STABILITY OF HYBRID CONTROL SYSTEMS: CRITERIA OF STABILITY AND RISK ANALYSIS

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Abstract A new approach of hybrid control systems, called *mathematical-heuristic modelling*, and its applications in analysis, simulation and stability of these systems, is presented. The developed stability criteria of hybrid control systems are useful for simulation and control of the now systems and for their stability evaluation. The case of a high complexity hydrological system, using a PC 586 or upgrade is illustrated.

Keywords: hybrid system, mathematical-heuristic model, stability criteria, risk analysis

1. INTRODUCTION

Classical hybrid control systems include a discrete event controller (Stiver&all, 1994; Cellier, 1993). This paper extends the concept of hybrid control system by including a fuzzy knowledge-based controller. This approach is based on *mathematical - heuristic model*, a hybrid model including a fuzzy model (Stanciulescu, 1995, 1999). The new approach brings about an important theoretical problem: the compatibility between the mathematical model of and the heuristic model, i.e. the fuzzy control model. This problem could be solved by means of a compatibility theorem (Stanciulescu, 2001). This approach let us to find a solution to the problem of stability of hybrid control systems by means of stability criteria.

2. MATHEMATICAL-HEURISTIC MODEL

The *mathematical-heuristic model* of a complex system is a model composed of interconnected (sub)models, i.e. a continuous simulation model and/or a discrete-time simulation model, a discrete event simulation model, and a heuristic and/or a fuzzy knowledge-based control model.

The structure of the mathematical-heuristic model is presented in Figure 1

Figure 1 The structure of the mathematical-heuristic model

A definition of the mathematical-heuristic model is: the *mathematical-heuristic model* is a symbolic representation of systems, consisting of two main parts, one is mathematical knowledge and other is heuristic knowledge; both mathematical and heuristic

knowledge are compatible.

2.1 The continuous processes simulation model

The continuous time simulation model of a complex system is a set of nonlinear differential equations:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + f_i(x_i, a_i) + v_i(x), \quad (1)$$

$$x_i(0) = x_{i0} \quad (2)$$

$$v_i(x) = \sum_{j=1, j \neq i}^n g_{ij}(x_j) \quad (3)$$

$$x_{i \min} \leq x_i \leq x_{i \max}, \quad (i=1, 2, \dots, n) \quad (4)$$

where: $x_i, v_i: R^{n_i} \rightarrow R$, $u_i: R^{m_i} \rightarrow R$ and where: A_i, B_i are matrices of state and control, f_i -a vectorial function describing nonlinearities of the process. The double inequality (4) interprets the expert desire that x_i belong to the interval $[x_{i \min}, x_{i \max}]$. The form (1)-(4) of the model describes the interconnected high complexity system.

2.2 Discrete time simulation model

The discrete time simulation model of a complex system is a set of nonlinear discrete-time equations, resulted from the discretization of a continuous simulation model:

The discrete time simulation model is

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k) + f_i(x_i(k), a_i) + v_i(x(k)) \quad (5)$$

also including the relations (2)-(4); k takes values from the discrete-time values set $\{k_0, k_1, \dots, k_i, \dots, k_f\}$

2.3 Discrete event processes simulation model

A discrete event simulation model will be included, which the standard form of is a Boolean differential equations set. The model of the discrete event system evolution is:

$$X_i(k+1) = F_i(X_1(k), X_2(k), \dots, X_n(k), U_1(k), U_2(k), \dots, U_m(k)) \quad (6)$$

$$Y_j(k+1) = G_j(X_1(k), X_2(k), \dots, X_n(k), U_1(k), U_2(k), \dots, U_m(k)), \quad (7)$$

where X_i and Y_j take values from the Boolean set $\{0, 1\}$, F_i is the discrete event state function, and G_j is the discrete event output function. Both F_i and G_j are Boolean functions. Let $x_i(t)$, ($i = 1, 2, \dots, n$) be states of

the continuous simulated system; we suppose that the variable t is discretised. So, we can define the discrete event variables:

$$X_i(k) = \begin{cases} 1, & \text{if } x_i(t) \in [x_{i \min}, x_{i \max}] \\ 0, & \text{other} \end{cases} \quad (8)$$

$$U_j(k) = \begin{cases} 1, & \text{if } u_j(t) \rightarrow x_i(t+1) \in [x_{i \min}, x_{i \max}] \\ 0, & \text{other} \end{cases} \quad (9)$$

where $[x_{i \min}, x_{i \max}]$ is the so-called suboptimality interval of $x_i(t)$ established by the expert.

