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HANKEL-NORM BASED INTERACTION MEASURE FOR INPUT-OUTPUT PAIRING

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Abstract: In this paper a new interaction measure for stable multivariable systems is introduced. The interaction measure, the Hankel Interaction Index, is a dynamic extension of the underlying idea in the classic Relative Gain Array (RGA) and its various extensions. The new index is based upon the Hankel norm of the SISO elementary subsystems built from the original MIMO system. The main advantage of the Hankel Interaction Index is its ability to quantify frequency dependent interactions and that it can be used for input-output pairing. Several examples are included to illustrate the new ideas presented.

Keywords: Interaction, Process control, Multivariable control, Sampled-data control, System design

1. INTRODUCTION

In the process industry, it is important to pair inputs and outputs to control the process and to use a set of SISO controllers. One of the main reasons is to be able to apply simple tuning strategies. Furthermore, full multivariable control is required only for special processes, since in many situations a good SISO design can strongly diminishes loop interaction.

To set the framework we consider a discrete time, $p \times p$ MIMO process with inputs $u_1[k], u_2[k], \ldots, u_p[k]$ and outputs $y_1[k], y_2[k], \ldots, y_p[k]$ related by

$$\begin{pmatrix} Y_1(z) \\ Y_2(z) \\ \vdots \\ Y_p(z) \end{pmatrix} = \mathbf{G}(z) \begin{pmatrix} U_1(z) \\ U_2(z) \\ \vdots \\ U_p(z) \end{pmatrix} \tag{1}$$

where

$$\mathbf{G}(z) = \begin{pmatrix} G_{11}(z) & G_{12}(z) & \cdots & G_{1p}(z) \\ G_{21}(z) & G_{22}(z) & \cdots & G_{2p}(z) \\ \vdots & \vdots & \ddots & \vdots \\ G_{p1}(z) & G_{p2}(z) & \cdots & G_{pp}(z) \end{pmatrix} \tag{2}$$

Although the ideas presented in this paper apply to both continuous-time and discrete-time MIMO systems, for simplicity, we will deal only with discrete-time systems.

Assume that we design p independent SISO control loops for $G_{11}(z), G_{22}(z), \ldots, G_{pp}(z)$. When those controllers are applied to the full MIMO plant (1), then the performance of the closed loop can be compared to the nominal decentralized design by comparing the nominal (diagonal) sensitivity, $\mathbf{S_o}(z)$, and the achieved sensitivity, $\mathbf{S}(z)$. They are related by (see e.g. Goodwin *et al.* (2000))

$$\mathbf{S}(z) = \mathbf{S_o}(z)[\mathbf{I} + \mathbf{H_T}(z)]^{-1}$$

where the element (i, j) of the matrix $\mathbf{H_T}(z)$ is given by

$$[\mathbf{H_T}(z)]_{ij} = \begin{cases} \dfrac{G_{ij}(z)}{G_{jj}(z)} T_j(z) & i \neq j, \\ 0 & i = j. \end{cases} \tag{3}$$

In (3), $T_j(z)$ is the nominal complementary sensitivity of the j:th loop. We observe that if the frequency responses of the off-diagonal terms, $[\mathbf{H_T}(z)]_{ij}$, are very small, then the resulting loops are well decoupled and almost behaves like p independent SISO control loops. This is achieved if $\mathbf{G}(z)$ is strongly (column) diagonally dominant, at least in the frequency bands of $T_1, T_2, \ldots, T_p$.

In this problem it is possible to distinguish several crucial elements, such as

- The (plant and loop) coupling characteristics, in general, are frequency dependent
- The (loop) coupling characteristics, in general, are dependent on the choices made for the SISO control designs.
- If the system steady-state gain is known, steady-state decoupling can be implemented leading to low frequency decoupled control loops.
- Different dominance characteristics may lead to a different pairing of inputs and outputs.

