DYNAMIC OUTPUT CONTROLLER MPC VIA LMI

Jacques Bernussou [a], Ernesto Granado [b], William Colmenares [b], Germain García [a]

[a] *LAAS-CNRS 7, Av. du Colonel Roche, 31077 Toulouse cedex 4, France.*
[b] *Universidad Simón Bolívar, Dpto. Procesos y Sistemas, Caracas 1080, Venezuela.*

Abstract: This paper extends some previous work on state feedback Model Predictive Control (MPC) to dynamic output-feedback in MPC methodology. The controller is updated at each sampling time by minimizing an upper bound of an infinite horizon quadratic cost. The optimization problem and its associated constraints, on input and output, are expressed by LMI (Linear Matrix Inequality). Only measurable output and *a priori* fixed range values of the non measurable states, are used to determine the controller. Stability of the closed loop system is proven. An extension to the robust case is discussed.

Keywords: Predictive control; Output-feedback; Linear matrix inequalities; Parametric uncertainty; Stability; Performance.

1. INTRODUCTION

During the past few years, MPC has emerged as one of the most popular multivariable control technique used in industrial process (Camacho and Bordons, 1998). MPC has also received in recent years considerable interest in the research community and consequently, important theoretical developments have been derived (Mayne, *et al.*, 2000; Bemporad, *et al.*, 2002). Among the reasons for the popularity, it can be argued the fact that it enables to directly deal with inputs, states and outputs constraints, which can be explicitly taken into account in the optimization. MPC technology can now be found in a wide variety of application areas including chemicals, food processing, automotive, and aerospace applications (Qin and Badgwell, 2003).

MPC solves an on-line optimization problem at each sampling time to compute the control law. Although more than one input move is computed, the controller only implements the first one. At the next sampling time, the optimization problem is solved again with new measurements, and the control input is updated (Maciejowski, 2002).

The MPC was introduced in the 70's, and since then much research has been done in the subject dealing with feasibility and stability (Rawlings and Muske, 1993; Scokaert, 1997; Zheng and Morari, 1995). In the approach proposed below, stability is based on the use of an infinite horizon quadratic cost problem and the verification under some assumptions that, at each step, the optimization solution remains feasible.

The technique proposed in this paper, is an extension to the output-feedback case of the methodology described in Kothare, *et al.*, (1996), where all states are measurable. The optimization problem is formulated in terms of LMIs (Boyd, *et al.*, 1994). There exist already dedicated powerful (LMI) algorithms that allow to think that an on line solution is achievable, at least, for not too fast processes. The method allows to include input/output constraints. Using the concept of invariant ellipsoid, those are translated in the form of additional LMIs which, then, appear as constraints in the optimization problem. It is also possible to include other performance specifications, such as, H_2 or H_∞ norms, in the form of LMIs constraints.

The controller determination is based on the measurable states and *a priori* assumption on the non measurable ones which are assumed to lie in a given range. A kind of robustness with respect to initial conditions is then achieved. Also, an extension of the results to the case of systems with parametric uncertainty is featured.

2. PROBLEM STATEMENT

Consider the discrete linear time invariant system:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned}, \qquad (1)$$

where $x(k) \in \Re^n$, $u(k) \in \Re^m$ and $y(k) \in \Re^q$ are respectively, the state, input and output of the system. $A \in \Re^{n\,x\,n}$, $B \in \Re^{n\,x\,m}$ and $C \in \Re^{q\,x\,n}$ are the dynamic, input and output matrices.

At each sampling time k, in the MPC framework, a control law based on the output measurement is defined through a dynamic output controller:

$$\begin{aligned} x_c(k+1) &= A_c x_c(k) + B_c y(k) \\ u(k) &= C_c x_c(k) \end{aligned}, \qquad (2)$$

where $x_c(k) \in \Re^n$ and the triplet A_c, B_c, C_c, are the dynamic controller matrices, i.e., at each sampling time, the triplet (A_c, B_c, C_c) has to be determined from the actual output measurements and the *a priori* assumptions to be presented later.

