DESIGN OF OBSERVER-BASED CONTROLLER FOR LINEAR NEUTRAL SYSTEMS

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Abstract: In this paper, the problem of observer-based state-feedback controller design for linear neutral systems is investigated. Employing Lyapunov method and quadratic stability theory, a new delay-independent stability criterion is obtained in the form of a linear matrix inequality which can be easily solved by well-known interior-point algorithms. A numerical example is introduced to demonstrate the effectiveness of the proposed method through simulation studies.

Keywords: time-delay, full state-observer, closed-loop stability, linear matrix inequalities.

1. INTRODUCTION

Time delays inevitably exist in dynamical and physical systems because of the lack of sufficient information processing rate and data transmission capability throughout the different parts of the system. The main effect of the time delay arises as the instability of the system behaviour and often causes poor and deteriorated performance. This situation motivates the researchers to study the stabilization of time delay systems. A particular class of time delay systems in which the time delay also exists in the derivative of the states is called time delay systems of neutral type. The robust stabilization problem of neutral time delay systems has been investigated in the literature (Xu et al., 2002; Xu et al., 2003; Park, 2003a; Park, 2003b; Xu et al., 2004). The stabilization methods reported by the above authors are all based on memoryless state-feedback controllers. This situation preassumes that all the states are available or accessible through measurements. In order to avoid the requirement of all the states to be available which is not so realistic in practice for many physical systems, the only remedy is to design an observer-based controller.

The design problem of state-observers for time delay systems has also been studied by a number of researchers (Trinh, 1999; Boutayeb, 2001; Hou et al., 2002; Wang, 2002; Mahmoud and Zribi, 2003; Trinh et al., 2004). However, these authors have generally considered delay differential systems of only retarded type. There exist quite a few number of work (Wang et al., 2002; Chen et al., 2004; Park, 2004) in which state observer design for neutral type of time delay systems has been investigated.

The problem of observer design for a class of time delay nonlinear systems with parameter uncertainties is considered by Wang and Unbehauen (2000). An algebraic parametrized approach based on a Riccati matrix equation is exploited to characterize the existence conditions and the set of expected robust nonlinear observers. However, the parametrized formulation of the observer gain matrix involves computational complexity due to the requirement of predetermined design parameters. A full-order observer that guarantees the exponential stability of the error dynamic system has been considered by (Wang et al., 2002) where they developed a matrix equation approach to solve the problem. (Chen et al., 2004) addressed the problem of guaranteed cost control for a class of neutral delay systems by using an observer-based memory state-feedback controller. (Park, 2004) developed an observer-based controller for a class of linear differential systems of neutral type. However, one of the stabilization criterion given in (Park, 2004) for the existence of the observer-based controller can not be expressed in the form of linear matrix inequalities. This implies that the matrix inequalities for the stabilization criteria cannot be solved simultaneously by using convex optimization methods. The only way is to put some additional conditions in order to convert matrix inequalities into linear matrix inequalities (LMIs). However, this situation may induce additional conservatism into the system.

In this paper, we consider the observer-based memoryless state-feedback controller design problem for a class of time delay systems of neutral type. The delay in the states and state derivatives is assumed to be constant. A full-order state-observer is constructed and both the error dynamics and closed-loop system dynamics are taken into consideration. A delay-independent stabilization criterion is obtained in terms of a linear matrix inequality. Unlike the method of (Park, 2004), our stabilization criterion can be easily solved by LMI optimization techniques in the LMI control toolbox with no requirement to make further assumptions. A numerical example is given in order to demonstrate the utilization of the observer-based stabilization method proposed in this note. Some numerical simulations are also presented.

