MISSILE GUIDANCE DESIGN USING OPTIMAL TRAJECTORY SHAPING AND NEURAL NETWORK

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Abstract: Preliminary studies have shown that the aspect angle of the interceptor at lock-on near 180 degree is a fundamental requirement for an aerodynamically controlled missile achieving small miss distance against a faster target. An optimal midcourse guidance law based on the optimal trajectory shaping is developed to meet this prerequisite. Neural networks are incorporated with the PN guidance at the terminal phase to improve the tracking accuracy. The whole defensible volume in the 3-dimesional space is characterized and the performance robustness is also verified.

Keywords: Optimal guidance law, missile, robustness, neural network

1. INTRODUCTION

In the area of theoretical study, optimality-based guidance designs for guided missiles have drawn the considerable attention of many researchers in the last three decades (Bryson and Ho, 1969; Lin, 1991; Zarchan, 1994). With regard to this theme, it has recently been indicated that successfully intercepting a high speed target with a relatively low speed missile, the aspect angle between the missile and target flight path is extremely limited to within 180-degree plus or minus few degrees (Kuroda and Imado, 1988; Imado and Kuroda, 1992). Such a geometric arrangement will minimize the required lateral acceleration for interceptors to effectively engage hypersonic targets.

Speed plays an important role in determining missile aerodynamic maneuverability. Based on this requirement and the preceding investigation, the function of midcourse guidance law is developed here to maximizing the missile velocity and bring the target aspect angle to 180-degree at handover as far as possible. In our design, the trajectory shaping is determined by the predicted lock-on point relative to the missile and the missile's final flight path angle.
It is also known from the optimal control theory that a straightforward solution of the optimal trajectory shaping problem leads to a two-point boundary-value problem (TPBVP) (Bryson and Ho; 1969) which is too complex for real-time onboard implementation. In this study, we extend the solution procedure proposed by Lin and Tsai (1987) to solve the optimal trajectory shaping problem subject to the counterattack condition.

We also introduce a neural network which can be used to compensate the tracking error resulting from modeling uncertainties or undesired disturbance during the midcourse and terminal phases. The study of neural networks and their wide applications has been existence for the recent decade in identification, signal processing, and control (Miller *et al*., 1991; Narendra and Parthasarthy, 1990).

In the proposed guidance scheme, the missile trajectory in the midcourse is designed using the optimal trajectory shaping guidance to intercept the target with a near 180-degree aspect angle. During the terminal phase, a neural network constructs a specialized on-line control architecture, which offers a means of synthesizing closed-loop guidance laws for aiding the guidance command generated by proportional navigation (PN) guidance. The neural network cannot only perform very well for tracking performance, but it even extends the effectively defensive volume.

2. SYSTEM MODELS

Consider the three-dimensional translational equations of motion, used to illustrate the missile trajectory, shown as in Figs. 1 and 2. The missile is modeled as a point mass and the equations of motion are described by

$$\dot{v}_m = (T\cos\alpha - D)/m - g\sin\gamma \quad (1a)$$

$$\dot{\gamma} = (L + T\sin\alpha)\cos\phi/(mv_m) - g\cos\gamma/v_m \quad (1b)$$

$$\dot{\psi} = (L + T\sin\alpha)\sin\phi/(mv_m\cos\gamma) \quad (1c)$$

$$\dot{x}_m = v_m\cos\gamma\cos\psi \quad (1d)$$

$$\dot{y}_m = v_m\cos\gamma\sin\psi \quad (1e)$$

$$\dot{h}_m = v_m\sin\gamma \quad (1f)$$

where the state variables are the position coordinates x, y, and h; pitch angle γ; yaw angle ψ; angle-of-attack α; roll angle ϕ; and the thrust and mass are predefined functions of time. The v_m denotes the total missile speed. Moreover, the aerodynamic forces of the missile are evaluated as the following expressions:

$$L = \frac{1}{2}\rho v_m^2 s_m C_{mL}, \qquad D = \frac{1}{2}\rho v_m^2 s_m C_{mD}. \quad (2)$$