2.4 Heuristic knowledge-based control model

We classify the heuristic rules in three main groups: *Behavioral heuristic rules*, shows the qualitative dependence of the state x_i on parameters α_i :

$$\langle \text{If } C_i(\alpha_i(k)), \text{Then } S_i(x_i(k)) \rangle, \quad (i=1, 2, \dots, n; k \in \{k_0, k_1, \dots, k_f\}) \quad (10)$$

Control heuristic rules, shows the qualitative dependence of the state x_i , on the control variable u_i :

$$\langle \text{If } S_i(x_i(k)) \wedge (u_i(k+1) \leftarrow u_i(k) + \Delta u_i(k)), \text{Then } S_i(x_i(k+1)) \rangle \quad (11)$$

Decision heuristic rules, shows the qualitative modification of the control variable $u_i(k)$, able to reset the state variable $x_i(k)$ into the suboptimality interval $[x_{i \min}, x_{i \max}]$.

$$\langle \text{If } (x_i(k) < x_{i \min}) \vee (x_i(k) > x_{i \max}), \text{Then } (u_i(k+1) \leftarrow u_i(k) \boxplus \Delta u_i(k)) \rangle \quad (12)$$

(in (11) and (12) " $\leftarrow$ " is the *assigning operator*)

In the heuristic rules (10)-(12) $x_i(k)$, (as well as $\Delta u_i(k)$) takes values from the set Q_5 ; the derivatives $x'(k)$ take values from the set Q'_5 ; both the set $Q_5 = \{-L, -S, 0, +S, +L\}$ and $Q'_5 = \{DS, Ds, ST, Is, IS\}$, together with the qualitative universe $U_0 = \{-, 0, +, ?\}$, and the qualitative operations: $\approx$ (qualitative equality), $\oplus$ (qualitative addition) and $\otimes$ (qualitative product), made up the qualitative algebra, $Q = \{Q_5, Q'_5, U_0, \approx, \oplus, \otimes\}$. The above symbols define the following: L = Large, S = Small, DS = decrease strong, Ds = decrease slowly, ST = stationary, Is = increase slowly, IS = increase strong.

2.5 Fuzzy knowledge-based control model

The control problem is to maintain the state variables $x_i(k)$, $i=1, 2, \dots, n$; $k=0, 1, \dots, k_f$ into the (sub)optimality intervals $[x_{i \min}, x_{i \max}]$. The control model is a fuzzy

one (Zadeh, 1996; Negoita, 1988). The state variables $x_i(k)$ belong to one or another of the intervals given in the Table 1, as $dx_i(k) = x_i(k) - x_i(k-1)$, where: VS = Very Small, S = Small, O = (sub) Optimal, L = Large, VL = Very Large, Z = Zero, are qualitative values (in the acceptance of fuzzy set theory). The state variables x_i belong to set: $\chi = \{VS, S, ST, L, VL\}$ while the derivatives dx_i belong to set: $d\chi = \{DS, Ds, Z, Is, IS\}$. Computation of membership functions $\mu_i(x_i)$ is made by trapezoidal representation-Figure 2.

Figure 2 Trapezoidal representation of the membership function

From the Figure 2 we can infer:

$$m_i(x_i) = \begin{cases} 1, & \text{if } x_i \in [x_{imin}, x_{imax}] \\ 0, & \text{if } x_i < x_{imin} - \varepsilon, \text{ or } x_i > x_{imax} + \delta, \\ \frac{1}{\varepsilon} x_i - \frac{x_{imin} - \varepsilon}{\varepsilon}, & \text{if } x_i \in [x_{imin} - \varepsilon, x_{imin}] \\ -\frac{1}{\delta} x_i + \frac{x_{imax} + \delta}{\delta}, & \text{if } x_i \in [x_{imax}, x_{imax} + \delta] \end{cases} \quad (13)$$

