CONSTRAINED ROBUST MODEL PREDICTIVE CONTROL BASED ON PERIODIC INVARIANCE

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Abstract: The dual-mode strategy has been adopted in many constrained MPC methods. The size of stabilizable regions of states of MPC methods depends on the size of underlying feasible and positively invariant set and number of control moves. These results, however, could be conservative because the definition of positive invariance does not allow temporal leave of states from the set. In this paper, a concept of periodic invariance is introduced in which states are allowed to leave a set temporarily but return into the set in finite time steps. This approach is novel in the sense that a set of different state feedback gains can be used to steer the state back into the starting set. These facts make it possible for the periodically invariant sets to be considerably larger than ordinary invariant sets. The periodic invariance can be defined for systems with polyhedral model uncertainties. We derive computationally very efficient MPC methods based on these periodically invariant sets. Some numerical examples are given to show that the use of periodic invariance yields considerably larger stabilizable sets with better performance than the case of using ordinary invariance.

Keywords: Input Constraints, Model Predictive Control, Periodic Invariant Sets, Model Uncertainty

1. INTRODUCTION

The 'dual-mode paradigm' is known to be an effective way to handle physical constraints in actuators.(Fukushima and Bitmead, 2005)(Lee and Kouvaritakis, 2000) (Kothare et al., 1996) The basic idea of the dual-mode paradigm is to use feasible control moves to steer the current state into a feasible and invariant set in finite time steps. A constant state feedback control is assumed to be used after the state belongs to the positively invariant set. The feasible and positive invariance of a set is defined with respect to a state feedback gain and it requires that the state feedback control should satisfy the input constraints for all the states in the set and states should remain in the set when the state feedback control is applied. This dual-mode strategy has been adopted in many constrained MPC methods. The size of stabilizable regions of states of MPC methods depends on the size of underlying feasible and positively invariant set and number of control moves. These results, however, could be conservative because the definition of positive invariance does not allow temporal leave of states from the set. In the recent work of (Cuzzola et al, 2002), a parameter dependent Lyapunov function has been used to reduce the conservativeness of the dual mode approach. But these works still assume the use of a single feedback gain and requires strict invariance in the definition of a positively invariant set. Motivated by these considerations, the concept of quasi-invariant sets was introduced in (Lee, 2004), based on polyhedral type sets, which allows the state to leave the set temporarily. The result given in (Lee, 2004), however, provides an analysis tool rather than a synthesis tool *i.e.* no systematic way of obtaining underlying state feedback gain is provided.

In this paper, a concept of periodic invariance is introduced based on ellipsoidal type sets, in which states are allowed to leave a set temporarily but return into the set in finite time steps. Moreover, the periodic invariance involves the use of more than one state feedback gains as it was done for unconstrained deterministic systems in Yan and Bitmead(1991). These facts make it possible for the periodically invariant sets to be considerably larger than ordinary invariant sets and to yield better performance.

The periodic invariance was deployed for LPV systems in Lee, Kouvaritakis and Cannon(2005), however, here we focus considerations on systems with polyhedral model uncertainties. Applying the concept of periodic invariance to uncertain systems can enjoy novel characteristics that can not be obtained for LPV cases. If a periodically invariant target set is searched online. It can be shown to be equivalent to using the convex hull of all of the periodically invariant sets as a target set and the resulting stabilizable region becomes large. Instead of computing periodically invariant sets online, we can use the convex-hull of pre-computed periodically invariant sets as a target set and the computational burden can be much reduced without affecting the size of stabilizable region much. The effect of computation reduction becomes significant for high order plants. In both of the methods, the cost indices are state dependent and this fact allows using tight control to yield good performance

In section 2, the periodic invariance is defined and a MPC method that uses the convex hull of periodically invariant sets as a target was developed in Section 3. In section 4, an novel method of reducing the computational burden is proposed and an illustrative numerical example is given in Section 5 to show that the use of periodic-invariance yields much larger stabilizable set with better performance.