The Relative Gain Array (RGA), Bristol (1966), is a basic tool to decide on input-output pairing in multivariable processes. The advantage of the RGA and, at the same time, its disadvantage is that it only relies on the steady-state process gain. The RGA gives indications about the difficulties to control the process with only SISO controllers and, in many cases, suggests the most suitable input-output pairings. Different modifications have been proposed, see e.g. Niederlinski (1971) and Chiu and Arkun (1991). Also, attempts have been made to take the frequency response of the process into account, such as those reported in Gagnon *et al.* (1999), Zhu (1996), and Yang *et al.* (1999). Moore (1986) uses the singular values of $K = G(1) = U\Sigma V^T$ and the vectors of U and V to make the pairing. Structured singular values are used in Grosdidier and Morari (1986) to analyze interaction. In Stanley *et al.* (1985) the Relative Disturbance Gain is introduced, which includes the influence of the disturbances of the process. Witcher and McAvoy (1977) proposed a dynamic RGA (DRGA) as a natural extension of Bristol's RGA, namely

$$DRGA(\mathbf{G}) = \mathbf{G}(e^{j\omega}) \times \mathbf{G}(e^{j\omega})^{-T}$$

where the operation symbol $\times$ denotes element by element multiplication. Although this measure may reveal some hidden interaction features, it can be hard for the designer to make decisions regarding input-output pairing based upon the comparative analysis of $p \times p$ frequency responses. A good survey on input-output pairing can be found in Kinnaert (1995).

In Conley and Salgado (2000) a new gramian based interaction measure is introduced. This new measure is based upon the sum of the squared Hankel singular values for the elementary subsystems of the process. In this paper, a modification to that approach is proposed where we only use the Hankel norm of the subsystems. With this modification, we achieve further physical insight in terms of the controllability and observability of the different subsystems. The new measure deals with the full frequency range of the plant. However, it can also be used to investigate suitable bandwidths for the different control loops.

The idea of using gramians has already been used in some methods to solve the input/output selection problem, as reported, for instance, in van de Wal and de Jager (2001). However, in those cases the gramians serve to determine which inputs and outputs are to be used to control a process. In this paper, we assume that the input/output selection has already been made and we deal with how to pair those inputs and outputs to build a decentralized controller.

2. PROCESS DESCRIPTION AND PROBLEM FORMULATION

Assume that the (completely controllable and completely observable) process is described by its square pulse transfer function matrix $\mathbf{G}(z)$ given by (2) where $G_{ij}(z) \in \mathbb{C}$ denotes the transfer function from the j:th input to the i:th output. It is further assumed that the process is stable.

We note that $G_{ij}(z)$ describes the completely controllable and completely observable part of the elementary subsystem (i, j). The (minimal) realization for the whole process is given by the 4-tuple ($\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$) while the (not necessarily minimal) realization associated to the elementary subsystem (i, j) is given by the 4-tuple ($\mathbf{A}$, $\mathbf{B_j}$, $\mathbf{C_i}$, D_{ij}), where $\mathbf{B_j}$ is the j:th column of $\mathbf{B}$, $\mathbf{C_i}$ is the i:th row of $\mathbf{C}$ and where D_{ij} is the (i, j) element of matrix $\mathbf{D}$.

The main issue addressed in this paper is how to find the most suitable pairing of inputs and outputs. We also want to consider the dynamic properties of the system, which implies that we are looking for a more sophisticated measure than the RGA, since this only considers the zero frequency characteristics of the process, i.e. deals only with $\mathbf{G}(1)$ or at some other predefined frequency related to the desired closed loop bandwidth. Our goal naturally implies that we must be prepared to perform more elaborate computations to determine the new interaction index.

When the above defined goal is satisfactorily achieved, a sensible input-output pairing is found. This defines a controller structure, which can be used to design and to tune a decentralized controller. With some abuse of language this will be called a diagonal structure, since the controller structure can be made diagonal by permutations of the inputs and/or the outputs. We also want to have indications if a diagonal structure is insufficient to provide a good control.