2. PROBLEM STATEMENT

Let us consider a class of linear neutral time delay system of the following form:

$$\begin{aligned} &\dot{x}(t) - C\dot{x}(t-\tau) = Ax(t) + A_d x(t-h) + Bu(t) \\ &y(t) = Fx(t) \end{aligned} \quad (1)$$

with the initial condition function

$$x(t+\theta) = \Phi(\theta), \quad \forall \theta \in \left[-\bar{h}, 0\right] \quad (2)$$

where $x(t) \in \Re^n$ is the state vector; $u(t) \in \Re^m$ is the control input vector; $y(t) \in \Re^p$ is the output vector; $A, A_d, C \in \Re^{n\times n}, B \in \Re^{n\times m}, F \in \Re^{p\times n}$ are constant known system matrices; τ, h are known nonnegative constant scalars denoting the neutral and discrete delays, respectively; $\bar{h}$ is $\max(\tau, h)$ and $\Phi(\cdot)$ is the given continuously differentiable function on $\left[-\bar{h}, 0\right]$.

Assumption 1. We assume that the pairs (A, B) and (A, C) are controllable and observable, respectively. Let us define the following type of an operator as

$$D(x_t) = x(t) - Cx(t-\tau) \quad (3)$$

it follows from Lemma 1 given by Ivanescu et al. (2003) that the operator $D(x_t)$ is guaranteed to be stable if the matrix C is stable in the sense of Schur-Cohn. Moreover, this situation also enables to assure the stability of (3) for any neutral time-delay satisfying $0 \le \tau < \infty$. We can consider the following type of a full-order memory state-observer for the system described in (1), (2)

$$\begin{aligned} &\dot{\tilde{x}}(t) - C\dot{\tilde{x}}(t-\tau) = A\tilde{x}(t) + A_d\tilde{x}(t-h) \\ &+ Bu(t) + L\left[y(t) - F\tilde{x}(t)\right] \end{aligned} \quad (4)$$

and we choose the state-feedback control law as follows

$$u(t) = -K\tilde{x}(t) \quad (5)$$

where $\tilde{x}(t) \in \Re^n$ is the estimated state vector; $K \in \Re^{m\times n}$ is the constant feedback gain matrix and $L \in \Re^{n\times p}$ is the constant observer-gain matrix to be selected.

In order to obtain the error dynamics, we can define the estimation error as

$$e(t) = x(t) - \tilde{x}(t) \quad (6)$$

where $e(t)$ is the estimation error vector. Then we can formulate the error dynamics as follows

$$\begin{aligned} \dot{e}(t) &= \dot{x}(t) - \dot{\tilde{x}}(t) = Ax(t) + A_d x(t-h) \\ &+ C\dot{x}(t-\tau) + Bu(t) - A\tilde{x}(t) - A_d\tilde{x}(t-h) \\ &- C\dot{\tilde{x}}(t-\tau) - Bu(t) - LF\left[x(t) - \tilde{x}(t)\right] \\ &= (A - LF)e(t) + A_d e(t-h) + C\dot{e}(t-\tau) \end{aligned} \quad (7)$$

or we can rewrite (7) in the form of neutral type delay-differential equation as

$$\dot{e}(t) - C\dot{e}(t-\tau) = (A - LF)e(t) + A_d e(t-h) \quad (8)$$

Moreover, substituting the control law (5) into system (1) gives the closed-loop system dynamics as

$$\begin{aligned} &\dot{x}(t) - C\dot{x}(t-\tau) = Ax(t) + A_d x(t-h) - BK\tilde{x}(t) \\ &= Ax(t) + A_d x(t-h) - BK\left[x(t) - e(t)\right] \\ &= (A - BK)x(t) + A_d x(t-h) + BKe(t) \end{aligned} \quad (9)$$

The objective of this paper is to design an observer-based state-feedback controller $u(t)$ for the neutral system (1) such that the error dynamics (7) and the resulting closed-loop neutral system (9) are asymptotically stable.

3. MAIN RESULT

In this section we develop a delay-independent criterion presented in the form of a linear matrix inequality assuring the asymptotic stabilizability of the neutral delay-differential system (1) with the memory observer-based state-feedback controller (5). The following theorem summarizes the main result of this note.