2. PERIODIC INVARIANCE AND FEASIBILITY

Consider the following input constrained linear uncertain system:

$$\mathbf{x}(k+1) = \tilde{A}\mathbf{x}(k) + \tilde{B}\mathbf{u}(k), \ |\mathbf{u}(k)| \leq \overline{\mathbf{u}}, \quad (1)$$

where the matrix functions $\tilde{A}$ and $\tilde{B}$ belong to the polyhedral uncertainty class:

$$\Omega = \left\{ (\tilde{A}, \tilde{B}) | (\tilde{A}, \tilde{B}) = \sum_{l=1}^{n_p} \eta_l (A_l, B_l), \ \eta_l \geq 0, \sum_{l=1}^{n_p} \eta_l = 1 \right\}, \quad (2)$$

We will consider a time-varying state feedback control law as:

$$\mathbf{u}(k) = K(k)\mathbf{x}(k), \quad (3)$$

which requires

$$|\mathbf{u}(k)| = |K(k)\mathbf{x}(k)| \leq \overline{\mathbf{u}}. \quad (4)$$

Provided that (4) is satisfied, use of $\mathbf{u}(k) = K(k)\mathbf{x}(k)$ would yield

$$\mathbf{x}(k+1) = \tilde{\Phi}(k)\mathbf{x}(k), \quad \tilde{\Phi}(k) := \tilde{A} + \tilde{B}K(k). \quad (5)$$

Consider the uncertain linear system described by (1) and (2). A set $\mathcal{S}_0$ is defined to be feasible and **periodic-invariant** with respect to the time varying feedback control $\mathbf{u}(k) = K(k)\mathbf{x}(k)$ of (3) if there exists a finite positive number $\mathbf{v}$ such that for any initial state $\mathbf{x}(k) \in \mathcal{S}_0$, the future states $\mathbf{x}(k+i)$ $(i = 1, \cdots, \mathbf{v})$ of the system (5) satisfy the input constraint (4)(feasible) and $\mathbf{x}(k+\mathbf{v})$ belongs to $\mathcal{S}_0$(periodic-invariant).

Consider an ellipsoidal set defined as:

$$\mathcal{S}_0 = \{\mathbf{x} | \mathbf{x}' P_0 \mathbf{x} \leq 1\}. \quad (6)$$

The periodic-invariance of $\mathcal{S}_0$ would be checked by considering propagation of the states in terms of ellipsoidal sets. Assume that the closed-loop dynamics of (5) makes $\mathbf{x}(k+1) \in \mathcal{S}_1$ for any $\mathbf{x}(k) \in \mathcal{S}_0$, where

$$\mathcal{S}_1 := \{\mathbf{x} | \mathbf{x}' P_1 \mathbf{x} \leq 1\}. \quad (7)$$

It is easy to see that the following relation:

$$P_0 - \Phi_l(k)' P_1 \Phi_l(k) > 0, (l = 1, \cdots, n_p) \quad (8)$$

guarantees that $\mathbf{x}(k+1) \in \mathcal{S}_1$ for any $\mathbf{x}(k) \in \mathcal{S}_0$ and $(\tilde{A}, \tilde{B}) \in \Omega$, where $\Phi_l(k) := A_l + B_l K_0$. Similarly, an ellipsoidal set $\mathcal{S}_2$ can be defined for the ellipsoidal set $\mathcal{S}_1$. Relations

$$P_1 - \Phi_l(k+1)' P_2 \Phi_l(k+1) > 0, (l = 1, \cdots, n_p) \quad (9)$$

would guarantee that $\mathbf{x}(k+2) \in \mathcal{S}_2$ for any $\mathbf{x}(k+1) \in \mathcal{S}_1$ and $(\tilde{A}, \tilde{B}) \in \Omega$.

The above argument can be applied recursively to yield ellipsoidal sets of states:

$$\mathcal{S}_j = \{\mathbf{x} | \mathbf{x}' P_j \mathbf{x} \leq 1\}, \quad (10)$$

and relations

$$P_j - \Phi_l(k+j)' P_{j+1} \Phi_l(k+j) > 0, \quad (11)$$

for $l = 1, 2, \cdots, n_p$ and $j = 1, 2, \cdots, \mathbf{v} - 1$ with $P_{\mathbf{v}} := P_0$. On the other hand, it should be noted that the above arguments hold true for the system (1) provided that

$$|K_j \mathbf{x}| \leq \overline{\mathbf{u}}, \quad \forall \mathbf{x} \in \mathcal{S}_j, \quad j = 0, 1, \cdots, (\mathbf{v} - 1) \quad (12)$$

