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ON RECONFIGURABILITY WITH RESPECT TO ACTUATOR FAILURES

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Abstract: This papers considers the problem of fault tolerance with respect to actuator failures. Accommodation and reconfiguration strategies are presented, and the system reconfigurability is analysed under possible energy limitation constraints.

Keywords: Fault tolerant control, reconfiguration, actuator faults

1. INTRODUCTION

Fault tolerant control (FTC) is a research area of increasing importance, due to the growing demand for system safety and availability. Reliability analysis provides probabilistic evaluations of the system ability to perform correctly and allows the design of systems with given reliability performances (Blanke, 1996). As far as real time control is concerned, FTC can be achieved through fault accommodation or through control reconfiguration (Staroswiecki and Gehin, 2000). Fault accommodation is the problem of controlling the faulty system, which rests on fault detection, isolation and estimation schemes. It is strongly related to robust control, whose framework can be used for the simultaneous design of control and diagnosis schemes (Stoustrup and Grimble, 1997), (Jacobson and Nett, 1991). Reconfiguration is the problem of replacing the faulty part of the system by a non-faulty one, so as to still achieve the control objectives. Reconfiguration strategies only need fault detection and isolation procedures.

Recoverability is concerned with the possibility either to accommodate the faults or to reconfigure the system when faults occur. It has been considered from the point of view of the system structural properties, e.g. observability or controllability, extending the evaluation of these properties to the faulty system. For example, the smallest second order mode, first introduced in (Moore, 1981), has been proposed as a reconfigurability measure in (Wu *et al.*, 2000), while the determinants of the controllability and observability Gramians were preferably suggested in (Frei *et al.*, 1999). However, these measures characterize the performance of the system actuation and measurement scheme by itself : rather than a measure of its *fault tolerance*, they provide a measure of its *admissibility* with respect to energetic constraints. In (Hoblos *et al.*, 2000) and (Staroswiecki and Aitouche, 1999), two reconfigurability measures have been proposed to evaluate the size of the set of fault tolerant situations, namely the number of recoverable failures (redundancy degrees) and the mean time until a non-recoverable failure occurs.

The recoverability problem can also be considered from the point of view of a specific objective, as analysed in (Staroswiecki and Gehin, 2000), by considering the question : "can the objective be achieved either through fault accommodation or through system reconfiguration?"

The aim of this paper is to analyse the fault tolerance property in the context of the control problem in the presence of actuator failures (the analysis of the estimation problem in the presence of sensor failures is not developed, being dual).

The control and the fault tolerant control problem are presented in sections 2 and 3, and the accommodation and reconfiguration points of view are defined when actuator faults occur. Due to their simplicity and the low requirements they impose upon the FDI algorithms, reconfiguration strategies are further analysed. In section 4, fault tolerant actuation schemes are characterized and the energy based reconfigurability measure is discussed. Section 5 gives some concluding remarks.

2. CONTROL PROBLEM

Consider the LTI deterministic system modelled by

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ &= Ax(t) + \sum_{i \in I} B_i u_i(t)\end{aligned} \tag{1}$$

$x \in X \subset R^n$ is the state vector and $u \in U \subset R^m$ is the control vector (I is the set of the actuators in nominal operation mode, $u_i(t) \in R^{m_i}$ is the input of actuator n°$i \in I$, and $m = \sum_{i \in I} m_i$). A and B are constant matrices of suitable dimensions, and it is assumed that the pair (A, B) is controllable. In order to characterize the set of actuators I, the following standard optimal control problem is considered:

1) Objective : transfer the system state from $x(0) = \gamma$ to $x(\infty) = 0$, where $\gamma \in R^n$, and $x(\infty)$ stands for $lim_{t \longrightarrow \infty} x(t)$,

2) Constraints : eqn. (1) is satisfied $\forall t \in [0, \infty)$, $x(t)$ and $u(t)$ are continuous functions of time, and $X = R^n$, $U = R^m$,

3) Criterion : minimize the functional

$$Q(u, \gamma) = \int_0^\infty \|u(t)\|^2 \, dt \tag{2}$$

where $\|.\|$ is the Euclidian norm (other criteria could of course be used; the problem above only provides a "standard" analysis frame).