Theorem 1. Given a nonnegative constant scalar, $\alpha > 0$, if there exist symmetric and positive definite matrices X, Q, R, S, T, Z and an arbitrary matrix W, an arbitrary vector K, all with appropriate dimensions satisfying

$$\Sigma < 0 \quad (10)$$

where

$\Sigma_{11} = A^T X + XA + \alpha(A + A^T) - \alpha K^T B^T - \alpha BK + Q + R$

$\Sigma_{21}^T = \Sigma_{12} = \alpha A_d + XA_d$,

$\Sigma_{31}^T = \Sigma_{13} = XAC + \alpha AC - \alpha BKC + (Q + R)C$,

$\Sigma_{41}^T = \Sigma_{14} = \alpha BK$, $\Sigma_{61}^T = \Sigma_{16} = \alpha BKC$,

$\Sigma_{71}^T = \Sigma_{17} = 2XB$, $\Sigma_{81}^T = \Sigma_{18} = K^T$, $\Sigma_{22} = -Q$,

$\Sigma_{33} = -R + C^T(Q + R)C$, $\Sigma_{93}^T = \Sigma_{39} = C^T K^T$,

$\Sigma_{44} = A^T S + SA - F^T W^T - WF + T + Z$,

$\Sigma_{54}^T = \Sigma_{45} = SA_d$, $\Sigma_{64}^T = \Sigma_{46} = SAC - WFC$,

$\Sigma_{10,4}^T \Sigma_{4,10} = K^T$, $\Sigma_{55} = -T$,

$\Sigma_{66} = -Z + C^T(T + Z)C$, $\Sigma_{11,6}^T = \Sigma_{6,11} = C^T K^T$,

$\Sigma_{77} = -I$, $\Sigma_{88} = -I$, $\Sigma_{99} = -I$, $\Sigma_{10,10} = -I$, $\Sigma_{11,11} = -I$.
and the remaining entries are zero, then system (1), (2) is asymptotically stabilizable with the observer-based controller $u(t) = -K\hat{x}(t)$.

Proof. Let us choose a candidate Lyapunov-Krasovskii functional as

$$V(x(t)) = \sum_{i=1}^{6} V_i \tag{11}$$

where

$$\begin{aligned} V_1 &= [D(x_t)]^T PD(x_t) \\ V_2 &= \int_{t-h}^{t} x^T(s)Qx(s)ds \\ V_3 &= \int_{t-\tau}^{t} x^T(s)Rx(s)ds \\ V_4 &= [D(e_t)]^T SD(e_t) \\ V_5 &= \int_{t-h}^{t} e^T(s)Te(s)ds \\ V_6 &= \int_{t-\tau}^{t} e^T(s)Ze(s)ds \end{aligned} \tag{12}$$

with P, Q, R, S, T, Z being symmetric and positive definite matrices all to be selected appropriately. Computing the time derivative of V_i $(i = 1,\dots 6)$ along the state and error trajectories of (8) and (9) respectively yields,

$$\dot{V}_1 = [\dot{x}(t) - C\dot{x}(t-\tau)]PD(x_t) + [D(x_t)]^T P \times [\dot{x}(t) - C\dot{x}(t-\tau)] \tag{13}$$

Substituting (9) and the relation of $x(t) = D(x_t) + Cx(t-\tau)$ into (13) gives

$$\begin{aligned} \dot{V}_1 &= \{(A-BK)[D(x_t) + Cx(t-\tau)] + A_d x(t-h) \\ &+ BKe(t)\}^T PD(x_t) + [D(x_t)]^T P\{(A-BK) \\ &\times [D(x_t) + Cx(t-\tau)] + A_d x(t-h) + BKe(t)\} \\ &= [D(x_t)]^T [(A-BK)^T P + P(A-BK)]D(x_t) \\ &+ 2[D(x_t)]^T P(A-BK)Cx(t-\tau) + 2[D(x_t)]^T P \\ &\times A_d x(t-h) + 2[D(x_t)]^T PBKe(t) \end{aligned} \tag{14}$$

