15th Triennial World Congress, Barcelona, Spain

BACKLASH COMPENSATION OF NONLINEAR SYSTEMS USING FUZZY LOGIC

Jun Oh Jang*, Pyeong Gi Lee*, Hee Tae Chung, and In Soo Lee*****

** Department of Computer Control Engineering, Uiduk University*
Kyongju, 780-713, South Korea, Email : jojang@mail.uiduk.ac.kr
*** Department of Electronic Engineering, Pusan University of Foreign Studies*
**** School of Electronic and Electrical Engineering, Sangju National University*

Abstract: A backlash compensator is designed for nonlinear systems using the fuzzy logic. The classification property of fuzzy logic systems makes them a natural candidate for the rejection of errors induced by the backlash, which has regions in which it behaves differently. A tuning algorithm is given for the fuzzy logic parameters, so that the backlash compensation scheme becomes adaptive, guaranteeing small tracking errors and bounded parameter estimates. Formal nonlinear stability proofs are given to show that the tracking error is small. The fuzzy logic backlash compensator is simulated on a nonlinear system to show its efficacy. *Copyright © 2002 IFAC*

Keywords: Backlash compensation, Fuzzy logic, Nonlinear systems, Inversion, Tracking performance
.

1. INTRODUCTION

Very accurate control is required in mechanical devices such as xy positioning tables (Li and Cheng, 1994), overhead crane mechanisms (Mahfouf, et al., 2000), robot manipulators (Jang and Jeon, 1999), etc. For many of these devices, the performance is limited by deadzone, friction, and backlash. Precise positioning, in particular, control of very small displacement is an especially difficult problem for micro positioning devices. Due to the nonanalytic nature of the actuator nonlinearities and the fact that their exact parameters are unknown such systems present a challenge for the control design engineer. A number of control strategies have been developed to overcome the problems caused by the backlash effects. Backlash characteristics are common in control system components such as mechanical connections and electromagnetic devices with hysteresis (Krasnoselskii and Pokrovskii, 1989). They are non differentiable nonlinearities and have been among the factors severely limiting the performance of feedback systems. A backlash element is itself a dynamic system with memory and characterized by parameters. The key feature of a backlash control scheme is to use a dynamic backlash inverse to cancel the effect of the backlash characteristic so that a linear controller structure can be employed to achieve the control objective.

Recently, in seminal work several rigorously derived adaptive schemes have been given for actuator nonlinearity compensation (Tao and Kokotovic, 1996). Backlash compensation using adaptive inverse method is considered in (Tao and Kokotovic, 1995a). Dynamic inversion using a neural network is presented in (Selmic and Lewis, 1999) and (Campos, et al., 2000) for discrete time, where the neural network is used for cancellation of the inversion error. Backlash compensation of systems using fuzzy logic is presented in (Jang, et al., 2001).

The use of fuzzy logic systems has accelerated in recent years in many areas, including feedback control (Jamshidi, et al., 1993). Fuzzy logic deadzone compensation schemes are provided in (Kim, et al., 1994; Lewis, et al., 1999; Jang, 2001). Particularly important in fuzzy logic control are the universal function approximation capabilities of fuzzy logic systems (Kosko, 1994; Wang and Mendel, 1992). The fuzzy logic systems offer significant advantages over adaptive control, including no requirement for linearity in the parameters assumptions and no need to compute a regression matrix for each specific system. Actuator nonlinearities are typically defined in terms of piecewise linear functions according to the region to which the argument belongs. The fuzzy logic function approximation properties and ability of fuzzy logic systems to discriminate information based on regions of the input variables, makes them an ideal candidate for compensation of non-analytic actuator nonlinearities.

In this paper, authors present the backlash compensation method of nonlinear systems using fuzzy logic. A rigorous design procedure with proofs is given that results in a PD tracking loop with an adaptive fuzzy logic system in the feedforward loop for backlash compensation. Authors derive a practical bound on tracking error from the analysis of the tracking error dynamics and investigate the performance of the fuzzy logic backlash compensator in a nonlinear system through the computer simulations.