The solution of problem (2) is well known from the classical theory of optimal control, and the optimal value of the criterion is given by

$$Q(I, \gamma) = \tilde{\gamma} W_c^{-1}(I) \gamma \tag{3}$$

where $W_c(I)$ is the controllability Gramian, which is invertible since the pair (A, B) is controllable (M being some matrix, the notation $\tilde{M}$ stands for the transpose of M).

Eqn. (3) shows that the performance of the actuator set I depends on the control objective. Indeed,

$$\Gamma = \{\gamma \in R^n, \text{ s.t. } \tilde{\gamma} W_c^{-1}(I) \gamma \leq 1\}$$

represents the set of points in the state space from which the origin can be reached with control energy less than 1. The characterization of the actuation scheme I independently of the control objective γ leads to consider the worst control problem from the energetic point of view : transfer the system state from $x(0) = \gamma^*$ to $x(\infty) = 0$, where

$$\gamma^* = \arg \max_{\|\gamma\|=1} Q(I, \gamma)$$

The set of actuators I is thus characterized by the maximum eigenvalue of $W_c^{-1}(I)$ which is interpreted as the maximum energy which might be required to transfer the system state from $x(0) = \gamma$ to $x(\infty) = 0$ for some $\gamma \in R^n$ such that $\|\gamma\| = 1$,

$$Q(I) = Q(I, \gamma^*) = \lambda_{\max} \left[W_c^{-1}(I) \right] \tag{4}$$

3. FAULT TOLERANT CONTROL

The FTC problem is concerned with the existence of *admissible* solutions to the control problem (in a sense which will be defined later) when actuators fail. Let $I = I_N(t) \cup I_F(t)$ where $I_N(t)$ is the subset of the normal actuators while $I_F(t)$ is the subset of the faulty ones at some given time t. Since the changes in the sets $I_N(t)$ and $I_F(t)$ only occur when actuators fail (repair operations are not considered here), they are rare events with respect to the system dynamics, and the notation is simplified into I_N and I_F. The faulty system behaviour is described by

$$\dot{x}(t) = Ax(t) + \sum_{i \in I_N} B_i u_i(t) + \sum_{i \in I_F} \beta_i(u_i(t), \theta_i) \tag{5}$$

where $\beta_i(u_i(t), \theta_i)$ describes the contribution of the faulty actuator i. This vector may be known, or known with unknown parameters θ_i or completely unknown, depending on the faults which are considered, and of the capability of the FDI algorithm to estimate them. The objective, constraints and criterion of the FTC problem are identical to those of the control problem, with the exception of constraint (1) being replaced by constraint (5).

Analysing the FTC problem rises two questions, namely

1) can this problem be properly stated, and

2) can this problem be solved.

FTC problem statement. The identification of the subset I_F of faulty actuators is normally done by the FDI algorithm, which detects and isolates the faults. The statement of the constraints resumes to the identification of the functions $\beta_i(u_i(t), \theta_i), i \in I_F$. This is not usually done by FDI algorithms, and could be referred to as a *diagnostic* possibility, which rests on fault modelling and on fault parameter identification, and it could be - or not - provided by the FDI system, thus leading to distinguish two approaches, namely *fault accommodation* and *system reconfiguration.*

Problem solution. The problem solution exists provided the objective $x(\infty) = 0$ can still be reached from the initial state $x(0) = \gamma$ (the cost might however be prohibitive; this point will be addressed later). When solutions are needed to exist for any objective γ, it is necessary that system (5) be still controllable.