and we also compute the following

$$\dot{V}_2 = x^T(t)Qx(t) - x^T(t-h)Qx(t-h) \tag{15}$$

$$\dot{V}_3 = x^T(t)Rx(t) - x^T(t-\tau)Qx(t-\tau) \tag{16}$$

and in a similar manner we obtain

$$\dot{V}_4 = [\dot{e}(t) - C\dot{e}(t-\tau)]SD(e_t) + [D(e_t)]^T S \times [\dot{e}(t) - C\dot{e}(t-\tau)]$$

Using (7) and replacing $e(t)$ with $D(e_t) + Ce(t-\tau)$ in the above equation gives

$$\begin{aligned} \dot{V}_4 &= \{(A-LF)[D(e_t) + Ce(t-\tau)] + A_d e(t-h)\}^T SD(e_t) \\ &+ [D(e_t)]^T S\{(A-LF)[D(e_t) + Ce(t-\tau)] + A_d e(t-h)\} \\ &= [D(e_t)]^T [(A-LF)^T S + S(A-LF)]D(e_t) + 2[D(e_t)]^T \\ &\times S(A-LF)Ce(t-\tau) + 2[D(e_t)]^T SA_d e(t-h) \end{aligned} \tag{17}$$

Finally, we get the following derivatives

$$\dot{V}_5 = e^T(t)Te(t) - e^T(t-h)Te(t-h) \tag{18}$$

$$\dot{V}_6 = e^T(t)Ze(t) - e^T(t-\tau)Ze(t-\tau) \tag{19}$$

Summing up $\dot{V}_i$ $(i = 1,\dots 6)$ in (14)-(19) to get $\dot{V}(x(t))$ as

$$\begin{aligned} \dot{V}(x(t)) &= [D(x_t)]^T [(A-BK)^T P + P(A-BK)]D(x_t) \\ &+ 2[D(x_t)]^T P(A-BK)Cx(t-\tau) + 2[D(x_t)]^T P \\ &\times A_d x(t-h) + 2[D(x_t)]^T PBKe(t) + x^T(t)(Q+R)x(t) \\ &- x^T(t-h)Qx(t-h) - x^T(t-\tau)Rx(t-\tau) \\ &+ [D(e_t)]^T [(A-LF)^T S + S(A-LF)]D(e_t) \\ &+ 2[D(e_t)]^T S(A-LF)Ce(t-\tau) + 2[D(e_t)]^T S \\ &\times A_d e(t-h) + e^T(t)(T+Z)e(t) \\ &- e^T(t-h)Te(t-h) - e^T(t-\tau)Ze(t-\tau) \end{aligned} \tag{20}$$

We can express the quadratic terms involving $x(t)$ and $e(t)$ in terms of $D(x_t)$ and $D(e_t)$, respectively.

$$\begin{aligned} x^T(t)(Q+R)x(t) &= [D(x_t) + Cx(t-\tau)]^T (Q+R) \\ &\times [D(x_t) + Cx(t-\tau)] = [D(x_t)]^T (Q+R)D(x_t) \\ &+ 2[D(x_t)]^T (Q+R)Cx(t-\tau) + x^T(t-\tau)C^T \\ &\times (Q+R)Cx(t-\tau) \end{aligned} \tag{21}$$

$$\begin{aligned} e^T(t)(T+Z)e(t) &= [D(e_t) + Ce(t-\tau)]^T (T+Z) \\ &\times [D(e_t) + Ce(t-\tau)] = [D(e_t)]^T (T+Z)D(e_t) \\ &+ 2[D(e_t)]^T (T+Z)Ce(t-\tau) + e^T(t-\tau)C^T \\ &\times (T+Z)Ce(t-\tau) \end{aligned} \tag{22}$$

$$\begin{aligned} 2[D(x_t)]^T PBKe(t) &= 2[D(x_t)]^T PBK[D(e_t) + Ce(t-\tau)] \\ &= 2[D(x_t)]^T PBKD(e_t) + 2[D(x_t)]^T PBKCe(t-\tau) \end{aligned} \tag{23}$$