2. FUZZY LOGIC COMPENSATION OF BACKLASH NONLINEARITY

In this section a fuzzy logic precompensator is designed for the non-symmetric backlash nonlinearity. It is shown that the fuzzy logic approach includes and subsumes approaches based on switching logic and indicator functions (Tao and Kokotovic, 1995b; Recker, et al., 1991). This brings these references very close to fuzzy logic work in (Kim, et al., 1994), and potentially allows for more exotic compensation schemes for actuator nonlinearities using more complex decision (e. g. membership) functions. This section provides a rigorous framework for fuzzy logic applications in backlash compensation for a broad class of systems.

The backlash model and a simple backlash example are shown in Fig. 1. The backlash characteristic $B(\cdot)$ with input $u(t)$ and output $T(t)$: $T(t) = B(u(t))$ is described by two parallel straight lines, upward and downword sides of $B(\cdot)$, connected with horizontal line segments. Mathematically, the backlash is modeled as

$$\dot{T}(t) = B(T, u, \dot{u}) \\ = \begin{cases} \dot{u}(t), & \text{if} \quad \dot{u}(t) > 0 \quad \text{and} \quad T(t) = u(t) - d_+ \\ & \text{or} \quad \dot{u}(t) < 0 \quad \text{and} \quad T(t) = u(t) - d_- \\ 0, & \text{otherwise} \end{cases} . \tag{1}$$

One can see that backlash is a first order velocity driven dynamic system, with inputs u and $\dot{u}$, and state T. It contains its own dynamics, therefore its compensation requires the design of the dynamic

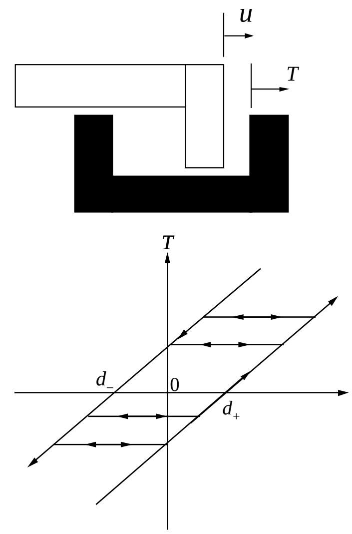

Fig. 1. Backlash Nonlinearity.

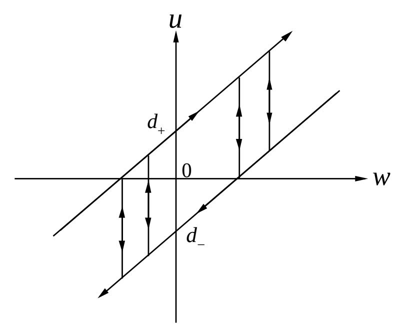

Fig. 2. Backlash inverse.

compensator. Whenever the motion $u(t)$ changes its direction, the motion $T(t)$ is delayed from motion of $u(t)$. The objective of a backlash compensator is to make this delay as small as possible, i.e. to make the $T(t)$ to closely follow $u(t)$. In order to cancel the effect of backlash in the system, the backlash precompensator needs to generate inverse of the backlash nonlinearity. The backlash inverse function is shown in Fig. 2. The dynamics of the backlash inverse is given by

$$\dot{u}(t) = \\ \begin{cases} \dot{w}(t) & \text{if} \quad \dot{w}(t) > 0 \quad \text{and} \quad u(t) = w(t) + d_+ \\ & \text{or} \quad \dot{w}(t) < 0 \quad \text{and} \quad u(t) = w(t) + d_- \\ 0 & \text{if} \quad \dot{w}(t) = 0 \\ g(t,t) & \text{if} \quad \dot{w}(t) > 0 \quad \text{and} \quad u(t) = w(t) + d_- \\ -g(t,t) & \text{if} \quad \dot{w}(t) < 0 \quad \text{and} \quad u(t) = w(t) + d_+ \end{cases} \tag{2}$$

where $g(\tau, t) = \delta(\tau - t)(d_+ - d_-)$ with $\delta(t)$ being the Dirac δ function. In this definition the inverse of a horizontal segment of the backlash characteristic is a vertical jump of a distance $d_+ - d_-$.

To offset the deleterious effects of backlash, one may place a precompensator as illustrated in Fig. 3.

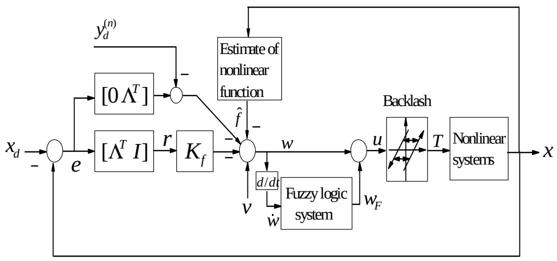

Fig. 3. Fuzzy backlash compensation of a nonlinear system.