Computation formula of the new control variables, $u_i(k+1)$, in order to reset the state $x_i(k)$ into the (sub)optimality intervals is:

$$u_i(k+1) = u_i(k) + K_{ui} \Delta u_i(k) \quad (14)$$

where $\Delta u_i(k)$ is computed by using the formula

$$\Delta u_i(k) = \frac{\sum_{i=1}^n m_i(x_i) \cdot u_i(k)}{\sum_{i=1}^n m_i(x_i)}, \quad (i=1, 2, \dots, n) \quad (15)$$

The set of fuzzy knowledge-based rules, useful in control of the complex system result

<If $(x_i(k) \text{ is } \chi_i) \wedge (dx_i(k) \text{ is } d\chi_i)$, Then the control law becomes $\langle u_i(k+1) = u_i(k) + K_{ui} \Delta u_i(k) \rangle (i=1, 2, \dots, n)$ (16)

Starting from rule (16) and using both Table 1 and $\Delta u_i(k)$ given by (15) the fuzzy control rules can be automatically generated. For each state variable $x_i(k)$ there are 25 fuzzy control rules. The set of fuzzy control rules for $x_i(k)$ can be viewed as generator of all fuzzy control rules $(i=1, 2, \dots, n)$.

The set of the 25 fuzzy control rules of the state variable $x_i(k)$ is:

< If $(x_i(k) \text{ is } VS) \wedge (dx_i(k) \text{ is } DS)$, Then the new control law is: $\langle u_i(k+1) = u_i(k) + K_{11} \Delta u_i(k) \rangle$,

< If $(x_i(k) \text{ is } VS) \wedge (dx_i(k) \text{ is } Ds)$, Then the new control law is: $\langle u_i(k+1) = u_i(k) + K_{21} \Delta u_i(k) \rangle$,

< If $(x_i(k) \text{ is } VL) \wedge (dx_i(k) \text{ is } IS)$, Then the new control law is: $\langle u_i(k+1) = u_i(k) + K_{55} \Delta u_i(k) \rangle$,

Remark. The mathematical model of the controlled process and the fuzzy control model are not a priorically automatically compatible. A theorem of compatibility has been produced; it constitutes the theoretical underlies of the hybrid control algorithm.

3. STABILITY OF HYBRID CONTROL SYSTEMS DESCRIBED BY MATHEMATICAL-HEURISTIC MODELS.

3.1. A criterion of stability of hybrid control systems

The mathematical-heuristic approach of high complexity systems leads to an important theoretical problem: the stability of systems described by means of mathematical-heuristic models and the compatibility of mathematical and heuristic/fuzzy models. The problem of the compatibility between the numerical simulation model and the fuzzy model was solved in terms of a theorem of compatibility (Stanculescu, 1999). Based on the same reasoning we can obtain a *criterion of stability of hybrid control systems*. Let:

$$\begin{aligned} x_i(k+1) &= A_i x_i(k) + B_i u_i(k) + f_i(x_i(k), a_i) + v_i(x(k)), \\ x_i(0) &= x_{i0}, \quad (i=1, 2, \dots, n). \end{aligned}$$

Table 1 Intervals and qualitative values of x_i and dx_i

Interval x_i	$< x_{imin} - \varepsilon$	$[x_{imin} - \varepsilon, x_{imin}]$	$[x_{imax}, x_{imax}]$	$(x_{imax}, x_{imax} + \delta]$	$> x_{imax} + \delta$
x_i	VS	S	O	L	VL
Interval dx_i	$< -\alpha$	$[-\alpha, 0)$	0	$(0, \beta]$	$> \beta$
dx_i	DS	Ds	ST	Is	IS

be the numerical simulation model. The first step is to construct the row of functions

$$x_1(0), x_1(1), x_1(2), x_1(3), x_1(4), x_1(k), x_1(k+1), x_1(n)$$

Using the method of complete induction, the formula for computation of $x_i(k)$ can be given:

$$x_i(k) = A_i^k x_{i0} + \sum_{j=0}^{k-1} A_i^{k-j-1} B_i u_{ij} + \sum_{j=0}^{k-1} A_i^{k-j-1} f_{ij} + \sum_{j=0}^{k-1} A_i^{k-j-1} v_{ij}, \quad (17)$$