3. HANKEL SINGULAR VALUES

The controllability gramian, $\mathbf{\Gamma}_c$, and the observability gramian, $\mathbf{\Gamma}_o$, associated to system (1) satisfy the Lyapunov equations

$$\mathbf{\Gamma}_c - \mathbf{A}\mathbf{\Gamma}_c\mathbf{A}^T - \mathbf{B}\,\mathbf{B}^T = 0$$

$$\mathbf{\Gamma}_o - \mathbf{A}^T \mathbf{\Gamma}_o \mathbf{A} - \mathbf{C}^T \mathbf{C} = 0$$

Furthermore the corresponding gramians, $\mathbf{\Gamma}_c^{(j)}$ and $\mathbf{\Gamma}_o^{(i)}$, associated to the elementary subsystem (i, j) satisfy

$$\mathbf{\Gamma}_c^{(j)} - \mathbf{A}\mathbf{\Gamma}_c^{(j)} \mathbf{A}^T - \mathbf{B_j} \mathbf{B_j}^T = 0 \quad (4)$$

$$\mathbf{\Gamma}_o^{(i)} - \mathbf{A}^T \mathbf{\Gamma}_o^{(i)} \mathbf{A} - \mathbf{C_i}^T \mathbf{C_i} = 0 \quad (5)$$

The controllability and observability gramians of a system quantify the difficulty to control and to observe the system state. For instance, the ranks of matrices $\mathbf{\Gamma}_c$ and $\mathbf{\Gamma}_o$ are the dimensions of the controllable and the observable subspaces, respectively. Gramians vary with state similarity transformations. However, the eigenvalues of the product of the gramians $\mathbf{\Gamma}_c$ and $\mathbf{\Gamma}_o$ are invariant with respect to those transformations (see, e.g. Glover (1984)).

If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues of $\mathbf{\Gamma}_c\mathbf{\Gamma}_o$, then the system Hankel singular values σ_i^H are

$$\sigma_i^H(\mathbf{G}(z)) = \sqrt{\lambda_i} \qquad i = 1, 2, \ldots, n$$

where, by convention, $\sigma_i^H \geq \sigma_{i+1}^H \geq 0$. The Hankel singular values (HSV) are fundamental invariants that are related to both the gain and the dynamic complexity of the system. The number r ($r \leq n$) of nonzero Hankel singular values is the dimension of the controllable and observable subspace. The HSV also play an important role when making balanced realizations and model reduction. We will use the HSV for a different purpose.

The Hankel norm of a system with transfer function $\mathbf{G}(z)$ is defined as

$$\|\mathbf{G}(z)\|_H = \sqrt{\lambda_{max}(\mathbf{\Gamma}_c\mathbf{\Gamma}_o)} = \sigma_1^H$$

The Hankel norm is thus the maximum HSV. The Hankel norm can also be interpreted in the following way. Consider a system input $u[k] = 0 \ \forall k > 0$ and a system output $y[k]$, then

$$\|\mathbf{G}(z)\|_H = \sup_{u \in \ell_2(-\infty,0)} \frac{\|y\|_{\ell_2(0,\infty)}}{\|u\|_{\ell_2(-\infty,0)}} \quad (6)$$

where $\ell_2(0,\infty)$ is the space of square summable vector sequences in the interval $[0,\infty)$. An analogous definition applies to $\ell_2(-\infty,0)$.

The Hankel norm gives the ℓ_2 gain from past inputs to future outputs. Equation (6) can be interpreted as a measure of how significant is the effect of an input on the state and how much that effect is reflected in the output. The Hankel norm can thus be interpreted as a controllability *and* observability measure of the system. Summarizing, the Hankel norm has two fundamental properties: is a controllability-observability measure and, secondly is independent of the state space representation of the system.