There, the desired function of the precompensator is to cause the composite throughput from w to T to be unity. The power of fuzzy logic systems is to that they allow one to use intuition based on experience to design control systems, then provide the mathematical machinery for rigorous analysis and modification of the intuitive knowledge, for example through learning or adaptation, to give guaranteed performance, as will be shown in Section 3. Due to the fuzzy logic classification property, they are particularly powerful when the nonlinearity depends on the region in which the argument u of the nonlinearity is located, as in the non-symmetric backlash.

A backlash precompensator using dynamic inversion would be discontinuous and depend on the region within which $\dot{w}\,(= dw/dt)$ occurs. It would be naturally described using the rules

$$\begin{aligned}&\text{If (} \dot{w} \text{ is positive) then (} u = w + \hat{d}_+ \text{)}\\ &\text{If (} \dot{w} \text{ is zero) then (} u = w + \hat{d}_0 \text{)}\\ &\text{If (} \dot{w} \text{ is negative) then (} u = w + \hat{d}_- \text{)}\end{aligned} \qquad (3)$$

where $\hat{d} = [\hat{d}_+\ \hat{d}_0\ \hat{d}_-]^T$ is an estimate of the backlash width parameter vector $d = [d_+\, d_0\, d_-]^T$. To make this intuitive notion mathematically precise for analysis define the membership function's

$$X_+(\dot{w}) = \begin{cases} 0, & \dot{w} < 0 \\ 1, & 0 < \dot{w} \end{cases}$$

$$X_0(\dot{w}) = \begin{cases} 0, & \dot{w} \neq 0 \\ 1, & \dot{w} = 0 \end{cases} \qquad (4)$$

$$X_-(\dot{w}) = \begin{cases} 1, & \dot{w} < 0 \\ 0, & 0 < \dot{w} \end{cases}.$$

One may write the precompensator as

$$u = w + w_F \qquad (5)$$

where w_F is given by the rule base

$$\begin{aligned}&\text{If (} \dot{w} \in X_+(\dot{w}) \text{) then (} w_F = \hat{d}_+ \text{)}\\ &\text{If (} \dot{w} \in X_0(\dot{w}) \text{) then (} w_F = \hat{d}_0 \text{)}\\ &\text{If (} \dot{w} \in X_-(\dot{w}) \text{) then (} w_F = \hat{d}_- \text{).}\end{aligned} \qquad (6)$$

The output of the fuzzy logic system with this rule base is given by

$$w_F = \frac{\hat{d}_+ X_+(\dot{w}) + \hat{d}_0 X_0(\dot{w}) + \hat{d}_- X_-(\dot{w})}{X_+(\dot{w}) + X_0(\dot{w}) + X_-(\dot{w})}. \qquad (7)$$

The estimates $\hat{d}_+$, $\hat{d}_0$, $\hat{d}_-$ are, respectively, the control representive value of $X_+(\dot{w})$, $X_0(\dot{w})$, and $X_-(\dot{w})$. This may be written (note $X_+(\dot{w}) + X_0(\dot{w}) + X_-(\dot{w}) = 1$) as

$$w_F = \hat{d}^T X(\dot{w}) \qquad (8)$$

where the fuzzy logic basis function vector given by

$$X(\dot{w}) = \begin{bmatrix} X_+(\dot{w}) \\ X_0(\dot{w}) \\ X_-(\dot{w}) \end{bmatrix} \qquad (9)$$

is easily computed given any value of $\dot{w}$. It should be noted that the membership functions (4) are the indicator functions and $X(\dot{w})$ is similar to the regressor (Tao and Kokotovic, 1995b; Recker, et al., 1991). The fuzzy logic compensator may be expressed as follows

$$\begin{aligned} u &= w + w_F \\ &= w + \hat{d}^T X(\dot{w}) \end{aligned} \qquad (10)$$

where $\hat{d}$ is estimated backlash widths.

Given the fuzzy logic compensator with rulebase (6), the throughput of the compensator plus backlash is given by

$$T = w + \tilde{d}^T X(\dot{w}) - \tilde{d}^T \delta \qquad (11)$$

where the backlash width estimation error is given by

$$\tilde{d} = d - \hat{d} \qquad (12)$$

and the modeling mismatch term δ is bounded so that $|\delta| < \delta_M$ for some scalar δ_M.