Thus, substituting (21)-(23) into (20) yields,

$$\begin{aligned} \dot{V}(x(t)) &= [D(x_t)]^T [(A-BK)^T P + P(A-BK) + Q + R] \\ &\times D(x_t) + 2[D(x_t)]^T PA_d x(t-h) \\ &+ 2[D(x_t)]^T [P(A-BK)C + (Q+R)C]x(t-\tau) \\ &+ 2[D(x_t)]^T PBKD(e_t) + 2[D(x_t)]^T PBKCe(t-\tau) \\ &- x^T(t-h)Qx(t-h) - x^T(t-\tau)[-R + C^T(Q+R)C] \\ &\times x(t-\tau) + [D(e_t)]^T [(A-LF)^T S + S(A-LF) + T + Z] \\ &\times D(e_t) + 2[D(e_t)]^T SA_d e(t-h) + 2[D(e_t)]^T \\ &\times [S(A-LF)C + (T+Z)C]e(t-\tau) - e^T(t-h)Te(t-h) \\ &- e^T(t-\tau)[Z - C^T(T+Z)C]e(t-\tau) \end{aligned} \tag{24}$$

We can rewrite (24) in quadratic form as follows

$$\dot{V}(x(t)) = \chi^T(t)\Omega\chi(t) \tag{25}$$

where

$$\chi(t) = [D(x_t)^T \quad x^T(t-h) \quad x^T(t-\tau) \quad D(e_t)^T \quad e^T(t-h) \quad e^T(t-\tau)]^T$$

$\Omega_{11} = (A-BK)^T P + P(A-BK) + Q + R$,
$\Omega_{21}^T = \Omega_{12} = PA_d$,
$\Omega_{31}^T = \Omega_{13} = P(A-BK)C + (Q+R)C$,
$\Omega_{41}^T = \Omega_{14} = PBK$, $\Omega_{61}^T = \Omega_{16} = PBKC$, $\Omega_{22} = -Q$,
$\Omega_{33} = -R + C^T(Q+R)C$,
$\Omega_{44} = (A-LF)^T S + S(A-LF) + T + Z$
$\Omega_{54}^T = \Omega_{45} = SA_d$, $\Omega_{64}^T = \Omega_{46} = S(A-LF)C$
$+(T+Z)C$, $\Omega_{55} = -T$, $\Omega_{66} = -Z + C^T(T+Z)C$.

and the remaining entries are all zero. Hence, in order to guarantee the asymptotic stability of system (1) with the memory observer-based controller (5), one needs to satisfy the following inequality

$$\dot{V}(x(t)) = \chi^T(t)\Omega\chi(t) < 0$$

which implies that

$$\Omega < 0 \quad (26)$$

Note that, the matrix inequality given in (26) is not in the form of linear matrix inequalities. We can employ matrix decomposition technique (Bartholomeus et al., 1997) to be able to reexpress (26) in the form of a linear matrix inequality.

Given a constant nonnegative scalar, $\alpha > 0$, we assume that P is chosen such thatbe decomposed as

$$P = \alpha I + X \quad (27)$$

where X is a symmetric and positive definite matrix. Then we can recompute the entries of Ω that involve P by substituting (27) appropriately

$(A - BK)^T P + P(A - BK) + Q + R$

$= A^T X + XA + \alpha(A + A^T) - \alpha K^T B^T - \alpha BK + Q + R$

$- K^T B^T X - XBK$, and $P(A - BK)C + (Q + R)C$

$= XAC + \alpha AC - \alpha BKC + (Q + R)C - XBKC$, and $PBK = \alpha BK + XBK$, and $PBKC = \alpha BKC + XBKC$, and $PA_d = \alpha A_d + XA_d$.