4. HANKEL INTERACTION INDEX

We will next use the Hankel norm to build an interaction index. For each elementary subsystem ($\mathbf{A}$, $\mathbf{B_j}$, $\mathbf{C_i}$, D_{ij}), we use the Hankel norm to quantify the ability of input u_j to control output y_i. These norms are collected into a matrix $\bar{\mathbf{\Sigma}}_H$, where the element (i, j), $[\bar{\mathbf{\Sigma}}_H]_{ij}$, is defined by

$$[\bar{\mathbf{\Sigma}}_H]_{ij} = \|G_{ij}(z)\|_H$$

The $\bar{\mathbf{\Sigma}}_H$ array is very similar to the Participation Matrix, $\mathbf{\Phi}$, proposed in Conley and Salgado (2000), where the elements of the matrix are the trace of $\mathbf{\Gamma}_c^{(j)}\mathbf{\Gamma}_o^{(i)}$. The elements in $\mathbf{\Phi}$ can easily be bounded using the Hankel norm for the corresponding elementary subsystem. Furthermore, the $\bar{\mathbf{\Sigma}}_H$ has a structure which is the transpose of the structure of the Participation Matrix. This and equation (6) allow us to interpret $\bar{\mathbf{\Sigma}}_H$ as a gain matrix and we may then write the relation

$$y_H = \bar{\mathbf{\Sigma}}_H u_H$$

which provides an easy way to visualize different connections between inputs and outputs. The $\bar{\Sigma}_H$ array is thus a gain measure in the same way as the RGA.

To get rid of difficulties arising from scaling we can normalize the matrix in different ways. One idea to introduce a scaling is to use the same method as for the RGA, i.e. to use an index

$$\bar{\mathbf{\Sigma}}_H \times \left(\bar{\mathbf{\Sigma}}_H^T\right)^{-1} \quad (7)$$

where the operator $\times$ denotes the element-by-element multiplication of the two matrices. This normalization results in a matrix where the sum of all elements in a row or column is equal to one, although (7) does not have the same nice physical interpretation as for the RGA. Another way of making the normalization is to express every input and every output in percentage of full scale. This normalization supports the idea of scaling all elements in $\bar{\Sigma}_H$ so that their sum is equal to one. The latter normalization is preferred and we choose the following normalized Hankel Interaction Index Array

$$[\mathbf{\Sigma}_H]_{ij} = \frac{\|G_{ij}(z)\|_H}{\sum_{i,j}\|G_{ij}(z)\|_H} \quad (8)$$

Using the same methodology as for the RGA we determine the input-output pairing by finding in each row i the largest element (i, j). Input j is then selected to control output i.

To compute the Hankel Interaction Array we have to solve the $2p$ Lyapunov equations (4) and (5) and compute the eigenvalues of the p^2 products $\mathbf{\Gamma}_c^{(j)}\mathbf{\Gamma}_o^{(i)}$ to obtain the Hankel norm.

If $G_{ij}(z) = 0$ for a given pair (i, j), then $\mathbf{\Gamma}_c^{(j)}\mathbf{\Gamma}_o^{(i)} = 0$, leading to $[\mathbf{\Sigma}_H]_{ij} = 0$. This implies that a block diagonal $\mathbf{G}(z)$ gives a block diagonal $\mathbf{\Sigma}_H$ matrix, with the same structure. This is consistent with intuition, since, in those cases, the $\mathbf{\Sigma}_H$ matrix will suggest the right controller structure. It is important to observe that the $\mathbf{\Sigma}_H$ takes the full dynamic effects of the system into account and not only the steady-state performance as the RGA or the behavior at a single frequency.