Conditions (8), (9), (11) and (12) can be transformed into LMIs using technique proposed in (Boyd et al., 1994) and used in (Kothare et al., 1996), which technique is well known and the corresponding LMIs are summarized in the following theorem without proof:

THEOREM *1.* Consider the constrained uncertain system (1-2). An ellipsoidal set:

$$\mathcal{S}_0 = \{\mathbf{x} | \mathbf{x}' P_0 \mathbf{x} \leq 1\} \quad (13)$$

is feasible and periodic-invariant with respect to the time-varying control (3) provided that there exist matrices $Q_j := P_j^{-1} (> 0) (j = 0, 1, 2, \cdots, \mathbf{v})$, and Y_j, $X_j (j = 0, 1, 2, \cdots, \mathbf{v} - 1)$ such that $Y_j = K_j \cdot Q_j$,

$$\begin{bmatrix} Q_{j-1} & (A_l Q_{j-1} + B_l Y_{j-1})^T \\ (A_l Q_{j-1} + B_l Y_{j-1}) & Q_j \end{bmatrix} > 0 \quad (14)$$

for $l = 1, 2, \cdots, n_p$ and $j = 1, 2, \cdots, \mathbf{v}$ with $Q_{\mathbf{v}} := Q_0$, and

$$\begin{bmatrix} X_j & Y_j \\ Y_j^T & Q_j \end{bmatrix} > 0, \; X_{j,ii} \leq \overline{\mathbf{u}}_i^2, \quad (15)$$

for $j = 0, 1, 2, \cdots, \mathbf{v} - 1$, where $X_{j,ii}$ and $\overline{\mathbf{u}}_i$ represent the i^{th} (diagonal) element of X_j and $\overline{\mathbf{u}}$, respectively.

■

From the periodicity of the relation between the sets, *i.e.* $\mathcal{S}_0 \to \mathcal{S}_1 \to \cdots \to \mathcal{S}_{\mathbf{v}-1} \to \mathcal{S}_0$, all the sets $\mathcal{S}_j$, $j = 0, 1, \cdots, \mathbf{v} - 1$ are actually periodic-invariant sets. Thus, we have the following Corollary:

COROLLARY *1.* Consider the constrained uncertain system (1-2). If the set $\mathcal{S}_0$ of (13) is periodic-invariant with subsequent series of sets $\mathcal{S}_j$, $j = 1, 2, \cdots, \mathbf{v} - 1$, which are defined as (10) with $P_j = Q_j^{-1}$ satisfying (14-15). Then, each of the sets $\mathcal{S}_j$, $j = 1, 2, \cdots, \mathbf{v} - 1$ are also periodic-invariant.

Corollary 1 means that once a state found to be inside of a periodic-invariant set, then it can be steered into another periodic-invariant set.

REMARK *1.* Assume that $\mathcal{S}_0^1$ and $\mathcal{S}_0^2$ are two different periodic-invariant sets with corresponding positive definite matrices $P_0^1 (= Q_0^{1^{-1}})$ and $P_0^2 (= Q_0^{2^{-1}})$, respectively. Then another periodic-invariant set $\mathcal{S}_0^3$ defined with the matrix $P_0^3 = (\lambda_1 Q_0^1 + \lambda_2 Q_0^2)^{-1}$ is also periodic-invariant from the relations (14-15), where $\lambda_1 + \lambda_2 = 1$ and $\lambda_1, \; \lambda_2 \geq 0$. Thus, we can conclude that periodic-invariant sets can be combined to produce another periodic-invariant sets. ■

3. RECEDING HORIZON CONTROL BASED ON THE PERIODIC INVARIANCE

We would like to propose a MPC method using $\mathcal{S}_0$ as a target set following the dual-mode paradigm. Our control strategy is to steer the current state $\mathbf{x}(k)$ into $\mathcal{S}_0$ using a feasible control move $\mathbf{u}(k)$, which requires:

$$\mathbf{x}(k+1|k)^T P_0 \mathbf{x}(k+1|k) \leq 1, \quad (16)$$

where $\mathbf{x}(k+1|k) := \tilde{A}\mathbf{x}(k) + \tilde{B}\mathbf{u}(k)$. Relation (16) can be transformed into LMIs which is affine in $(\tilde{A}, \tilde{B})$ and they are equivalent to the following LMIs.