The closed loop system from (1) and (2) is:

$$\begin{aligned} x_e(k+1) &= \hat{A}(k) x_e(k) + \hat{B} u(k) \\ u(k) &= \hat{K}(k) x_e(k) \\ y(k) &= \hat{C} x_e(k) \end{aligned} \qquad (3)$$

where $x_e(k) = \begin{bmatrix} x(k)^T & x_c(k)^T \end{bmatrix}^T \in \Re^{2n}$ and

$$\begin{aligned} \hat{A}(k) &= \begin{bmatrix} A & 0 \\ B_c(k)C & A_c(k) \end{bmatrix}, \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \\ \hat{K}(k) &= \begin{bmatrix} 0 & C_c(k) \end{bmatrix}, \hat{C} = \begin{bmatrix} C & 0 \end{bmatrix} \end{aligned} \qquad (4)$$

In terms of the closed loop system variables, the quadratic cost function is written as:

$$J_\infty(k) = \sum_{i=1}^{\infty} \left(\begin{bmatrix} x_e(k+i/k) \\ u(k+i/k) \end{bmatrix}^T \hat{Q} \begin{bmatrix} x_e(k+i/k) \\ u(k+i/k) \end{bmatrix} \right), \qquad (5)$$

where $\hat{Q} = \text{blocdiag}\left(Q, C_c^T R C_c\right)$, $Q > 0$, $R > 0$ and $x_e(k+i/k)$ represents the prediction of x_e at instant $k+i$, given $x_e(k/k)$. Obviously, $x_e(k/k) = x_e(k)$. A way to overcome the problem caused by the uncertainty on the non measured state, is to follow a kind of guaranteed cost approach. For that let

$$V_L(x_e(k/k)) = x_e(k/k)^T P x_e(k/k), P > 0, \qquad (6)$$

which is determined such that at each instant k and $i \geq 0$, the following condition is satisfied,

$$\begin{aligned} &\Delta V_L\left(x_e(k+i/k)\right) = \\ &\quad V_L(x_e(k+i+1/k)) - V_L(x_e(k+i/k)) \leq \\ &-\left(x_e(k+i/k)^T Q x_e(k+i/k) + u(k+i/k)^T R u(k+i/k)\right) \end{aligned} \qquad (7)$$

Summing inequality (7) from $i = 0$ up to $i = \infty$ with the assumption of asymptotic stability $x_e(\infty/k) = 0$, one gets:

$$V_L(x_e(k/k)) \geq J_\infty(k), \qquad (8)$$

that is, (6) is an upper bound for the objective function (5). The proposed algorithm, much as in Kothare, *et al.*, (1996), is that of minimizing (6) subject to conditions that satisfy (7), at each sampling time.

2.1 Characterization of the non measurable states

Classically, the optimization approaches for controller determination, need *a priori* information about all the states. A more practical assumption is that of partial state information and output feedback control is likely more practically oriented than the state feedback one.

For the non measurable states when no filtering or observer function is chosen it is sometimes assumed that they are random variables with given statistics and the optimization concerns the mathematical expectation of the cost. The formulation here is different and a range for the non measured variables is defined, assuming that previous information can be used for that purpose. Consider the partition of the state vector $x(k) = \begin{bmatrix} x_r(k)^T & x_m(k)^T \end{bmatrix}^T \in \Re^n$ where $x_m(k) \in \Re^p$ represents the components accessible to measurement and $x_r(k) \in \Re^{n-p}$ is the non measured sub-vector.

