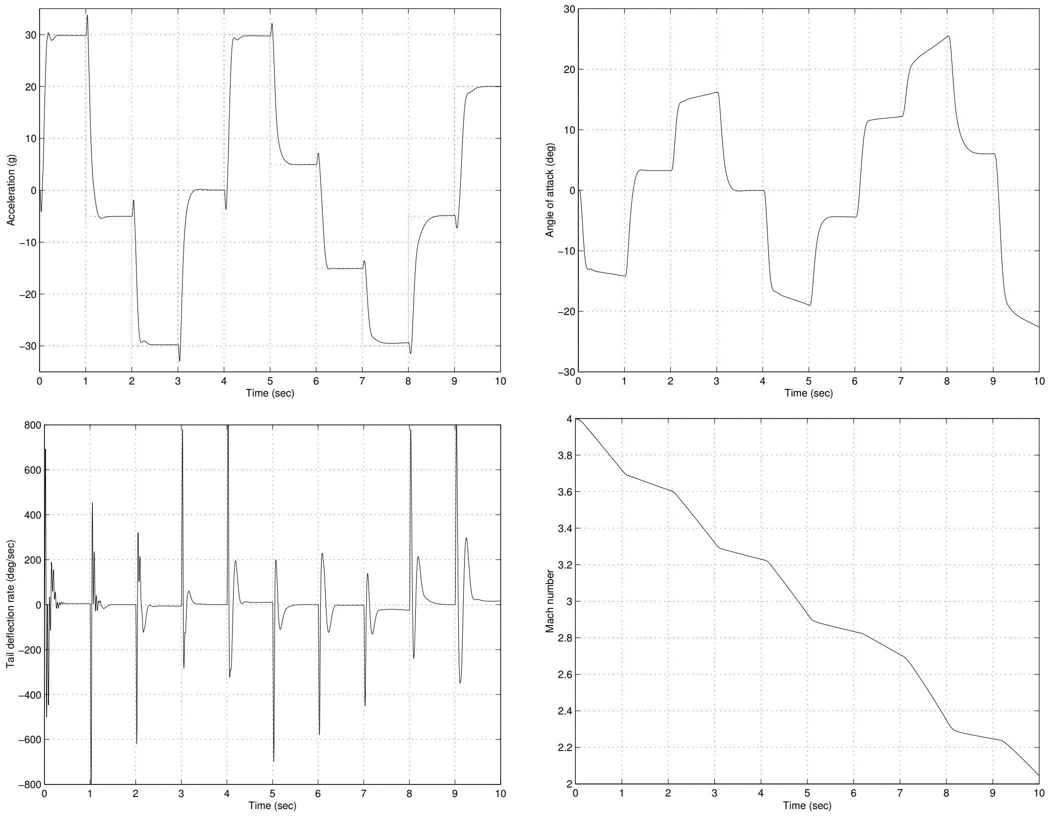

Fig. 4. Nonlinear closed-loop response using (K_2, Δ_{K_2}) for time-varying M

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