$$\begin{bmatrix} 1 & (A_l \mathbf{x}(k) + B_l \mathbf{u}(k))^T \\ (A_l \mathbf{x}(k) + B_l \mathbf{u}(k)) & Q_0 \end{bmatrix} > 0$$
$$for \; l = 1, 2, \cdots, n_p \quad (17)$$

and

$$diag(\overline{\mathbf{u}} - \mathbf{u}(k)) > 0. \quad (18)$$

The periodic-invariant set $\mathcal{S}_0$ has good properties mentioned in Corollary 1 and Remark 1. Thus the actual target set would be the convex-hull of every periodic-invariant sets for the system (1) when the target periodic-invariant set is searched on-line. From the fact that ordinary invariant sets are special cases of periodic-invariant sets, we can expect that the size of target set (in turn the size of stabilizable set) of our approach would be much larger than those of earlier works based on ordinary invariant sets.

The control input $\mathbf{u}(k)$ satisfying (17) and (14-15) would not be unique. We would like to determine the current control $\mathbf{u}(k)$ so that it steers $\mathbf{x}(k+1|k)$ as close as possible to the origin while (14-15) are satisfied. So, we introduce an additional variable α such as:

$$\alpha > \mathbf{x}(k+1|k)^T P_0 \mathbf{x}(k+1|k), \qquad (19)$$

which can be transformed into the following LMIs:

$$\begin{bmatrix} \alpha & (A_l\mathbf{x}(k) + B_l\mathbf{u}(k))^T \\ (A_l\mathbf{x}(k) + B_l\mathbf{u}(k)) & Q_0 \end{bmatrix} > 0 \qquad (20)$$

for $l = 1, 2, \cdots, n_p$. Note that (17) can be replaced by (20) provided that $\alpha < 1$. The control move $\mathbf{u}(k)$ will be determined as:

$$\mathbf{u}^*(k) = arg \left\{ \begin{array}{cc} \underset{\substack{\mathbf{u}(k) \\ Q_j, Y_j}}{min} & \alpha \end{array} \right\} \qquad (21)$$
$$subject\ to\ (14-15)(18)\ and\ (20).$$

We will use the control $\mathbf{u}^*(k)$ of (21) in receding horizon manner. Closed-loop stability of the receding horizon control based on $\mathbf{u}^*(k)$ of (21) can be established as per the following theorem:

THEOREM *2.* Consider the uncertain system (1-2). Assume that the problem of obtaining $\mathbf{u}^*(k)$ of (21) was initially feasible with $\alpha < 1$ then the problem remains feasible and the use of the optimal control $\mathbf{u}^*(k)$ obtained at each time steps guarantees the asymptotic stability of the closed-loop system.

Proof : Feasibility : Because of the periodicity of the relation (14), the problem remain feasible once the state is steered into $\mathcal{S}_0$ at initial time steps.
Stability : Assume that the problem of (21) is feasible at time step k with feasible solution $\{Q_j,\ and\ K_j = Y_j Q_j^{-1},\ \ j = 0, 1, \cdots, \mathbf{v}-1\}$ with the upper bound α . From (20), we have:

$$\alpha > \mathbf{x}(k+1|k)^T Q_0^{-1} \mathbf{x}(k+1|k). \qquad (22)$$

Because of the periodicity of (14), $\{\ Q_{j+1},\ K_{j+1} = Y_{j+1} Q_{j+1}^{-1},\ \ j = 0, 1, \cdots, \mathbf{v}-1\}$ will provide another feasible solutions for (14-15). Consider the control input $\mathbf{u}(k+1|k) = K_0 \mathbf{x}(k+1|k)$ at time step $k+1$, then we have:

$$\mathbf{x}(k+2|k) = (\tilde{A} + \tilde{B} K_0) \mathbf{x}(k+1|k) \qquad (23)$$

and

$$\mathbf{x}(k+1|k)^T Q_0^{-1} \mathbf{x}(k+1|k) \qquad (24)$$
$$> \mathbf{x}(k+2|k)^T Q_1^{-1} \mathbf{x}(k+2|k)$$

from (14). From (24), it is clear that we could have an upper bound $\hat{\alpha}$ such that $\hat{\alpha} < \alpha$ at time step $k+1$. The above argument can be applied recursively and the upper bound α can be made monotonically decreasing, which in turn guarantees the closed-loop stability. ■