At initial time ($k = 0$), it is assumed that $x_r(0)$ belongs to the polyhedral convex domain:

$$x_{r,i}^{\min} \leq x_{r,i}(0) \leq x_{r,i}^{\max}, \; i = 1, \ldots, n-p, \qquad (9)$$

Let

$$x_r^j(0), \; j = 1, \ldots, l, \; l = 2^{n-p} \qquad (10)$$

be the vertices of the membership domain for $x_r(0)$, i.e., the i^{th} component of $x_r^j(0)$ is $x_{r,i}^{\min}$ or $x_{r,i}^{\max}$, $\forall i$. Denote

$$\chi^j(k) = \begin{bmatrix} x_r^j(k)^T & x_m(k)^T \end{bmatrix}^T \in \Re^n, j = 1, \ldots, l \qquad (11)$$

Defining

$$\Omega(0) = co\left\{ \chi_e^j(0) = \begin{bmatrix} x_r^j(0) \\ x_m(0) \\ x_c(0) \end{bmatrix}, \; j = 1,\ldots,l \right\}$$

$$\text{let } \chi_e(0) = \begin{bmatrix} x_r(0) \\ x_m(0) \\ x_c(0) \end{bmatrix} \in \Omega(0), \tag{12}$$

the actual value of the augmented state at initial time. $\Omega(0)$ is the membership domain of the augmented vector at initial time. Observe that:

$$\chi_e(0) = \sum_{j=1}^{l} \lambda_i \chi_e^j(0), \; \lambda_i \in \Gamma,$$
$$\Gamma = \left\{ \lambda_i : \lambda_i \geq 0, \; \sum_{i=1}^{l} \lambda_i = 1 \right\}. \tag{13}$$

In the next iteration, the system being linear

$$\chi_e(1) \in \Omega(1)$$
$$\text{provided } \chi_e(0) \in \Omega(0) \tag{14}$$

where $\Omega(1)$ is the membership domain of the vector state at time $k = 1$, obviously defined by

$$\Omega(1) = Co\left\{ \chi_e^j(1) = \left(\hat{A}(1) + \hat{B}\hat{K}(1)\right)\chi_e^j(0) \right\}, \tag{15}$$

or equivalently,

$$\Omega(1) = \left(\hat{A}(1) + \hat{B}\hat{K}(1)\right)\Omega(0). \tag{16}$$

Thus, recursively at each sampling time of the MPC algorithm, it is possible, by simple propagation of the uncertainty (of membership) domain for the vector, to define polyhedral domains defined by

$$\Omega(k+1) = \left(\hat{A}(k) + \hat{B}\hat{K}(k)\right)\Omega(k) \tag{17}$$

so that:

$$\chi_e^j(k+1) \in \Omega(k+1) \text{ if } \chi_e^j(k) \in \Omega(k) \tag{18}$$

This way "of propagating" the field of membership of the non measured state is in coherence with the previous work of Kothare, *et al.*, (1996).

For practical purposes, it is interesting also to cope with input and output constraints. Input constraints may represent physical limits (such as valve saturation, power limitation, etc.), for instance:

$$\left\| u(k+i/k) \right\|_2 \leq u_{\max}, i \geq 0 \tag{19}$$

Output constraints may represent performance requirements or safety constraints, let $\left\| y(k+i/k) \right\|_2 \leq y_{\max}, i \geq 1$. Vector $y(k+i/k)$, represents system's predicted output at time $k+i$, based on the output at time k, $y(k/k)$.

3. MAIN RESULT

Theorem 1. *Let* $\chi^j(k) = \chi^j(k/k), j = 1,\ldots,l$ *and* $x_c(k) = x_c(k/k)$ *be the information of the state of system (1) and controller (2) at sampling time* k*, and given* $y_{\max}$ *and* $u_{\max}$*, system (1) is stabilized by a dynamic controller (2), which minimize an upper bound for the objective function (5) under constraints, if there exist symmetric positive definite matrices* $X, Y \in \Re^{n}$*, and matrices* $F \in \Re^{n\,x\,q}$*,* $L \in \Re^{m\,x\,n}$*,* $Z \in \Re^{n\,x\,n}$*,* $V \in \Re^{n\,x\,n}$*, solutions of the following optimization problem:*