3. ADAPTIVE FUZZY LOGIC BACKLASH COMPENSATION OF NONLINEA SYSTEMS

In this section authors show how to provide the fuzzy logic backlash compensation for backlash in nonlinear systems. The fuzzy logic backlash compensator is given by (10). Authors show to tune or learn the backlash width estimates $\hat{d}$ on-line so that the tracking error is guaranteed small and all internal states are bound. This turns the backlash compensator into an adaptive fuzzy logic backlash compensator. It is assumed, of course, that the backlash output $T(t)$ is not measurable.

The dynamics of a large class of single input nonlinear systems can be written in the Brunovsky form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\dots \\ \dot{x}_n &= f(x) + T(k) + T_d(k) \\ y &= x_1 \end{aligned} \qquad (13)$$

where the output is $y(t)$, the state is $x = [x_1\, x_2 \ldots x_n]^T$, T_d is the bounded unknown disturbance, and T is the actuator output, and $f(x)$

presents system nonlinearities like friction etc. The actuator output $T(t)$ is related to the control input $u(t)$ through the backlash nonlinearity (1). Therefore overall dynamics of the system consists of (13) and backlash dynamics (1).

It is assumed that $|T_d| < \tau_d$, with τ_d, a known positive constant. The unknown backlash widths are bound so that $|d| < d_M$ for some scalar d_M and are constant so that $\dot{d} = 0$. The nonlinear function $f(x)$ is assumed to be unknown, but a fixed estimate $\hat{f}(x)$ is assumed known such that the functional estimation error, $\tilde{f}(x) = f(x) - \hat{f}(x)$, satisfies $\|\tilde{f}(x)\| \leq f_M(x)$, for some known bounding $f_M(x)$.

To design a motion controller that causes the system output, $y(t)$, to track a smooth prescribed trajectory, $y_d(t)$, authors define the desired state as

$$x_d(t) = [y_d \ \dot{y}_d \cdots y_d^{(n-1)}]^T \tag{14}$$

with $y_d^{(n-1)}$ the (n-1)st derivative. Authors define the error by

$$e = x - x_d \tag{15}$$

and the tracking error by

$$r = [\lambda_1 \quad \lambda_2 \quad \ldots \quad \lambda_{n-1} \quad 1]e \equiv \Lambda^T e \equiv [\Lambda^T \quad 1]e, \tag{16}$$

with Λ a gain parameter vector selected so that $e(t) \to 0$ exponentially as $r(t) \to 0$. Then (16) is stable system so $e(t)$ is bounded as long as controller guaranties that the tracking error $r(t)$ is bounded.

Differentiating tracking error and using (13), the dynamics may be written in terms of the tracking error as:

$$\dot{r} = f(x) + Y_d + T_d + T, \tag{17}$$

where

$$Y_d = -y_d^{(n)} + [0 \quad \Lambda^T]e \tag{18}$$

is known function of the desired trajectory and actual states. The desired trajectory is bounded so that $\|x_d(t)\| \leq X_d$, where X_d is a known constant.

A robust compensation scheme for unknown terms in $f(x)$ is provided by selecting the tracking controller

$$w = -\hat{f}(x) - K_f r - Y_d + v \tag{19}$$

with $\hat{f}(x)$, an estimate for the nonlinear terms $f(x)$, $v(t)$ a robustifying term. The feeedback gain $K_f > 0$ is often selected diagonal. Backlash compensation is provided using

$$u = w + \hat{d}^T X(\dot{w}) \tag{20}$$

with $X(\dot{w})$ given by (9), which gives the overall feedfordward throughout (11). The control structure implied by this scheme is shown in Fig. 3. The controller has a Proportional-Derivative(PD) tracking loop where the backlash effect is ameliorated by a feedfordward compensator. The estimate $\hat{f}(x)$ is computed by an inner nonlinear control loop (Jang and Jeon, 2000; Jang and Lee, 2000).

Substituting (19) and (11) into (17) yields the closed-loop error dynamics

$$\dot{r} = -K_f r + \tilde{d}^T X(\dot{w}) - \tilde{d}^T \delta + [\tilde{f} + T_d + v] \tag{21}$$

where the nonlinear functional estimation error is given by $\tilde{f} = f(x) - \hat{f}(x)$.