Suppose that $x_i(k) \notin [x_{i1}, x_{i2}]$. By using the fuzzy knowledge-based control model a new control variable (able to reset the state $x_i(k)$ into the suboptimality interval) can be found: $u_i(k+1) = u_i(k) + K_{ui} \Delta u_i(k)$, where K_{ui} is given by the expert and $\Delta u_i(k)$ is computed by using the formula (15). The state variable x_i , at the time moment $k+1$, results:

$$x_i(k+1) = A_i^{k+1} x_{i0} + B_i \left\{ \sum_{j=0}^k A_i^{k-j} u_{ij} + K_{ui} \frac{\sum_{j=1, j \neq i}^n \mu_j(x_j) u_j(k)}{\sum_{j=1, j \neq i}^n \mu_j(x_j)} \right\} + \sum_{j=0}^k A_i^{k-j} f_{ij} + \sum_{j=0}^k A_i^{k-j} v_{ij} \quad (18)$$

We set the condition that the constraint (4) be accomplished i.e. that $x_i(k) \in [x_{imin}, x_{imax}]$:

$$x_{imin} \leq A_i^{k+1} x_{i0} + B_i \left\{ \sum_{j=0}^k A_i^{k-j} u_{ij} + K_{ui} \frac{\sum_{j=1, j \neq i}^n \mu_j(x_j) u_j(k)}{\sum_{j=1, j \neq i}^n \mu_j(x_j)} \right\} + \sum_{j=0}^k A_i^{k-j} f_{ij} + \sum_{j=0}^k A_i^{k-j} v_{ij} \leq x_{imax} \quad (19)$$

Note that the expert gives x_{imin} and x_{imax} and ω_i is computable, since the value of x_{i0} and f_{ij} are given and v_{ij} are computable at each step. From (19) here results

$$B_i^{-1} (x_{imin} - \omega_i) \leq \sum_{j=0}^k A_i^{k-j} u_{ij} + K_{ui} \frac{\sum_{j=1, j \neq i}^n \mu_j(x_j) u_j(k)}{\sum_{j=1, j \neq i}^n \mu_j(x_j)} \leq B_i^{-1} (x_{imax} - \omega_i), \quad (20)$$

where:

$$\omega_i = A_i^{k+1} x_{i0} + \sum_{j=0}^k A_i^{k-j} f_{ij} + \sum_{j=0}^k A_i^{k-j} v_{ij} \quad (21)$$

The criterion of stability can be stated as

Theorem 1. The hybrid control system composed of the discrete-time controlled process, described by eqs. (2)-(12) and the fuzzy controller described by eqs. (13)-(16), is stable if and only if the double inequality (20) holds.

3.2. A criterion of structural stability of hybrid control systems

The stability criterion of hybrid control system imposes the strict constraint that state variable $x_i(k)$ must belong to suboptimality interval $[x_{i \min}, x_{i \max}]$; this interval can be very narrow and consequently the control $u_i(k)$, able to maintain $x_i(k)$ into interval $[x_{i \min}, x_{i \max}]$, can be rather difficult to obtain. Sometimes it is possible to relax the suboptimality interval, i.e. to replace this interval by an admissibility interval, so that the fuzzy control system be stable for small perturbation in initial conditions x_{i0} and/or system's parameters a_i ($i = 1, 2, n$). Let $x_{i0} + \delta x_{i0}$ and $a_i + \delta a_i$ ($i = 1, 2, \dots, n$), the new values of the initial conditions and system's parameters. By replacing x_{i0} and a_i by $x_{i0} + \delta x_{i0}$ and $a_i + \delta a_i$ respectively in the state equation (5) there results:

$$\tilde{x}_i(k+1) = A_i \tilde{x}_i(k) + B_i u_i(k) + f_i(x_i(k), \alpha_i + \delta \alpha_i) + v_i(x(k)),$$

$$\tilde{x}_i(0) = x_{i0} + \delta x_{i0}, \quad (i = 1, 2, \dots, n).$$

Taking the same procedure we obtain the expression of the state variable $x_{i, k+1}$ is obtained:

$$\tilde{x}_i(k+1) = A_i^{k+1} (x_{i0} + \delta x_{i0}) + B_i \left\{ \sum_{j=0}^k A_i^{k-j} u_{ij} + K_{ui} \frac{\sum_{j=1, j \neq i}^n m_j(x_j) u_j(k)}{\sum_{j=1, j \neq i}^n m_j(x_j)} \right\} + \sum_{j=0}^k A_i^{k-j} \tilde{f}_{ij} + \sum_{j=0}^k A_i^{k-j} \tilde{v}_{ij} \quad (22)$$