Suppose that the ballistic target system model in radar coordinates centered at radar site is expressed as

$$\begin{aligned} \dot{v}_{tx} &= -\frac{\rho v_t^2}{2\beta_t} g\cos\gamma_1\sin\gamma_2 + a_{tx} \\ \dot{v}_{ty} &= -\frac{\rho v_t^2}{2\beta_t} g\cos\gamma_1\cos\gamma_2 + a_{ty} \\ \dot{v}_{th} &= -\frac{\rho v_t^2}{2\beta_t} g\sin\gamma_1 - g + a_{th} \end{aligned} \quad (3)$$

where v_{tx}, v_{ty} and v_{th} denote the velocity components of v_t along X, Y, and H axes, respectively; $a_{tx}(t), a_{ty}(t)$ and $a_{th}(t)$ are uncertain accelerations due to model uncertainties and ballistic target's maneuvering, γ_1, γ_2 and the ballistic coefficient β_t are defined as

$$\gamma_1(t) = \tan^{-1}\left(-\frac{v_{th}}{\sqrt{v_{tx}^2 + v_{ty}^2 + v_{th}^2}}\right) \quad (4)$$

$$\gamma_2(t) = \tan^{-1}\left(\frac{v_{tx}}{v_{ty}}\right), \; \beta_t = \frac{W}{s_t C_{tD0}}$$

where s_t, W and C_{tD0} represent the reference area, weight and zero-lift drag coefficient of the target, respectively.

3. ATTACK STRATEGY

The guidance scheme proposed consists of midcourse, shaping and terminal phases shown as in Fig. 3. To design the midcourse guidance law so that the missile attains near head-on geometry, we define the preset lock-on point range as

$$R_p = R + R_{lock} \quad (5)$$

where R_{lock} is the seeker lock-on range which is assumed to be constant and R is the relative range between the missile and target. The required time-to-go for the target attaining the preset lock-on point can be estimated as

$$t_{gop} = -\frac{R_P}{\dot{R}_P} = \frac{R_p R}{R_{mtx}v_{rx} + R_{mty}v_{ry} + R_{mth}v_{rh}} \quad (6)$$

where

$$R = \sqrt{R_{mtx}^2 + R_{mty}^2 + R_{mth}^2}$$

Based on the t_{gop} obtained, the predicted lock-on point is estimated as follows

$$x_f = x_t + v_{tx}t_{gop}, \; y_f = y_t + v_{ty}t_{gop}, \; h_f = h_t + v_{th}t_{gop} \quad (7)$$

To attain the near head-on geometry, we specify the desired flight-path angle as

$$\hat{\gamma}_f = \tan^{-1}\left(\frac{h_f - h_m}{x_f - x_m}\right) \quad (8)$$

$$\hat{\psi}_f = \tan^{-1}\left(\frac{y_f - y_m}{x_f - x_m}\right) \quad (9)$$

The predicted line-of-slight (LOS) angle with respect to the lock-on point is given by

$$\hat{\theta}_v = \tan^{-1}\left(\frac{\hat{h}_t - h_m}{\hat{x}_t - x_m}\right) \quad (10)$$

$$\hat{\theta}_h = \tan^{-1}\left(\frac{\hat{y}_t - y_m}{\hat{x}_t - x_m}\right) \quad (11)$$

where the subscripts v and h denote, respectively, the vertical and horizontal planes and the following estimations were used:

$$\hat{x}_t = x_t + v_{tx}\tilde{t}, \; \hat{y}_t = y_t + v_{ty}\tilde{t}, \; \hat{h}_t = h_t + v_{th}\tilde{t}$$

with

$$\tilde{t} = -\frac{R_{lock}}{\dot{R}_p} = \frac{R_{lock}R_{mt}}{R_{mtx}v_{rx} + R_{mty}v_{ry} + R_{mth}v_{rh}} \quad (12)$$