5. CLOSED LOOP BANDWIDTH EFFECT

When a measure of dynamic interaction is built, attention should be paid to the relevant frequency range. This has, for instance, been proposed in Witcher and McAvoy (1977) and Gagnepain and Seborg (1982). Specifically, interactions are meaningful in control design only in a frequency band where the plant input has significant energy. One way to introduce this element into the building of the interaction index is to observe that the (vector) plant input, $\mathbf{u}[k]$ is connected to the reference signals and to the output disturbances through the control sensitivity $\mathbf{S}_u(z)$, see Goodwin *et al.* (2000), through the expression

$$\mathbf{S}_u(z) = (\mathbf{G}(z))^{-1}\mathbf{T}(z)$$

The frequency response of $\mathbf{S}_u(z)$ depends on the relationship between the plant bandwidth and the closed loop bandwidth. To gain insight, we consider the SISO case with a biproper controller. In that simpler situation, we observe that, when the closed loop bandwidth is larger than the plant bandwidth, the control sensitivity has larger magnitude at high frequencies than at low frequencies. In the reverse situation, the control sensitivity has a low-pass characteristic. This also applies to MIMO systems. However, it has to be applied with caution since there may be a non-unique closed loop bandwidth. Even with this caution, there are many cases when this approach will provide a useful information if we apply the Hankel Interaction Index to a modified system, $\tilde{\mathbf{G}}(z)$, obtained by filtering the plant through a (scalar) case-dependent filter, $F(z)$, i.e.

$$\tilde{\mathbf{G}}(z) = \mathbf{G}(z)F(z)$$

The effect of this filtering will be illustrated in the following section.

6. EXAMPLES

Some examples are used to illustrate the usefulness of the Hankel Interaction Index.

EXAMPLE 1

Consider the system with the pulse transfer function

$$\mathbf{G}(z) = \begin{pmatrix} \dfrac{0.1}{(z-0.8)(z-0.5)} & \dfrac{0.08}{z-0.8} \\ \dfrac{-0.24}{z-0.5} & \dfrac{0.1(z-0.1)}{(z-0.8)(z-0.5)} \end{pmatrix} \quad (9)$$

The Hankel Interaction Index Array (8) is

$$\mathbf{\Sigma}_H = \begin{pmatrix} 0.3755 & 0.1872 \\ 0.1300 & 0.3073 \end{pmatrix}$$

This result suggests that a diagonal controller should be chosen with the pairs (u_1, y_1) and (u_2, y_2), since the largest elements are the elements $(1,1)$ and $(2,2)$.

For the MIMO process (9), the RGA is

$$\text{RGA}(\mathbf{G}) = \begin{pmatrix} 0.8242 & 0.1758 \\ 0.1758 & 0.8242 \end{pmatrix}$$

In this example the RGA leads to the same conclusion regarding the pairing of inputs and outputs. □

EXAMPLE 2

The system in this case has a transfer function $\mathbf{G}(z)$ given by

$$\begin{pmatrix} \dfrac{0.1021}{z-0.9048} & \dfrac{0.3707z-0.3535}{z^2-1.724z+0.7408} \\ \dfrac{-0.192z+0.1826}{z^2-1.869z+0.8781} & \dfrac{0.09516}{z-0.9048} \end{pmatrix} \quad (10)$$

If the RGA is computed we obtain

$$\text{RGA}(\mathbf{G}) = \begin{pmatrix} 0.5033 & 0.4967 \\ 0.4967 & 0.5033 \end{pmatrix}$$

We observe that the RGA suggests, very weakly, that the pairing should be (u_1, y_1) and (u_2, y_2). On the other hand, the Hankel Interaction Index Array is

$$\mathbf{\Sigma}_H = \begin{pmatrix} 0.1936 & 0.3281 \\ 0.2978 & 0.1805 \end{pmatrix} \quad (11)$$

The Hankel Interaction Index provides a clear suggestion. It directs the designer to pair (u_1, y_2) and (u_2, y_1). The reason for this difference is that the Hankel Interaction Index Array takes into account the dynamic features of the interaction.