REMARK *2.* One of the key points of this approach is that it provides more degrees of freedom *i.e.* $P_j,\ j = 0, 1, \cdots, \mathbf{v}-1$ than (Kothare et al., 1996) in laying the upper bound, which would result in non-conservative upper bound and better performance. As the value $\mathbf{v}$ increases, we would have better performances in addition to larger stabilizable sets. ■

4. REDUCTION OF COMPUTATIONAL LOAD

The parameters to be searched in the problem of (21) would be $Q_j \in R^{n\times n}$, $Y_j \in R^{m\times n}$, $X_j \in R^{m\times m}$ and $\mathbf{u}(k)$ for $j = 0, 1, \cdots, \mathbf{v}$. The computational burden for obtaining these all parameters online might be excessive. Thus, in this section, we provide an effective method to reduce the online computational burden. The key idea is to compute a set of periodic invariant ellipsoidal sets offline and then to use the convex hull of the ellipsoidal sets a target set of our online MPC approach.

Assume that $Q_j,\ j = 0, 1, \cdots, \mathbf{v}-1$ satisfying (14-15) are given. Consider a set of states defined as:

$$\overline{\mathcal{S}} := \{\mathbf{x} \in R^n | \mathbf{x}' (\sum_{j=0}^{\mathbf{v}-1} \lambda_j Q_j)^{-1} \mathbf{x} < 1\}, \qquad (25)$$

where λ_js are variables satisfying the constraints:

$$\sum_{j=0}^{\mathbf{v}-1} \lambda_j = 1, \quad \lambda_j \geq 0. \qquad (26)$$

It is clear that $\overline{\mathcal{S}}$ contains all the $\mathcal{S}_j,\ j = 0, 1, \cdots, \mathbf{v}-1$ as its subsets. Now the cost index is defined as:

$$\alpha > \mathbf{x}(k+1|k)^T (\sum_{j=0}^{\mathbf{v}-1} \lambda_j Q_j)^{-1} \mathbf{x}(k+1|k), \qquad (27)$$

which can be transformed into the following LMIs:

$$\begin{bmatrix} \alpha & (A_l\mathbf{x}(k) + B_l\mathbf{u}(k))^T \\ (A_l\mathbf{x}(k) + B_l\mathbf{u}(k)) & \sum_{j=0}^{\mathbf{v}-1} \lambda_j Q_j \end{bmatrix} > 0, \qquad (28)$$

for $l = 1, 2, \cdots, n_p$. Now the new MPC algorithm can be formulated as:

$$\mathbf{u}^*(k) = arg \left\{ \begin{array}{l} min \\ \mathbf{u}(k) \\ \lambda_j \end{array} \alpha \right\} \quad (29)$$
$$subject\ to\ (18)\ (26)\ and\ (28).$$

We will use the control $\mathbf{u}^*(k)$ of (29) in receding horizon manner. The feasibility and stability property of this receding horizon control can be established as per the following theorem.

THEOREM *3.* Consider the uncertain system (1-2) and matrices $Q_j,\ Y_j$s satisfying (14-15). Assume that the problem of obtaining $\mathbf{u}^*(k)$ of (29) was initially feasible with $\alpha < 1$ and $\lambda_j(> 0)$s such that $\sum_{j=0}^{\mathbf{v}-1} \lambda_j = 1$ then the problem remain feasible and the use of the optimal control $\mathbf{u}^*(k)$ obtained at each time steps guarantees the asymptotic stability of the closed-loop system.