The next theorem provides an algorithm for tuning the backlash precompensator.

Theorem 1: Given the system (17), select the tracking control (19) plus backlash compensator (20), where $X(\dot{w})$ is given by (9). Choose the robustifying signal

$$v(t) = -(f_M(x) + \tau_d)\frac{r}{\|r\|}. \tag{22}$$

Let the estimated backlash widths be provided by the fuzzy logic system tuning algorithm

$$\dot{\hat{d}} = X(\dot{w})r^T - k\hat{d}\,\|r\| \tag{23}$$

where the scalar $k > 0$. Then the tracking error r evolves with a practical bound,

$$\|r\| \leq \frac{c_0^2}{4 \cdot K_{f\,\min} \cdot k}. \tag{24}$$

Proof : Define a Lyapunov function candidate for the error dynamics (17) as:

$$L = \frac{1}{2} r^T r + \frac{1}{2} \tilde{d}^T \tilde{d}. \tag{25}$$

Differentiating (25) yields:

$$\dot{L} = r^T \dot{r} + \tilde{d}^T \dot{\tilde{d}} \tag{26}$$

hence substitution of (21) yields

$$\dot{L} = -r^T K_f r + \tilde{d}^T (X(\dot{w})r^T - \delta r^T + \dot{\tilde{d}}) + r^T [\tilde{f} + T_d + v]. \tag{27}$$

Note that $\tilde{d} = d - \hat{d}$, and by (12), $\dot{\tilde{d}} = -\dot{\hat{d}}$. Therefore, substituting the tuning algorithm (23), robustifying term (22) gives

$$\begin{aligned}\dot{L} &= -r^T K_f r + \tilde{d}^T(-\delta r^T + k(d - \tilde{d})\|r\|) + r^T[\tilde{f} + T_d + v] \\ &\leq -K_{f\,\min}\|r\|^2 + \delta_M \|\tilde{d}\|\|r\| + kd_M\|\tilde{d}\|\|r\| - \\ &\quad k\|\tilde{d}\|^2\|r\|\end{aligned} \tag{28}$$

where $|\delta| < \delta_M$ for some scalar δ_M, $K_{f\,\min} = \sigma_{\min}(K_f)$, minimum singular value of K_f, and the bounding properties were used. Therefore

$$\dot{L} \leq -\|r\|[(K_{f\,\min}\|r\| - c_0\|\tilde{d}\| + k\|\tilde{d}\|^2] \tag{29}$$

with $c_0 \equiv \delta_M + kd_M$.

This is negative as long as the quantity in the brace is positive. To determine conditions for this, complete the square to see that $\dot{L}$ is negative as long as either

$$\|r\| > \frac{c_0^2}{4 \cdot K_{f\,\min} k} \tag{30}$$

or

$$\|\tilde{d}\| > \frac{c_0}{k}. \tag{31}$$

According to the standard Lyapunov theorem, the tracking error decreases as long as the error is bigger than the right-hand side of Eq. (30). This implies Eq. (32) gives a practical bound on the tracking error

$$\| r \| \leq \frac{c_0^2}{4 \cdot K_{f\,\min} \cdot k}. \tag{32}$$

◊

Also, Lyapunov extension shows that the backlash width bound, $\| \tilde{d} \|$, is bounded to a neighborhood of the right hand side of (31). Since a PD controller, K_f, is determined according to the design of a PD controller, K_f cannot be increased arbitrarily. However, large K_f may decrease the tracking error bound as long as the PD controller and the robust term maintain the stability of a control system.

4. SIMULATION RESULTS

In this section, authors illustrate the effectiveness of a fuzzy logic backlash compensator by computer simulations. One considers the nonlinear system:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{1}{T_M} x_2 + Max_2^2 \sin(x_1) + Mga\cos(x_1) + T \tag{33}$$

which represents a mechanical motion of robot like system with one link. The motor time constant is T_M, M is a net effective load mass, a a length, g the gravitational constant. Authors select $T_M = 1\sec$; $M = 1Kg$; $a = 2.5m$. The input T is passed through the additional backlash nonlinearity given by (1). The backlash set at $d_+ = 20$ and $d_- = -25$. The controller parameters are chosen as $\Lambda = 15$, $K_f = 10$. Fig. 4 shows the PD controller response without backlash. Authors simulate the system dynamics with backlash nonlinearity using a PD controller. The simulation result of a PD controller with backlash is shown in Fig. 5. The performance is degreaded by the backlash. Therefore authors use the PD controller with a backlash compensator in order to compensate