3.1 *Fault accommodation*

Accommodation strategies consider the control problem associated with the faulty system. In this situation the FTC problem has to be analysed replacing eqn. (5) by

$$\dot{x}(t) = Ax(t) + \sum_{i \in I_N} B_i u_i(t) + \sum_{i \in I_F} \hat{\beta}_i(u_i(t), \hat{\theta}_i) \tag{6}$$

where the functions $\hat{\beta}_i(u_i(t), \hat{\theta}_i)$ and parameters $\hat{\theta}_i$, $i \in I_F$ are known or estimated. This approach obviously needs some fault models to be defined, and the conclusions will certainly hold when the actual fault(s) obey that model, which is true in very few cases and for very specific kinds of faults (see (Demetriou and Polycarpou, 1998) for example of recent works using this approach).

3.2 *System reconfiguration*

Reconfiguration strategies set the control problem of a system in which the faulty part has been switched off. The choice of a reconfiguration strategy might follow from the impossibility of estimating the fault, or it can be deliberate, so as to implement fault tolerant strategies which provide guaranteed results, and are as simple and as understandable as possible by operators.

In many cases, reconfiguration is understood as the replacement of the faulty part by some non-faulty one. Considering the problem under investigation, this means that some actuators were not in service before the fault occurrence and that they can be switched on after the fault. Let I_{off} be the set of those actuators, which are assumed without loss of generality to be non-faulty. It obviously follows that considering from the beginning the whole set of actuators $I \cup I_{off}$ reduces the problem to that of reconfiguring the system $I_{off} \cup I_N \cup I_F$ by simply removing the faulty part. Thus, including I_{off} within I (namely, into I_N), one can go on with unchanged notations. In this situation, the FTC problem has to be analysed replacing eqn (5) by

$$\dot{x}(t) = Ax(t) + \sum_{i \in I_N} B_i u_i(t) \tag{7}$$

3.3 *Admissible solutions and the definition of fault tolerance*

Suppose some fault situation $I_F \subset I$ such that the FTC problem can be formulated (using either accommodation or reconfiguration) and has a solution, i.e. the system state can be transferred from $x(0) = \gamma$ to $x(\infty) = 0$, and let $Q(I_N, \gamma)$ be the corresponding (minimal) energy cost. Obviously, the fact that a solution exists does not mean that it is satisfactory. Indeed, two cases can be distinguished : in the first one, the control energy needed is of no importance provided the system objective is achieved in spite of the fault. In this case, the actuation scheme I is fault tolerant with respect to the situation I_F if and only if system (6) - when accommodation is used - or (7) - when reconfiguration is concerned - is controllable. In the second case, some energy limitation constraint is considered : indeed, the control energy, although optimal, might be too high, thus denying the actuation scheme I to deserve the "fault tolerant" label with respect to the situation I_F.

Definition 3.1. : Let I_F be a fault situation. The solution of the FTC problem is admissible with respect to the control objective γ if and only if

$$Q(I_N, \gamma) \leq \sigma(\gamma) \tag{8}$$

where $\sigma(\gamma)$ is some predefined function.

The quantity

$$\tau(\gamma) = \frac{\sigma(\gamma)}{\tilde{\gamma} W_c^{-1}(I) \gamma}$$

where $\tau(\gamma) \geq 1$ can be interpreted in terms of admissible loss of efficiency of the actual actuation scheme I_N with respect to the nominal one I when the objective γ is to be achieved. Four special choices of $\sigma(\gamma)$ will be discussed below :

- $\sigma(\gamma) = \infty, \forall \gamma \in R^n$. In this case, fault tolerance is only concerned with the existence of an optimal solution, whatever its cost, thus resuming the fault tolerance property to the permanence of the controllability property,
- $\sigma(\gamma) = \sigma < \infty, \forall \gamma \in R^n, \|\gamma\| \leq 1$ defines a uniform bound for the energy spent in controlling the faulty system, whatever the initial state in the unit sphere,
- $\sigma(\gamma) = \tau.\tilde{\gamma} W_c^{-1}(I)\gamma, \ \forall \gamma \in R^n$ defines a uniform bound for the loss of efficiency in the control of the faulty system, whatever the control objective.
- $\sigma(\gamma) = \tilde{\gamma} W_c^{-1}(I)\gamma + \Delta\sigma$ where $\Delta\sigma$ is some given constant is interpreted as the definition of a uniform bound for the overcost when controlling the faulty system, whatever the control objective (it will be seen below that this choice is not sound, unless the admissibility condition is required to hold only in a bounded region of R^n).