A deeper insight can be gained with a simple control design. Assume that we want that $y_1[k]$ tracks a step reference signal, $r_1[k] = \mu[k]$, and that $y_2[k]$ tracks another step reference signal, $r_2[k] = -\mu[k-5]$. We want to synthesize a dead-beat control which drives the error to zero after two time units. The synthesis method is the Youla parameterization of all stabilizing controllers (Goodwin *et al.* (2000)), i.e. the controller has a transfer function, $\mathbf{C}(z)$, given by

$$\mathbf{C}(z) = (\mathbf{I} - \mathbf{G}_o(z)\mathbf{Q}(z))^{-1}\mathbf{Q}(z)$$

where $\mathbf{Q}(z)$ is any stable transfer function matrix, and where $\mathbf{G}_o(z)$ is the nominal model. The dead-beat performance is *nominally* achieved if

$$\mathbf{Q}(z) = \frac{1}{z^2}(\mathbf{G}_o(z))^{-1}$$

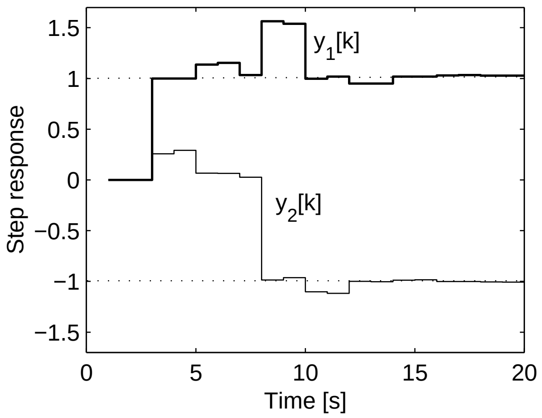

Fig. 1 Closed loop response to reference signals $r_1[k] = \mu[k]$ and $r_2[k] = -\mu[k-5]$ in Example 2.

provided that $\mathbf{G}_o(z)$ has all its zeros inside the open unit disk (as it does happen in this example).

If we abide by the RGA (weak) indication, the nominal model is $\mathbf{G}_o(s) = \mathrm{diag}(G_{11}(z), G_{22}(z))$; if, instead, we follow the Hankel Interaction Index suggestion, then the nominal model is $\mathbf{G}_o(s) = \mathrm{adiag}(G_{12}(z), G_{21}(z))$ where adiag stands for *anti-diagonal*.

When the controller is computed in each case, and the controller is used to control the full MIMO plant (10), we can then verify that for the RGA case, the closed loop system is unstable. However, when using the pairing suggested by the Hankel Interaction Index the result is a stable closed loop system, with the step response (for the given reference signals) shown in Fig. 1. Naturally, the prescribed dead-beat behavior has not been achieved, since the controller was applied to the full MIMO process.

A key issue in this simple design is that we have specified a very high bandwidth. This makes the RGA more likely to yield a bad control performance than other criteria where the dynamic nature of the interactions is accounted for. This is intimately connected to the ideas presented in Section 5, and it can be illustrated using a filter as suggested in that section. Consider first a low-pass filter $F(z)$ with transfer function $0.01/(z-0.99)$. We can then re-compute the Hankel Interaction Index Array for $\tilde{\mathbf{G}}(z)$. This yields

$$\mathbf{\Sigma}_H = \begin{pmatrix} 0.2706 & 0.2409 \\ 0.2362 & 0.2522 \end{pmatrix}$$

We observe that now the Hankel Interaction Index Array gets closer to the RGA in the sense that no clear conclusion can be drawn.