$$\min \gamma \tag{20}$$

subject to

$$\begin{bmatrix} Y & I & Y\chi^j(k) + Vx_c(k) \\ I & X & \chi^j(k) \\ \left(Y\chi^j(k) + Vx_c(k)\right)^T & \chi^j(k)^T & I \end{bmatrix} > 0$$
$$j = 1,\ldots,l \tag{21}$$

$$\begin{bmatrix} Y & I & YA+FC & \cdots & Z & 0 & 0 \\ I & X & A & & AX+BL & 0 & 0 \\ (YA+FC)^T & A^T & Y & & I & 0 & Q^{1/2} \\ Z^T & (AX+BL)^T & I & & X & L^T R^{1/2} & XQ^{1/2} \\ 0 & 0 & 0 & & R^{1/2}L & \gamma I & 0 \\ 0 & 0 & Q^{1/2} & \cdots & Q^{1/2}X & 0 & \gamma I \end{bmatrix} > 0 \tag{22}$$

$$\begin{bmatrix} Y & I & 0 \\ I & X & L^T \\ 0 & L & u_{\max}^2 I \end{bmatrix} > 0, \tag{23}$$

$$\begin{bmatrix} Y & I & (CA)^T \\ I & X & (CAX+CBL)^T \\ CA & CAX+CBL & y_{\max}^2 I \end{bmatrix} > 0. \tag{24}$$

Proof.

$$\min \gamma \tag{25}$$

subject to

$$x_e(k)^T P x_e(k) < \gamma \text{ or } x_e(k)^T \tilde{P}^{-1} x_e(k) < 1 \tag{26}$$

where $\tilde{P} = \gamma P^{-1}$. By Schur complement, the second inequality in (26) may be written as:

$$\begin{bmatrix} \tilde{P} & x_e(k) \\ x_e(k)^T & I \end{bmatrix} > 0 . \quad (27)$$

Let us partition matrices $\tilde{P}$ and $\tilde{P}^{-1}$:

$$\tilde{P} = \begin{bmatrix} X & U \\ U^T & \hat{X} \end{bmatrix}, \quad \tilde{P}^{-1} = \begin{bmatrix} Y & V \\ V^T & \hat{Y} \end{bmatrix} \quad (28)$$

and define the matrix:

$$T = \begin{bmatrix} Y & I \\ V^T & 0 \end{bmatrix}. \quad (29)$$

Without loss of generality, matrix V is chosen regular (Scherer, *et al.*, 1997), so that T is also regular. Pre multiplying (27) by T_1^T and post multiplying it by T_1, with $T_1 = \text{blocdiag}(T, I)$, yields:

$$\begin{bmatrix} Y & I & Yx(k)+Vx_c(k) \\ I & X & x(k) \\ (Yx(k)+Vx_c(k))^T & x(k)^T & I \end{bmatrix} > 0 , \quad (30)$$

Inequality (30) results from a convex combination of (21), for any possible $x(k)$. Hence (20) and (21) follow.

From (3), (6) and (7):

$$(\hat{A}(k)+\hat{B}\hat{K}(k))^T P(\hat{A}(k)+\hat{B}\hat{K}(k)) - P + \hat{Q} + \hat{K}(k)^T R\hat{K}(k) < 0 \quad (31)$$

With $\tilde{P} = \gamma P^{-1}$ and by Schur complement operation:

$$\begin{bmatrix} \tilde{P} & (\hat{A}(k)+\hat{B}\hat{K}(k))\tilde{P} & \cdots \\ \tilde{P}(\hat{A}(k)+\hat{B}\hat{K}(k))^T & \tilde{P} & \\ 0 & R^{1/2}\hat{K}(k)\tilde{P} & \\ 0 & \hat{Q}^{1/2}\tilde{P} & \end{bmatrix} \quad (32)$$

$$\begin{bmatrix} \cdots & 0 & 0 \\ & \tilde{P}\hat{K}(k)^T R^{1/2} & \tilde{P}\hat{Q}^{1/2} \\ & \gamma I & 0 \\ & 0 & \gamma I \end{bmatrix} > 0$$