Proof : The feasibility of the problem (29) at time step k and the monotonicity of the cost index α could be proved if we can show that there exist a state feedback gain K and parameters $\hat{\lambda}_j$s such that:

$$(\tilde{A} + \tilde{B}K)^T (\sum_{j=0}^{\mathbf{v}-1} \hat{\lambda}_j Q_j)^{-1} (\tilde{A} + \tilde{B}K) < (\sum_{j=0}^{\mathbf{v}-1} \lambda_j Q_j)^{-1} \quad (30)$$

$$\sum_{j=0}^{\mathbf{v}-1} \hat{\lambda}_j = 1,\ \hat{\lambda}_j > 0 \quad (31)$$

$$|K\mathbf{x}(k)| \le \overline{\mathbf{u}}. \quad (32)$$

The relation (30) can be transformed into:

$$\begin{bmatrix} \overline{Q}_0 & (A_l\overline{Q}_0 + B_l K \overline{Q}_0)^T \\ (A_l\overline{Q}_0 + B_l K \overline{Q}_0) & \overline{Q}_1 \end{bmatrix} > 0. \quad (33)$$

where $\overline{Q}_0 := \sum_{j=0}^{\mathbf{v}-1} \lambda_j Q_j$ and $\overline{Q}_1 := \sum_{j=0}^{\mathbf{v}-1} \hat{\lambda}_j Q_j$. On the other hand, summing up (14) for $j = 1, 2 \cdots, \mathbf{v}$ after multiplying λ_{j-1} to both sides it would yields:

$$\begin{bmatrix} \overline{Q}_0 & (A_l\overline{Q}_0 + B_l \overline{Y}_0)^T \\ (A_l\overline{Q}_0 + B_l K \overline{Y}_0) & \overline{Q}_1 \end{bmatrix} > 0, \quad (34)$$

where

$$\hat{\lambda}_j = \lambda_{j+1}\ (j = 0, 1, \cdots, \mathbf{v} - 2) \quad (35)$$

$$\hat{\lambda}_{\mathbf{v}-1} = \lambda_0 \quad (36)$$

and $\overline{Y}_0 := \sum_{j=0}^{\mathbf{v}-1} \lambda_j Y_j$. Comparing (33) and (34), we can see that assigning K as

$$K = \overline{Y}_0 \overline{Q}_0^{\,-1} \quad (37)$$

would make them equivalent. The relations (35-36) and (37) tells us how to choose $\hat{\lambda}_j$s and K so that conditions (30-31) are satisfied. Furthermore, summing up (15) for $j = 0, 1, \cdots, \mathbf{v} - 1$ after multiplying λ_j to both sides of it results in:

$$\begin{bmatrix} \overline{X}_0 & \overline{Y}_0 \\ \overline{Y}_0^T & \overline{Q}_0 \end{bmatrix} > 0,\quad \overline{X}_{0,ii} \le \overline{u}_i^2, \quad (38)$$

where $\overline{X}_0 := \sum_{j=1}^{\mathbf{v}-1} \lambda_j X_j$. From (37) and (38), we can see that $\mathbf{u}(k) = K\mathbf{x}(k)$ satisfy the input constraint (32) for $\mathbf{x}(k) \in \overline{\mathcal{S}}_0$. ■

Note that the number of the online parameters does not depend on the system order n. Thus, this method would be very effective for high order systems. Furthermore the online parameters $\lambda_j (j = 0, 1, \cdots, \mathbf{v} - 1)$ can be used to adjust the performance index so that the resulting control would be as tight as possible. Thus the resulting performance would not be much degraded compared with the method of (21) as will be shown in the simulation study.

5. NUMERICAL EXAMPLES

Consider the uncertain system (Lee and Kouvaritakis, 2000b)(Kothare et al., 1996) with polyhedral set Ω defined by (2) with $\overline{u} = 1$

$$A_1 = \begin{bmatrix} 0.9347 & 0.5194 \\ 0.3835 & 0.8310 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0.0591 & 0.2641 \\ 1.7971 & 0.8717 \end{bmatrix}$$
$$B = \begin{bmatrix} -1.4462 \\ -0.7012 \end{bmatrix}. \quad (39)$$

Fig.1 shows a series of ellipsoidal sets $\mathcal{S}_0, \mathcal{S}_1$ and $\mathcal{S}_2$ which correspond to a particular choice of feasible solutions Q_0, Q_1 and Q_2 with $n_{inv} = 3$. Stabilizable sets of the proposed MPC method with $n_{inv} = 1$, 3, and 5 are given in Fig.2. The stabilizable sets of the work (Kothare et al., 1996) is also given in Fig.2. This figure shows that we can get much larger stabilizable sets than (Kothare et al., 1996) does in any case.