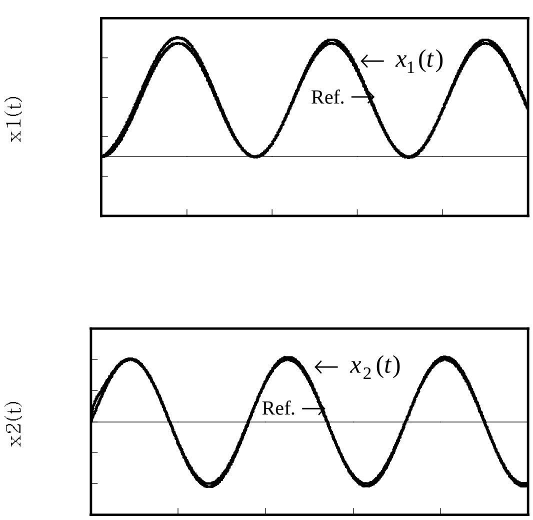

Fig. 4. State $x_1(t)$ and $x_2(t)$ without backlash.

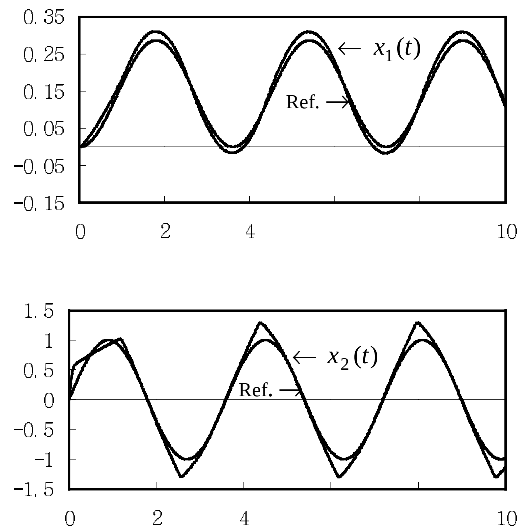

Fig. 5. State $x_1(t)$ and $x_2(t)$ with backlash.

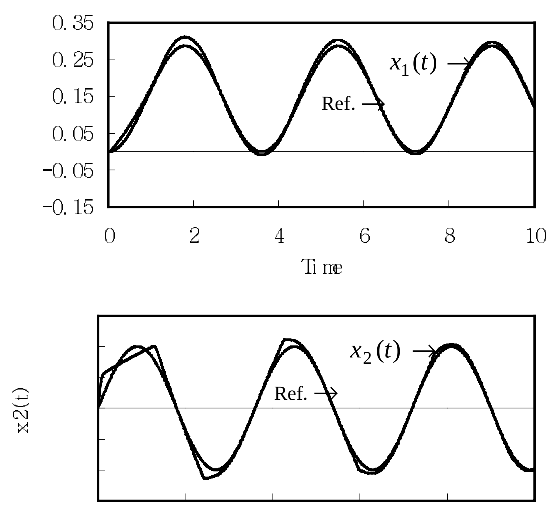

Fig. 6. State $x_1(t)$ and $x_2(t)$ with backlash compensation.

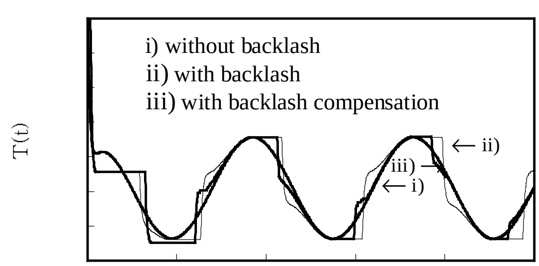

Fig. 7. Control inputs.

for backlash effects. The simulation result of the PD controller with a backlash compensator is shown in Fig. 6. The initial estimates for the backlash widths were selected as $d_+ = d_- = 0$. The proposed method exhibits an improvement in its response compared with the PD controller. The tracking is as good as it was without backlash after 7 seconds. Fig. 7 shows the control inputs with/without backlash compensation. The control input with backlash compensation converge to the control input without backlash, which means the desired function of the backlash compensator.