3.1Vertical Plane Guidance

A feedback form of guidance law deduced from the explicit guidance law is sought for the present problem. The guidance gains of the explicit guidance law are usually selected to shape the trajectory for the desired attributes (Wang, 1988; Wang *et al*., 1993) with the corresponding acceleration commands in inertial coordinates expressed in the form

$$\vec{a}_m = \frac{K_1}{t_{gop}}(\vec{v}_{mf} - \vec{v}_{m0}) + \frac{K_2}{t_{gop}^2}(\vec{r}_{mf} - \vec{r}_{m0} - \vec{v}_{m0}t_{gop}) \quad (13)$$

where $\vec{r}_m = [x_m \; y_m \; h_m]^T, \vec{v}_m = [v_{mx} \; v_{my} \; v_{mh}]^T$ and $\vec{a}_m = [a_{mx} \; a_{my} \; a_{mh}]^T$ are, respectively, the position, velocity and acceleration vectors. Traditionally, the gains K_1 and K_2 are obtained by minimizing

$$J(K_1, K_2) = \int_{t_0}^{t_f} \vec{a}_m^T \vec{a}_m dt \quad (14)$$

subject to

$$\dot{\vec{r}}_m = \vec{v}_m, \quad \vec{r}_m(t_0) = \vec{r}_{m0}, \quad \dot{\vec{v}}_m = \vec{a}_m, \quad \vec{v}_m(t_0) = \vec{v}_{m0}$$

and the given boundary conditions

$$\vec{r}_m(t_f) = \vec{r}_{mf}, \quad \vec{v}_m(t_f) = \vec{v}_{mf}$$

In the present problem, $t_f = t|_{R=R_{lock}} = t_0 + t_{gop}$. On the basis of this formulation we would like to find the optimal time varying K_1 and K_2 those maximize the missile velocity at hand-over and satisfy the constraint imposed on the solution.

Using the approximation $R = |\vec{r}_{mf} - \vec{r}| \cong v_m t_{gop}$ the normal acceleration can be represented as

$$a_{mp} = K_{1v}a_v + K_{2v}a_p \quad (15)$$

where

$$a_v = -\frac{v_m}{t_{gop}} \sin\delta_v = -\frac{v_m^2}{R} \sin\delta_v$$
$$a_p = \frac{R}{t_{gop}^2} \sin\sigma_v = \frac{v_m^2}{R} \sin\sigma_v$$
with the predicted velocity angle error between the present and final vectors
$$\delta_v = \hat{r}_f - \gamma \tag{16a}$$
and the heading error angle
$$\sigma_v = \gamma - \hat{\theta}_v \tag{16b}$$
See Fig. 4 for the definitions.

Based on this observation, the optimal midcourse guidance law should take the following form:
$$\alpha_m = f_m(\delta_v, \sigma_v) \tag{17}$$
For the terminal guidance law, we select the heading error angle σ_v and its rate $\dot{\sigma}_v$ as the guidance system input and correct the equivalent guidance gains accordingly. That is the terminal guidance law takes the following form:
$$\alpha_t = f_t(\sigma_v, \dot{\sigma}_v) \tag{18}$$

3.2 Horizontal Plane Guidance

The similar idea can be adopted for the horizontal plane guidance law except that all gravity terms are omitted. In the horizontal plane, the predicted velocity and heading error angles are given by
$$\delta_h = \hat{\psi}_f - \psi \tag{19a}$$
$$\sigma_h = \psi - \hat{\theta}_h \tag{19b}$$
(see Fig. 5). In addition, the control variable α becomes β (side slip angle), γ becomes ψ and θ becomes the inertial line-of-sight angle on the horizontal plane.