The problem of structural stability of the hybrid control system can be expressed as follows: the hybrid control system is structural stable, with respect to small changes of initial conditions and/or system parameters, if and only if the following limit holds:

$$\lim_{k \rightarrow \infty} (x_i(k+1) - \tilde{x}_i(k+1)) \rightarrow 0 \quad (23)$$

or, if here exists a number K so that for all $k > K$

$|x_i(k+1) - \tilde{x}_i(k+1)| < \varepsilon$, where ε is arbitrary small ($i = 1, 2, \dots, n$). By introducing the expression of

$$x_i(k+1) \text{ and } \tilde{x}_i(k+1) \text{ in (23),}$$

After some calculi here results:

$$|A_i^{k+1} \delta x_{i0} + \sum_{j=0}^k A_i^{k-j} (f_{ij} - \tilde{f}_{ij}) + \sum_{j=0}^k A_i^{k-j} (v_{ij} - \tilde{v}_{ij})| < \varepsilon \quad (24)$$

(i = 1, 2, ..., n)

Inequality (24) represents the final form of the structural stability criterion of hybrid control systems:

Theorem 2: *The hybrid control system is structural stable, with respect to small changes of initial conditions and/or system parameters, if and only if the inequality (24) holds.*

4. RISK EVALUATION IN HIGH COMPLEXITY SYSTEMS

The *mathematical-heuristic modelling* of a high complexity system (Siljak, 1983) includes sub-optimality intervals, associated with each state, so that: $x_i \in [x_{i \min}, x_{i \max}]$. It is important to evaluate the possibility that x_i is less than $x_{i \min}$, or bigger than $x_{i \max}$. We will note: $\delta x_{i \min}$, the risk that $x_i < x_{i \min}$, and $\delta x_{i \max}$ the risk that $x_i > x_{i \max}$.

Definition of risk $\delta x_{i \min}$

By definition we understand by risk $\delta x_{i \min}$

$$\delta x_{i \min} = \begin{cases} e^{-\frac{\|x_{i \min} - x_i(k)\|}{x_{i \min}}}, & \text{if } x_i \in [x_{i \min}, x_{i \max}] \\ 1, & \text{if } x_i < x_{i \min}; \end{cases} \quad (25)$$

The evaluation of risk $\delta x_{i \min}$ in % results, by using the norm :

$$\delta x_{i \min} \% = \begin{cases} \frac{-\min_{k \in [0, k]} \|x_{i \min} - x_i(k)\|}{x_{i \min}} * 100, & \text{if } x_i(k) \in [x_{i \min}, x_{i \max}] \\ 100, & \text{if } x_i(k) < x_{i \min} \end{cases} \quad (26)$$

Definition of risk $\delta x_{i \max}$

By definition we understand by the risk $\delta x_{i \max}$,

$$\delta x_{i \max} = \begin{cases} e^{-\frac{\|x_{i \max} - x_i(k)\|}{x_{i \max}}}, & \text{if } x_i \in [x_{i \min}, x_{i \max}] \\ 1, & \text{if } x_i(k) > x_{i \max}; \end{cases} \quad (27)$$

The evaluation of risk $\delta x_{i \max}$ in % results, by using the norm expression:

$$\delta x_{i \max} \% = \begin{cases} \frac{-\min_{k \in [0, k]} \|x_{i \max} - x_i(k)\|}{x_{i \max}} * 100, & \text{if } x_i(k) \in [x_{i \min}, x_{i \max}] \\ 100, & \text{if } x_i(k) > x_{i \max} \end{cases} \quad (28)$$

The above evaluation formulae of risk of a state variable of a discrete-time system let us computer study risk, in different fields of activity and in concrete applications.