Now we can rewrite (26) by substituting the above expressions to obtain

$$\Omega = \Phi + \Gamma + \Gamma^T < 0 \quad (28)$$

where

$\Phi_{11} = A^T X + XA + \alpha(A + A^T) - \alpha K^T B^T - \alpha BK + Q + R$

$\Phi_{21}^T = \Phi_{12} = \alpha A_d + XA_d$,

$\Phi_{31}^T = \Phi_{13} = XAC + \alpha AC - \alpha BKC + (Q + R)C$,

$\Phi_{41}^T = \Phi_{14} = \alpha BK$, $\Phi_{61}^T = \Phi_{16} = \alpha BKC$, $\Phi_{22} = -Q$,

$\Phi_{33} = -R + C^T (Q + R)C$,

$\Phi_{44} = (A - LF)^T S + S(A - LF) + T + Z$,

$\Phi_{54}^T = \Phi_{45} = SA_d$, $\Phi_{64}^T = \Phi_{46} = S(A - LF)C + (T + Z)C$,

$\Phi_{55} = -T$, $\Phi_{66} = -Z + C^T (T + Z)C$, $\Gamma_{11} = -XBK$,

$\Gamma_{13} = -XBKC$, $\Gamma_{14} = XBK$, $\Gamma_{16} = XBKC$ and the remaining entries of Φ and Γ are all zero.

Note that we can represent Γ in the following sum of product terms

$$\Gamma = \Gamma_0 \sum_{i=1}^{4} \Gamma_i \quad (29)$$

where

$\Gamma_0 = \begin{bmatrix} -B^T X & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$,

$\Gamma_1 = \begin{bmatrix} K & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$,

$\Gamma_2 = \begin{bmatrix} 0 & 0 & KC & 0 & 0 & 0 \end{bmatrix}$,

$\Gamma_3 = \begin{bmatrix} 0 & 0 & 0 & K & 0 & 0 \end{bmatrix}$,

$\Gamma_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & KC \end{bmatrix}$.

We can utilize Lemma 1 given in (Wang, 2002) as follows:

$$\Gamma + \Gamma^T \le 4\Gamma_0\Gamma_0^T + \Gamma_1^T\Gamma_1 + \Gamma_2^T\Gamma_2 + \Gamma_3^T\Gamma_3 + \Gamma_4^T\Gamma_4 \quad (30)$$

Substituting (30) appropriately into (28) allows to get

$$\Phi + \Gamma_1 + \Gamma_1^T < 0$$

$$\Leftrightarrow \Phi + 4\Gamma_0\Gamma_0^T + \Gamma_1^T\Gamma_1 + \Gamma_2^T\Gamma_2 + \Gamma_3^T\Gamma_3 + \Gamma_4^T\Gamma_4 = \Pi < 0 \quad (31)$$

where

$\Pi_{11} = A^T X + XA + \alpha(A + A^T) - \alpha K^T B^T - \alpha BK + Q + R$

$+ 4XBB^T X + K^T K$, $\Pi_{12} = \Phi_{12}$, $\Pi_{13} = \Phi_{13}$, $\Pi_{14} = \Phi_{14}$

$\Pi_{16} = \Phi_{16}$, $\Pi_{22} = \Phi_{22}$, $\Pi_{33} = \Phi_{33} + C^T K^T KC$,

$\Pi_{44} = \Phi_{44} + K^T K$, $\Pi_{45} = \Phi_{45}$, $\Pi_{46} = \Phi_{46}$, $\Pi_{55} = \Phi_{55}$,

$\Pi_{66} = \Phi_{66} + C^T K^T KC$. Choosing an arbitrary matrix W such that

$$L = S^{-1} W \quad (32)$$

and using Schur's complement (Boyd et al., 1994), we can represent (31) as a matrix inequality form given by (10). If the condition (10) is satisfied, then system (7), (9) are guaranteed to be asymptotically stable. Hence the proof is completed.