If we use a high pass filter $F(z)$ with transfer function $1.5z/(z+0.5)$ and re-compute the Hankel Interaction Index Array for $\tilde{\mathbf{G}}(z)$, we obtain

$$\mathbf{\Sigma}_H = \begin{pmatrix} 0.1932 & 0.3269 \\ 0.2998 & 0.1801 \end{pmatrix}$$

This is very close to the case when no filter was used (as seen in equation (11)). If intermediate filters were used one could observe a transition from the RGA to the Hankel Interaction Index Array given in (11), as we increase the frequency response of the filter at high frequencies. □

EXAMPLE 3

In this case we deal with a 3×3 MIMO system with the transfer function

$$\mathbf{G}(z) = \begin{pmatrix} \mathbf{G}_1(z) & \mathbf{G}_2(z) & \mathbf{G}_3(z) \end{pmatrix}$$

where

$$\mathbf{G}_1(z) = \begin{pmatrix} \dfrac{-0.7987z + 0.7673}{z^2 - 1.575z + 0.6065} \\ \dfrac{0.1813}{z - 0.8187} \\ \dfrac{-0.1814}{z - 0.7408} \end{pmatrix}$$

$$\mathbf{G}_2(z) = \begin{pmatrix} \dfrac{0.04758}{z - 0.9048} \\ \dfrac{1.517z - 1.457}{z^2 - 1.489z + 0.5488} \\ \dfrac{0.2592}{z - 0.7408} \end{pmatrix}$$

$$\mathbf{G}_3(z) = \begin{pmatrix} \dfrac{-0.09516}{z - 0.9048} \\ \dfrac{0.09063}{z - 0.8187} \\ \dfrac{2.163z - 2.077}{z^2 - 1.411z + 0.4966} \end{pmatrix}$$

The Hankel Interaction Index Array is given by

$$\mathbf{\Sigma}_H = \begin{pmatrix} 0.1429 & 0.0617 & 0.0451 \\ 0.0295 & 0.2437 & 0.0645 \\ 0.0589 & 0.0309 & 0.3228 \end{pmatrix}$$

and the RGA index is

$$\mathrm{RGA}(\mathbf{G}) = \begin{pmatrix} 0.1739 & 0.2348 & 0.5913 \\ 0.5217 & 0.5913 & -0.1130 \\ 0.3043 & 0.1739 & 0.5217 \end{pmatrix}$$

We observe that clear suggestions can be derived from both arrays. From the RGA the suggested pairing is anti-diagonal, e.g.(u_1,y_3), (u_2,y_2) and (u_3,y_1). On the other hand the Hankel Interaction Index suggests a diagonal pairing, e.g. (u_1,y_1), (u_2,y_2) and (u_3,y_3).

Again if, as in Example 2, we design a dead-beat control (in one step) we can verify that the suggested RGA pairing leads to an unstable closed loop system. However, a reasonable control is obtained if the pairing suggested by the Hankel Interaction Index is used. Since the dead beat control requires a high-pass control sensitivity, the result is not surprising, since we did not use any filter. We would like to investigate what happen if we apply the Hankel Interaction Index to a low-pass filtered plant, where the filter is chosen as $F(z) = 0.01/(z - 0.99)$. We then obtain

$$\mathbf{\Sigma}_H = \begin{pmatrix} 0.1228 & 0.1368 & 0.0946 \\ 0.0705 & 0.1166 & 0.1352 \\ 0.1411 & 0.0684 & 0.1140 \end{pmatrix}$$

It is now unclear what the pairing should be. To compare both alternatives we can compare the trace of the matrix $\mathbf{\Sigma}_H$ with the sum of the elements in the anti-diagonal. This yields 0.3534 against 0.3519. This implies that for a very low bandwidth, it is not clear how to do the pairing. □

7. CONCLUSIONS

A new interaction measure has been presented. This index is based upon the Hankel norm of the elementary subsystems. The proposed index includes the dynamic interaction and it is shown that it can be applied in conjunction with available information on closed loop frequency bands. The Hankel Interaction Index has been compared with the traditional RGA index. The new index exhibits a superior performance when the multivariable interaction has a non-monotonic behaviour in frequency. It is also simpler than other indices which measure dynamic interaction, since it does not require complex analysis of a (possibly very large) number of frequency responses.

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