Pre multiplying by $\hat{T}_2^T$ and post multiplying by $\hat{T}_2$:

$$\hat{T}_2 = \text{blocdiag}([T, T, I, I]), \quad (33)$$

yields:

$$\begin{bmatrix} T^T\tilde{P}T & T^T(\hat{A}(k)+\hat{B}\hat{K}(k))\tilde{P}T & \cdots \\ T^T\tilde{P}(\hat{A}(k)+\hat{B}\hat{K}(k))^T T & T^T\tilde{P}T & \\ 0 & R^{1/2}\hat{K}(k)\tilde{P}T & \\ 0 & \hat{Q}^{1/2}\tilde{P}T & \end{bmatrix}$$

$$\begin{bmatrix} \cdots & 0 & 0 \\ & T^T\tilde{P}\hat{K}(k)^T R^{1/2} & T^T\tilde{P}\hat{Q}^{1/2} \\ & \gamma I & 0 \\ & 0 & \gamma I \end{bmatrix} > 0 \quad (34)$$

With the following change of variables:

$$F = VB_c , \; L = C_c U^T , \; M = VA_c U^T , \quad Z = YAX + FCX + YBL + M , \quad (35)$$

(22) which is convex (linear) in the variables γ, X, Y, F, L and Z is obtained. If feasibility holds, every predicted state fulfills:

$$x_e(k+i/k)^T P \, x_e(k+i/k) < \gamma, \quad i \geq 1, \forall \; \chi(0) \in \Omega \quad (36)$$

that is, the ellipsoid $\varepsilon = \{z / z^T Pz < \gamma\}$ is an invariant ellipsoid for the predicted values of the states.

$$\begin{aligned} \max_{i\geq 0}\|u(k+i/k)\|_2^2 &= \max_{i\geq 0}\left\|\hat{K}(k)x_e(k+i/k)\right\|_2^2 \\ &\leq \max_{z\in\varepsilon}\left\|\hat{K}(k)z\right\|_2^2 \\ &= \lambda_{\max}\left(\tilde{P}^{1/2}\hat{K}(k)^T\hat{K}(k)\tilde{P}^{1/2}\right). \end{aligned} \quad (37)$$

By using Schur's complement, a sufficient condition for (19) to hold is:

$$\begin{bmatrix} \tilde{P} & \tilde{P}\hat{K}(k)^T \\ \hat{K}(k)\tilde{P} & u_{\max}^2 I \end{bmatrix} > 0 \quad (38)$$

which gives (23) by pre multiplication by T_1^T and post multiplication by T_1.

Similarly, considering the output:

$$\begin{aligned} &\max_{i\geq 0}\|y(k+i+1/k)\|_2^2 \\ &= \max_{i\geq 0}\left\|\hat{C}\left(\hat{A}+\hat{B}\hat{K}(k)\right)x_e(k+i/k)\right\|_2^2, \; i \geq 0 \\ &\leq \max_{z\in\varepsilon}\left\|\hat{C}\left(\hat{A}+\hat{B}\hat{K}(k)\right)z\right\|_2^2, \\ &= \lambda_{\max}\left(\tilde{P}^{1/2}\left(\hat{A}+\hat{B}\hat{K}(k)\right)^T\hat{C}^T\hat{C}\left(\hat{A}+\hat{B}\hat{K}(k)\right)\tilde{P}^{1/2}\right) \end{aligned} \quad (39)$$

then $\|y(k+i/k)\|_2 \leq y_{\max}, i \geq 1$ if:

$$\begin{bmatrix} \tilde{P} & \tilde{P}(\hat{A}+\hat{B}\hat{K}(k))^T\hat{C}^T \\ \hat{C}(\hat{A}+\hat{B}\hat{K}(k))\tilde{P} & y_{\max}^2 I \end{bmatrix} > 0 \quad (40)$$

pre multiplying (40) by T_1^T and post multiplying by T_1 yields (24). □

Remark 1. *The variables* γ, X, Y, L, F, Z *in the problem of optimization are calculated at every iteration* k.