Remark 1. Note that the inequality (10) is in the form of a linear matrix inequality which can be easily solved using interior point algorithms (Boyd et al., 1994).

Remark 2. The problem of how to choose the scalar parameter α can be handled by choosing repeatedly arbitrary positive values for α until the LMI control toolbox has given feasible solutions for the LMI (10).

4. NUMERICAL EXAMPLE

In this section we consider a numerical example in order to demonstrate the effectiveness of the proposed design approach.

Example. Consider the following linear neutral system:

$$\begin{aligned} &\dot{x}(t) - C\dot{x}(t - \tau) = Ax(t) + A_d x(t - h) + Bu(t) \\ &y(t) = Fx(t) \end{aligned} \quad (33)$$

with

$A = \begin{bmatrix} 0 & 0.5 \\ 1 & -1 \end{bmatrix}$, $A_d = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}$, $C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$,

$B = \begin{bmatrix} 0.3 \\ 0.3 \end{bmatrix}$, $F = \begin{bmatrix} 1 & 1 \end{bmatrix}$.

We first choose $\alpha = 50$ and solve (10) with interior-point algorithms in the LMI control toolbox. The feasible solutions are obtained as follows

$X = \begin{bmatrix} 85.0709 & -81.6913 \\ -81.6913 & 90.5383 \end{bmatrix}$,

$Q = \begin{bmatrix} 102.3275 & -54.3624 \\ -54.3624 & 148.2740 \end{bmatrix}$,

$R = \begin{bmatrix} 97.2577 & -60.2367 \\ -60.2367 & 145.2275 \end{bmatrix}$,

$S = \begin{bmatrix} 174.8857 & -36.3627 \\ -36.3627 & 183.3778 \end{bmatrix}$,

$T = \begin{bmatrix} 178.7015 & 2.5391 \\ 2.5391 & 181.4092 \end{bmatrix}$,

$Z = \begin{bmatrix} 184.7200 & 4.2524 \\ 4.2524 & 183.4694 \end{bmatrix}$,

$W = \begin{bmatrix} 397.2784 \\ 162.0864 \end{bmatrix}$, $K = \begin{bmatrix} 7.1512 & 5.3099 \end{bmatrix}$,

$L = \begin{bmatrix} 2.5610 \\ 1.3917 \end{bmatrix}$.

For the simulation case study, we chose the neutral and discrete delays as $\tau = 1\sec$ and $h = 3\sec$, respectively and using the design parameters K and L provided by the LMI control toolbox, we obtain the explicit dynamic equations for the closed-loop system as

$$\dot{x}_1(t) = 0.5x_2(t) + 0.3x_1(t-3) + 0.2x_2(t-3)$$
$$-2.1454\tilde{x}_1(t) - 1.5930\tilde{x}_2(t) + 0.1\dot{x}_1(t-1)$$

$$\dot{x}_2(t) = x_1(t) - x_2(t) + 0.2x_1(t-3) + 0.3x_2(t-3)$$
$$-2.1454\tilde{x}_1(t) - 1.5930\tilde{x}_2(t) + 0.1\dot{x}_2(t-1)$$

and similarly we get the state-observer dynamic equations

$$\dot{\tilde{x}}_1(t) = -4.7064\tilde{x}_1(t) - 3.6540\tilde{x}_2(t) + 0.3\tilde{x}_1(t-3)$$
$$+0.2\tilde{x}_2(t-3) + 0.1\dot{\tilde{x}}_1(t-1) + 2.5610x_1(t) + 2.5610x_2(t)$$

$$\dot{\tilde{x}}_2(t) = -2.5371\tilde{x}_1(t) - 3.9847\tilde{x}_2(t) + 0.2\tilde{x}_1(t-3)$$
$$+0.3\tilde{x}_2(t-3) + 0.1\dot{\tilde{x}}_2(t-1) + 1.3917x_1(t) + 1.3917x_2(t)$$

The simulation case study has been carried out with the estimated state-feedback control input signal which can be seen in Figure 1. Figure 2 depicts the response of the actual and estimated signals for $x_1(t)$.