6. CONCLUSIONS

A receding horizon control strategy was developed for input constrained linear uncertain systems based on periodically invariant sets. The definition of periodically invariant set allows state to leave the set temporarily. An ellipsoidal set is

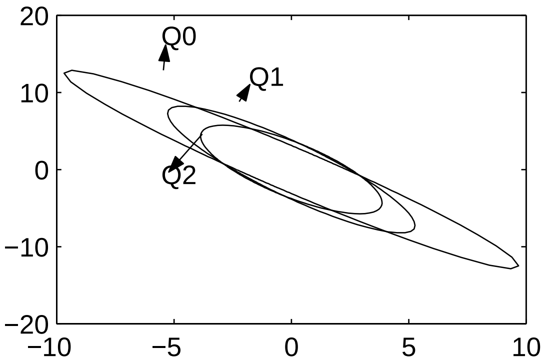

Fig. 1. Ellipsoidal sets $\mathcal{S}_0$, $\mathcal{S}_1$ and $\mathcal{S}_2$ corresponding to a set of feasible solutions Q_0, Q_1 and Q_2, respectively.

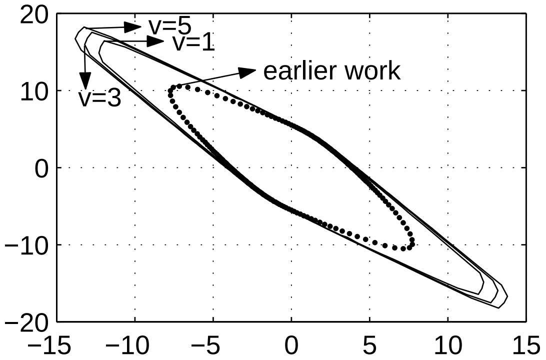

Fig. 2. Stabilizable sets with $\mathbf{v} = 1, 3$ and 5(solid lines) and stabilizable set of work (Kothare et al., 1996)(dotted line).

said to be periodically invariant if there is a series of feedback gains such that the use of these gains guarantees that all the states in the set return into the set in finite time steps.

A receding horizon control strategy in which the current state is steered into a periodically invariant set was proposed. This method is equivalent to use the convex hull of all of the periodically invariant sets as a target set and the resulting stabilizable region become large. Instead of computing periodically invariant sets online, we can use the convex-hull of pre-computed periodically invariant sets as a target set and the computational burden can be reduced considerably without affecting the size of stabilizable region much. In both of the methods, the cost indices are state dependent and this fact allows using tight control to yield good performance. The invariant set used in this paper contains the ellipsoidal invariant sets in the earlier works (Kothare. *et. al* 1996)(Lu and Arkun, 1999) as a special case. It will provide a larger invariant set and larger stabilizable set in turn.

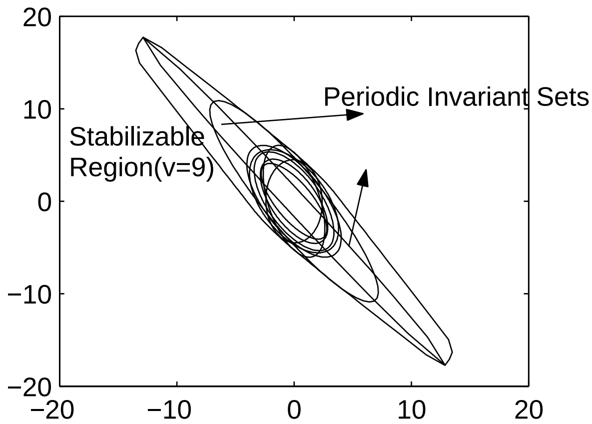

Fig. 3. Stabilizable region of the reduced computation method with $\mathbf{v} = 9$(Outer region) and corresponding 9 periodic invariant sets

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