3.1 Robust stability of the system

First, it is easy to see that, provided there exists a feasible solution, at $k=0$ for (21), (22), (23), (24), the given approach would determine a sequence of feasible solutions. Indeed, at $k=0$ solving an infinite time horizon with control and output restriction amounts to define a feasible control sequence for all subsequent time. The solution at each instant of the infinite time optimization problem would not destroy feasibility but determine a new feasible solution with a reduced cost.

Theorem 2. (Robust stability). *The dynamic receding horizon controller obtained from the solution of Theorem 1, ensures the asymptotic stability of the closed loop system.*

The stability is simply established by the fact that the proposed iterative procedure define a decreasing sequence of quadratic functions. Then $\tilde{P}(k)>0$ and $\lim_{k\to\infty} x_e(k/k)^T \tilde{P}(k)^{-1} x_e(k/k)=0$ implies $\lim_{k\to\infty} \|x_e(k/k)\|=0$.

Denoting $\tilde{P}(k)$ and $\tilde{P}(k+1)$ the values of $\tilde{P}$ obtained from the optimal solutions at instant k and $k+1$ respectively:

$$x_e(k+1/k+1)^T \tilde{P}(k)^{-1} x_e(k+1/k+1) \le x_e(k+1/k+1)^T \tilde{P}(k+1)^{-1} x_e(k+1/k+1) \quad (41)$$

since $\tilde{P}(k+1)$ is optimal whereas $\tilde{P}(k)$ is only feasible at time $k+1$.

The obtained controller is stabilizing and because $\tilde{P}(k)$ is a Lyapunov matrix, we have:

$$x_e(k+1/k)^T \tilde{P}(k)^{-1} x_e(k+1/k) \le x_e(k/k)^T \tilde{P}(k)^{-1} x_e(k/k) \quad (42)$$

If the obtained controller, at instant k is applied ($\tilde{x}(k+1/k+1)=\tilde{x}(k+1/k)$). Combining this with inequality (41) we conclude that:

$$x_e(k+1/k+1)^T \tilde{P}(k+1)^{-1} x_e(k+1/k+1) \le x_e(k/k)^T \tilde{P}(k)^{-1} x_e(k/k) \quad (43)$$

Thus, the sequence of functions $x_e(k/k)^T \tilde{P}(k)^{-1} x_e(k/k), \tilde{P}(k)>0$ is decreasing. □

3.2 Controller construction

Remark 2. *From the solution of (20)-(24) the* (A_C, B_C, C_C) *controller triplet can be recovered by inversion of the algebraic system (Geromel, et al.,1999). (35). The state space formulation of the controller depends on the choice of* V *which can be chosen regular. The input-output relation is unique.*

In the given procedure the state of the controller, which is needed in the optimization, is recursively determined from (3) using the updated (A_c, B_c, C_c) at each step. The choice of the initial $x_c(0)$ is left to the designer, $x_c(0)=0$ seems a reasonable one in order not to deteriorate the feasibility of the initial LMI set (21).

4. NUMERICAL EXAMPLE

The example is taken from Kothare, *et al.*, (1996), to which it has been added an output relation. The system is composed of two masses and a spring. The system model is represented by (1), where:

$$A=\begin{bmatrix} 1 & 0 & 0.1 & 0 \\ 0 & 1 & 0 & 0.1 \\ -0.1\frac{K}{m_1} & 0.1\frac{K}{m_1} & 1 & 0 \\ 0.1\frac{K}{m_2} & -0.1\frac{K}{m_2} & 0 & 1 \end{bmatrix},\ B=\begin{bmatrix} 0 \\ 0 \\ 0.1\frac{K}{m_1} \\ 0 \end{bmatrix},$$

$$C=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

m_1 and m_2 are the two masses and K is the constant of stiffness of the spring, x_1, x_2 are the position of the two masses, while x_3, x_4 are the velocities. The methodology is applied by considering: $m_1=m_2=K=1$, $x(0)=[1 \quad 1 \quad 0 \quad 0]^T$ and $x_c(0)=[0 \quad 0 \quad 0 \quad 0]^T$. The other data are $Q=I, R=1$ and $u_{\max}=1$. The non measured states will belong to the following interval: $-0.5 \le x_3, x_4 \le 0.5$. In Figure 1, it is depicted the behavior of y_1 and y_2 of the closed loop system which shows the asymptotic behavior.