Based on the definition of admissibility, fault tolerance can be defined as follows.

Definition 3.2. : The actuation scheme I is fault tolerant with respect to the fault situation I_F and the control objective γ if and only if the recovery problem has an admissible solution for γ.

4. FAULT TOLERANT ACTUATION SCHEMES

Due to their simplicity and the low requirements they impose upon the FDI algorithms, only reconfiguration strategies are now considered.

4.1 *Fault tolerance characterization*

The solution of the recovery problem under the constraints (7) exists and is admissible for any objective $\gamma \in R^n$ if and only if

1) (A, B_N) is controllable
2) $\tilde{\gamma} W_c^{-1}(I_N)\gamma \leq \sigma(\gamma) \qquad \forall \gamma \in R^n$

Obviously, as it has already been noticed, the second condition is always satisfied when $\sigma(\gamma) = \infty, \forall \gamma \in R^n$ is chosen. For the actuation scheme I to be fault tolerant through reconfiguration with respect to the situation I_F it is necessary and sufficient that the non-faulty actuators keep the system controllable (see (Hoblos *et al.*, 2000) for the determination of the subsets of actuators which enjoy this property).

The following results characterize the conditions under which the actuation scheme I is fault tolerant through reconfiguration with respect to the situation I_F under the other definitions of admissibility.

Theorem 4.1. : The actuation scheme I is fault tolerant through reconfiguration with respect to the situation I_F under the admissibility condition

$$\tilde{\gamma} W_c^{-1}(I_N)\gamma \leq \sigma \qquad \forall \gamma \in R^n, \|\gamma\| \leq 1$$

if and only if

1) (A, B_N) is controllable
2) $\mu_1 \leq \sigma$ where μ_1 is the maximal eigenvalue of $W_c^{-1}(I_N)$

Proof : evident from (4).

Theorem 4.2. : The actuation scheme I is fault tolerant through reconfiguration with respect to the situation I_F under the admissibility condition

$$\frac{\tilde{\gamma} W_c^{-1}(I_N)\gamma}{\tilde{\gamma} W_c^{-1}(I)\gamma} \leq \tau \qquad \forall \gamma \in R^n$$

if and only if

1) (A, B_N) is controllable
2) $\begin{cases} \mu_2 \leq \tau \text{ where } \mu_2 \text{ is the maximal root of :} \\ \det\left[W_c(I_N) - \mu_2 W_c(I)\right] = 0 \end{cases}$

Proof : the first condition is necessary and sufficient for solutions to exist for any objective $\gamma \in R^n$. The second condition means that these solutions are admissible. Indeed, the admissibility condition can be written

$$\max_{\gamma \in R^n} \frac{\tilde{\gamma} W_c^{-1}(I_N)\gamma}{\tilde{\gamma} W_c^{-1}(I)\gamma} \leq \tau$$

The well known solution of the maximization is provided by

$$\max_{\gamma \in R^n} \frac{\tilde{\gamma} W_c^{-1}(I_N)\gamma}{\tilde{\gamma} W_c^{-1}(I)\gamma} = \mu_2$$

where μ_2 is the maximal root of the matrix pencil

$$W_c^{-1}(I_N) - \mu_2 W_c^{-1}(I)$$

which is also the maximal root of the matrix pencil

$$W_c(I) - \mu_2 W_c(I_N) \tag{9}$$

(**end of proof**).

The result can also be stated in terms of the submatrix B_F of the faulty actuators. Indeed, by eventually re-arranging the components of u one has :

$$B = [B_N, B_F] \Longrightarrow B\tilde{B} = B_N \tilde{B}_N + B_F \tilde{B}_F$$

and:

$$W_c(I) = W_c(I_N) + W_c(I_F) \tag{10}$$

from which the matrix pencil (9) writes :

$$(1-\mu_2)W_c(I_N)+W_c(I_F)$$

leading to the admissibility condition :

$$\lambda \leq \tau - 1$$

where λ is the maximal eigenvalue of the matrix $W_c(I_F)W_c^{-1}(I_N)$.