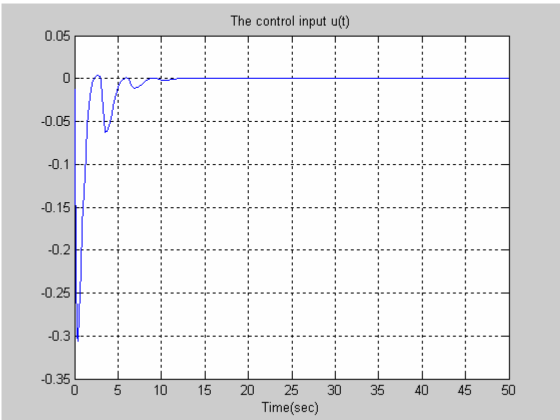

Figure 1. Input signal $u(t)$.

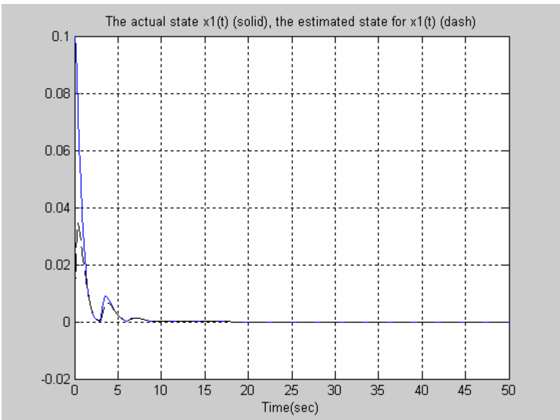

Figure 2. Response of the actual and estimated signal $x_1(t)$, $\tilde{x}_1(t)$, respectively.

The estimation error in the transient period is quite low and it rapidly converges to zero in the steady-state. The actual and estimated signals for $x_2(t)$ are shown in Figure 3 where a similar performance with that of $x_1(t)$ is noticed in the estimation of $x_2(t)$. Therefore, the simulation results indicate that the proposed delay-independent observer-based controller exhibits a satisfactory performance.

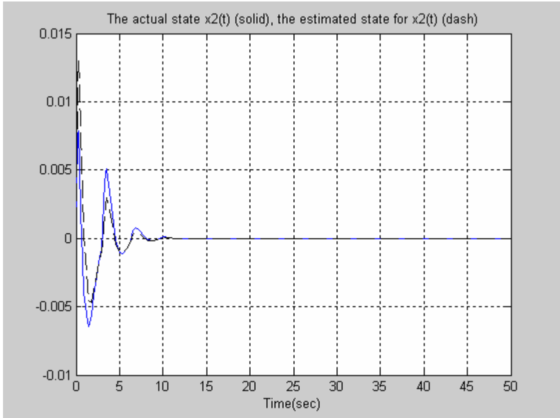

Figure 3. Response of the actual and estimated signal $x_2(t)$, $\tilde{x}_2(t)$, respectively.

5. CONCLUSION

In this paper, the memory observer-based stabilizability problem of a class of neutral time-delay systems is addressed. Based on Leibniz-Newton model transformation and Lyapunov quadratic stability theory, a new delay-independent linear matrix inequality form of stabilization criterion that ensures the asymptotic stability of both the error dynamics and the closed-loop systems dynamics is obtained. Unlike a recently given method by (Park, 2004), the proposed stabilizability condition for the existence of a memory observer-based controller can be directly solved using effective interior-point algorithms.

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