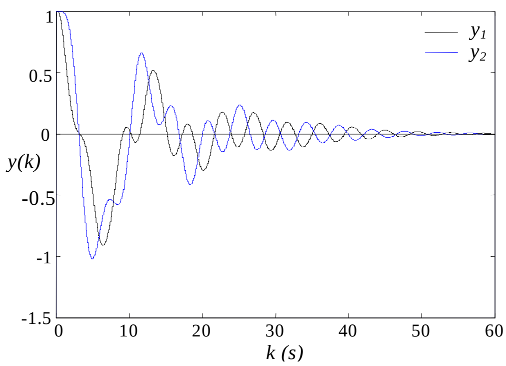

Fig. 1. Performance profile of the output.

5. DYNAMIC ROBUST CONTROLLER: POLYHEDRAL UNCERTAINTY

It is possible to extend the procedure to the case with polyhedral uncertainty. Consider the discrete linear time system represented by (1). A, B, C are uncertain matrices with polyhedral uncertainty defined by:

$$A_g = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}, \; \mathcal{D}_{\mathcal{A}g} = Co\ \{A_{g1}, A_{g2}, ..., A_{gN}\}, \quad (44)$$

The conditions of existence of the controller are given by the following theorem.

Theorem 3. *The condition in Theorem 1 are fulfilled in presence of model uncertainty if:*

$$\min \gamma \quad (45)$$

subject to

(21), (23)

$$\begin{bmatrix} Y & I & YA_i + FC_i & \tilde{Z}_i + M & 0 & 0 \\ I & X & A_i & A_i X + B_i L & 0 & 0 \\ (YA_i + FC_i)^T & A_t^T & Y & I & 0 & Q^{1/2} \\ (\tilde{Z}_i + M)^T & (A_i X + B_i L)^T & I & X & L^T R^{1/2} & XQ^{1/2} \\ 0 & 0 & 0 & R^{1/2} L & \gamma I & 0 \\ 0 & 0 & Q^{1/2} & Q^{1/2} X & 0 & \gamma I \end{bmatrix} > 0 \quad (46)$$

$\forall i = 1,2,\ldots,l$; $\tilde{Z}_i = YA_i X + FC_i X + YB_i L$,

$$\begin{bmatrix} Y & I & A_i^T C_i^T \\ I & X & XA_i^T C_i^T + L^T B_i^T C_i^T \\ C_i A_i & C_i A_i X + C_i B_i L & y_{\max}^2 I \end{bmatrix} > 0 \quad (47)$$

$\forall i = 1,2,\ldots,l$.

The problem is no longer linear but bilinear due to $\tilde{Z}_i$ term which is quadratically dependant on the X, Y, F, L unknown matrices. This, of course is a real complexity and, although relaxation techniques can be used to try to solve the problem in the SDP and LMI context it is likely that such an approach could not be used for large scale and dynamically fast systems.

6. CONCLUSIONS

In this paper, a method for the design of an output dynamic controller with constraints on the input and the output is presented within the MPC framework. At each sampling time, the parameters of the controller are updated, by solving a convex optimization problem based on linear matrix inequalities (LMI).

The concept of invariant ellipsoid is used to express the output and control constraints through LMI. It is shown that the controller ensures the stability of the closed loop system.

The design of the controller is based on real time knowledge of the measured state and the bounds of the non measured states. Other LMIs type constraints, such H_2 or H_∞ norms constraints, can be added to the procedure.

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