5. CONCLUSIONS

A fuzzy logic backlash compensator has been proposed for nonlinear systems. The classification property of fuzzy logic systems makes them a natural candidate for offsetting this sort of actuator nonlinearity having a strong dependence on the region in which the arguments occurs. It was shown how to tune the fuzzy logic parameters so that the unknown backlash parameters are learned on line, resulting an adaptive backlash compensator. Using nonlinear stability techniques, the bound on tracking error is derived from the tracking error dynamics. Simulation results show that significantly improved system performance can be achieved by the proposed adaptive fuzzy logic control schemes.

REFERENCES

Campos, J., F. L. Lewis and R. Selmic (2000). Backlash Compensation in discrete time nonlinear systems using dynamic inversion by neural networks. In *Proc. IEEE Conf. Robotics Automat.*, pp. 1289-1295. San Fransisco, CA.

Jang, J. O. (2001). A deadzone compensator of a DC motor system using fuzzy logic control. *IEEE Trans. Syst., Man, and Cybern. C*, **31(1)**, 42-48.

Jang, J. O. and G. J. Jeon (1999). Implementation of indirect neuro-control for nonlinear dynamic systems. *Mechatronics*, **9(6)**, 675-686.

Jang, J. O. and G. J. Jeon (2000). A parallel neuro-controller for DC motors containing nonlinear friction. *Neurocomputing*, **30**, 233-248.

Jang, J. O. and P. G. Lee (2000). Neuro-fuzzy control for DC motor friction compensation. In *Proc. IEEE Conf. Decision and Control*, pp. 3550-3555. Sydney, Australia.

Jang, J. O., P. G. Lee, S. B. Park and I. S. Ahn (2001). Backlash compensation of systems using fuzzy logic. In *Proc. American Contr. Conf.*, pp. 4788-4789. Arlington, VA.

Jamshidi, M., N. Vadiee and T. J. Ross (1993). *Fuzzy Logic and Control*, Prentice–Hall, Englewood Cliffs, NJ.

Kim, J. H., J. H. Park, S. W. Lee and E. K. P. Cheng (1994). A two layered fuzzy logic controller for systems with deadzones. *IEEE Trans. Ind. Electron.*, **41**, 155-162.

Kosko, B. (1994). Fuzzy systems as universal approximator. *IEEE Trans. Comput.*, **43**, 1-4.

Krasnoselskii, M. A. and A. V. Pokrovskii (1989). *Systems with Hysteresis*, Springer-Verlag, Berlin.

Lewis, F. L., W. K. Tim, L. Z. Wang and Z. X. Li (1999). Deadzone compensation in motion control systems using adaptive fuzzy logic control. *IEEE Trans. Contr. Syst. Technol.*, **7(6),** 731–742.

Li, W. and X. Cheng (1994). Adaptive high-precision control of positioning tables-Theory and experiment. *IEEE Trans. Contr. Syst. Technol.*, **2**, 265-270.

Mahfouf, M., C. H. Kee, M. F. Abbod and D. A. Linkens (2000). Fuzzy logic based anti-sway control design for overhead cranes. *Neural Computing & Applications*, **9**, 38-43.

Recker, D. A., P. V. Kokotovic, D. Rhode and J. Winkelman (1991). Adaptive nonlinear control of systems containing a deadzone. In *Proc. IEEE Conf. Decision and Control*, pp. 2111-2115. Brighton, UK.

Selmic, R. R. and F. L. Lewis (1999). Backlash compensation in nonlinear systems using dynamic inversion by neural networks. In *Proc. IEEE Conf. Control Applications*, pp. 1163-1168. Hawaii, HI.

Tao, G. and P. V. Kokotovic (1995a). Continuous time adaptive control of systems with unknown backlash. *IEEE Trans. Automat. Contr.*, **40(6)**, 1083-1087.

Tao, G. and P. V. Kokotovic (1995b). Discrete-time adaptive control of systems with unknown deadzones. *Int. J. Control*, **61(1)**, 1-17.

Tao, G. and P. V. Kokotovic (1996). *Adaptive Control of Systems With Actuator And Sensor Nonlinearity*, John Wiley & Sons, New York.

Wang, L. X. and J. M. Mendel (1992). Fuzzy basis functions, universal approximation, and orthogonal least-squares learning. *IEEE Trans. Neural Networks*, **3**, 807-814.