5. CASE STUDY: A HYDROLOGICAL SYSTEM

5.1. Simulation results

The theoretical approach has been applied to simulation and control of hydrological system of Danube River Delta. Using the mathematical-heuristic simulation model there results the matrix W , including numerical simulation results of the water refreshment factor $W_{i,k}$ ($i=0, 1, \dots, 5$; $k=0, 1, \dots, 11$):

	0	1	2	3
0	37.325	4.462	51.977	31.364
1	25.596	24.586	0	21.945
2	46.692	0	61.892	37.699
3	9.857	15.542	5.332	6.177
4	14.094	19.958	8.217	19.135
5	16.243	47.63	0	53.261

Figure 3 depicts the water refreshment factor $W_{i,k}$ of a big lake from a network of interconnected lakes.

Figure 3 Simulation of the water refreshment factor $W_{1,k}$ of a big lake of a River Delta

In Figure 3 the lower curve $W_{i,k}$ represents the dynamics of the water refreshment factor in a natural (uncontrolled) regime, while the upper curves $nW_{i,k}$ represent the dynamics of the controlled regime. Since $W_{i,k}$ sometimes exceeds both limits it is necessary to evaluate and control the risk that $W_{i,k}$ goes beyond the limits of the suboptimality intervals.

By using Mathcad Plus the risks $\delta W_{i\min}$ and $\delta W_{i\max}$ have been computed for $W_{\max} = 75$, $W_{\min} = 15$. The risk that W_i exceeds the lower limits is:

$\delta W_{i\min}$
89.807
48.786
36.788
93.568
94.143
100

The risk that W_i exceeds the upper limits is:

$\delta W_{i\max}$
77.301
56.191
89.062
46.551
54.404
100

5.2. Control of hydrological system by using the fuzzy knowledge-based control model

The simulation results and the risk evaluation have shown that the risk of the state variable $W_i(k)$, which exceeds the suboptimality interval $[W_{\min}, W_{\max}]$ and become unstable exists. So a control of these state variables, by using the fuzzy control model, in order to stabilise it, must be exerted. As the controlled state variables are $W_i(k)$ $i=1, 2, \dots, n$, we materialise the parameters from table 1: $x_{i\min}=0.2$, $x_{i\max}=0.5$, $\varepsilon=0.1$, $\delta=0.3$. The mathematical expression of the membership function $\mu_i(W_i(k))$ is:

$$\mu_i(W_i(k)) = \begin{cases} 1, & \text{if } W_i \in [0.2, 0.5], \\ 0, & \text{if } W_i \in [0, 0.1] \text{ or } W_i \in (0.8, 1], \\ 10W_i - 1, & \text{if } W_i \in [0.1, 0.2] \\ -\frac{10}{3}W_i + \frac{8}{3}, & \text{if } W_i \in (0.5, 0.8] \end{cases}$$

The new control variables $f_i(k)$ are computed as (see (12) and (13))

$$f_i(k+1) = f_i(k) + Kf_i \Delta f_i(k), \text{ where:} \\ \Delta f_i(k) = \sum_{i=1}^6 \mu_i(W_i) \cdot f_i(k) / \sum_{i=1}^6 \mu_i(W_i)$$

The expert gives the heuristic coefficients Kf_i . To illustrate the above a rule is given (see (16)):

<If $(W_i(k) = S) \wedge (dW_i(k) = -L)$, Then the law of fuzzy control is: $f_i(k+1) = f_i(k) - 0.35 \cdot \Delta f_i(k)$ (and other 24 fuzzy control rules).

Control results have been obtained by using the above described fuzzy control model, the Mathcad Plus under Windows, for simulation and control of a high complexity hydrological system.

6. CONCLUDING REMARKS

A new approach of hybrid control systems, called *mathematical-heuristic modelling*, including sub-optimality intervals, associated with each state, and its applications in analysis, simulation and control such systems has been presented. The *mathematical-heuristic model* of a complex system is a model composed of interconnected (sub)models, i.e. a continuous simulation model and/or a discrete-time simulation model, a discrete event simulation model, and a heuristic and/or a fuzzy knowledge-based control model. Stability criteria of hybrid control systems have been set up. *Mathematical - heuristic modelling* is used to simulate and control the behaviour of a high complexity system and to evaluate the stability of the system and the risk that the state variables exit from the suboptimality intervals. Computer experiment has been made to simulate a hydrological system, and evaluate the system stability, i.e. the risk that some ecological state variables exceed the lower and the upper limits. Comparing simulation and experimental results has validated the theoretical approach.

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