Consider now the last admissibility condition :

$$\tilde{\gamma}W_c^{-1}(I_N)\gamma \leq \tilde{\gamma}W_c^{-1}(I)\gamma + \Delta\sigma \qquad \forall\gamma \in R^n$$

Factorizing $W_c^{-1}(I_N)$ on the left and $W_c^{-1}(I)$ on the right, and making use of (10) gives :

$$\tilde{\gamma}W_c^{-1}(I_N)W_c(I_F)W_c^{-1}(I)\gamma \leq \Delta\sigma \qquad \forall\gamma \in R^n$$

which shows that such a definition is not sound, the inequality being inhomogeneous (multiplying γ by a scalar factor can give arbitrary large values to the left hand member). The interpretation is that the overcost indeed depends on the control objective, and cannot be bounded by a constant for any point in R^n. However, this can be required for a bounded region, e.g. a set defined by $\Gamma_Q = \left\{\gamma \in R^n, \frac{1}{2}\tilde{\gamma}Q\gamma \leq 1\right\}$ where Q is some symmetric positive definite matrix. In that case, the following result holds.

Theorem 4.3. : The actuation scheme I is fault tolerant through reconfiguration with respect to the situation I_N under the admissibility condition :

$$\tilde{\gamma}W_c^{-1}(I_N)\gamma \leq \tilde{\gamma}W_c^{-1}(I)\gamma + \Delta\sigma \qquad \forall\gamma \in \Gamma_Q \subset R^n$$

if and only if :

1) (A, B_N) is controllable
2) $\begin{cases} \mu_3 \leq \Delta\sigma \text{ where } \mu_3 \text{ is the maximal root of :} \\ \det\left[W_c(I_F) - \mu_3 W_c(I_N)QW(I)\right] = 0 \end{cases}$

Proof : it directly follows from the maximization of $\tilde{\gamma}W_c^{-1}(I_N)W_c(I_F)W_c^{-1}(I)\gamma$ under the quadratic constraint on γ (**end of proof**).

4.2 *Fault tolerance evaluation*

Structural and probabilistic measures of fault tolerance have been proposed in both situations where energy limitation constraints are, or are not, taken into account for the definition of admissibility in (Hoblos *et al.*, 2000), (Staroswiecki and Aitouche, 1999).

When $\sigma(\gamma) = \infty, \forall\gamma \in R^n$, the control energy, which acts no more as a constraint, has also been proposed as a measure of fault tolerance. Indeed let

$$S_N = \left\{I_N \in 2^I \text{ s.t. } (A, B_N) \text{ is controllable}\right\}$$

and let $Q(I_N) = \max_{\|\gamma\|\leq 1} Q(I_N, \gamma)$ be the worst case energy cost in some situation $I_N \in S_N$ (e.g. the quantities μ_1 and μ_2 in the preceding section). Since $(S_N, \subseteq)$ is a partial order, minimal elements exist, which are subsets of actuators through which the system is controllable, and such that controllability is lost when any of them is switched off. Let $S_{\min}$ be the subset of such minimal elements. From common sense considerations, one has

$$J_1 \subseteq J_2 \subseteq S_N \Longrightarrow Q(J_1) \geq Q(J_2)$$

since the energy cost needed to achieve the objective cannot decrease when actuators are lost. Thus, the worst fault situation from an energetic point of view has to belong to $S_{\min}$, and the energy-based measure of fault tolerance can be evaluated by

$$Q^* = \max_{J \in S_{\min}} Q(J)$$

The interpretation is as follows : the system fault tolerance is evaluated by means of the energy cost (in absolute or in relative terms) of the worst situation in which the system is still controllable. Since the control energy does not act as a constraint, the system is really fault tolerant in any situation of S_N, in the sense that the recoverability problem has an admissible solution. It follows that the energetic cost does not really measure the system reconfigurability, but it merely evaluates the quality of the solutions when they exist. This can also be seen by considering a non-fault tolerant situation, namely a set of actuators $J \in S_{\min}$. By definition of $S_{\min}$ the standard control problem has an admissible solution, and $Q(J)$ measures the energy cost for the worst objective γ such that $\|\gamma\| = 1$. Obviously, the system is not fault tolerant through reconfiguration, while any positive value of $Q(J)$ could be obtained, depending on the system parameter values.

5. EXAMPLE

Consider a MIMO system with 7 states, and 4 actuators : $I = \{a, b, c, d\}$. The matrices A and B are as follows,

$$A = diag\left\{-1, -0.5, -3, -4, -2, -1.5, -2.5\right\},$$

$$\tilde{B} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

The Gramian maximal eigenvalue is $\lambda_{\max}\left[W_c^{-1}(I)\right] = 0.4357$ energy units.

Assume that admissible solutions are defined such that the worst situation control cost should not exceed 1.125 energy units. Then, there are 10 fault situations in which the system is controllable, namely $\{a,b,c,d\}$, $\{b,c,d\}$, $\{a,c,d\}$, $\{a,b,d\}$, $\{a,b,c\}$, $\{c,d\}$, $\{b,c\}$, $\{a,d\}$, $\{a,b\}$, $\{a,c\}$, but only 6 of them are admissible when energy limitation is considered, as shown by Table 1.

Just for illustration, let us evaluate the fault tolerance by means of structural and probabilistic measures, as proposed in (Hoblos *et al.*, 2000), (Staroswiecki and Aitouche, 1999). The strong redundancy degree of $\{a,b,c,d\}$ is equal to one, which means that any single actuator can fail while the standard control problem still exhibits an admissible solution. Besides, assume that the actuators reliabilities are given by exponential laws with constant failure rates, all equal to $\lambda = 0.4 \times 10^{-5} H^{-1}$, and let $MTUFTL$ be the mean time until the fault tolerance property is lost. Table 1 shows the actuators subsets for which the system is controllable, and the associated $\lambda_{\max}$ and $MTUFTL$ (non admissibility is shown by bold characters).

Actuator subsets	$\lambda_{\max}$ (energy units)	$MTUFTL$ (10^5 hours)
$\{a,b,c,d\}$	0.4357	1.666
$\{b,c,d\}$	1.1197	0.833
$\{a,c,d\}$	0.4676	0.833
$\{a,b,d\}$	0.8274	1.25
$\{a,b,c\}$	0.4778	1.25
$\{\mathbf{c},\mathbf{d}\}$	**3.0201**	–
$\{\mathbf{b},\mathbf{c}\}$	**1.3948**	–
$\{\mathbf{a},\mathbf{d}\}$	**2.2576**	–
$\{a,b\}$	1.0612	1.25
$\{\mathbf{a},\mathbf{c}\}$	**1.1452**	–

Table 1 : Admissible actuators subsets and associated characteristics

6. CONCLUSION

In this paper, the fault tolerance problem has been analysed through accommodation and reconfiguration strategies. Investigation about the system reconfigurability has been developed taking (or not) into account energy considerations, and necessary and sufficient reconfigurability conditions have been given for different definitions of admissibility. Reconfigurability measures based on the controllability Gramian, which have been recently proposed in the literature have been shown not to constitute a fault tolerance measure, but to apply only in a limited frame to evaluate the quality of the control which can be achieved by a (fault tolerant or non fault tolerant) control scheme. This is obviously an *off-line* analysis, which rests on the definition of a *standard* problem, namely the minimum energy control problem on an infinite horizon. Other standard problems could be considered, e.g. the classical LQ problem with states and controls entering the cost functional. The *real-time control* problem has not been considered. Indeed, it should be stated in a completely different way, namely a hybrid system formulation, because the transitions from one operating set of actuators $I_N(t)$ to the next one $I_N(t+dt)$ need in that case to be explicitly considered.

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