4. GUIDANCE DESIGN

4.1Vertical Plane Guidance

Midcourse phase; optimal sense-based midcourse guidance. For the vertical plane guidance design, the equations of motion can be deduced from (1) as follows
$$\begin{aligned} \dot{v}_m &= \frac{1}{m}(T\cos\alpha - D) - g\sin\gamma \\ \dot{\gamma} &= \frac{1}{mv_m}(T\sin\alpha + L) - \frac{1}{v_m} g\cos\gamma \\ \dot{x}_m &= v_m \cos\gamma \\ \dot{h}_m &= v_m \sin\gamma \end{aligned} \tag{20}$$
Our aim is to minimize energy lose and bring the velocity error to zero at handover. Mathematically, the objective is equivalent to finding the optimal time-varying gains K_1 and K_2 in (14) those maximized the terminal speed at the lock-on point. The performance index is thus defined as
$$J_1(K_1, K_2) = \max v_{mf} \tag{21}$$
subject to the boundary conditions: $\sigma_v(t_f) = 0$ and $\delta_v(t_f) = 0$. A convenient way to derive the optimal gains K_{1v} and K_{2v} with respect to J_1 is to replace (14) with the instantaneous curvature $\kappa = a_{mp} / v_m^2$, i.e.
$$\kappa = -\frac{K_{1v}}{R}\sin\delta_v + \frac{K_{2v}}{R}\sin\sigma_v \tag{22}$$
$1/\kappa$ is known as radius curvature. Maximizing the missile terminal velocity of (21) is equivalent to maximizing
$$J_2(K_{1v}, K_{2v}) = \int_R^{R_{lock}} \frac{dv_m}{dR} dR \tag{23}$$
For aerodynamically controlled missiles with the equations of motion governed by (20), J_2 can be further transformed into the maximization of the following cost function (Lin and Tsai, 1987)
$$J_3(K_{1v}, K_{2v}) = \int_R^{R_{lock}} \left(1 + \frac{\kappa^2}{2F_v^2}\right) \sec\sigma_v dR \tag{24}$$
where F_v is the trajectory-shaping coefficient defined by
$$F_v = \sqrt{\frac{D_0 \tilde{L}_\alpha (T/\tilde{L}_\alpha + 1)^2}{m^2 v_m^4 (2\mu + T/\tilde{L}_\alpha)}}$$
with $\tilde{L}_\alpha = qs_t C_{mL\alpha}^2$, subject to the constraint
$$\frac{d\gamma}{dR} = -\kappa \sec\sigma_v \tag{25a}$$
$$\frac{d\sigma_v}{dR} = -\kappa\sec\sigma_v - \frac{1}{R}\tan\sigma_v \tag{25b}$$
and the boundary conditions
$$\sigma_v(t_f) = 0,\quad \gamma(t_f) = \hat{\gamma}_f$$
Note that the state equations (25) governing γ and σ_v are derived from the following state equations
$$\dot{R} = -v_m \cos\sigma_v \tag{26a}$$
$$\dot{\theta}_v = \frac{v_m \sin\sigma_v}{R} \tag{26b}$$

As in (Lin and Tsai, 1987), it is possible to derive a closed-form of the optimal closed-loop feedback control gains K_1 and K_2 by applying the optimal control theory. First define the Hamiltonian H_v as
$$H_v = \left(1 + \frac{\kappa^2}{2F_v^2}\right)\sec\sigma_v - \lambda_{\sigma_v}\frac{1}{R}\tan\sigma_v - (\lambda_{\sigma_v} + \lambda_\gamma)\kappa\sec\sigma_v \tag{27}$$
where λ_{σ_v} and λ_γ are the Largrange multipliers which correspond, respectively, to the constraints (25a) and (25b). The optimal curvature κ satisfies the Euler-Largrange equations:
$$\frac{\partial H_v}{\partial\kappa} = 0,\ \frac{d\lambda_\gamma}{dR} = -\frac{\partial H_v}{\partial\gamma},\ \frac{d\lambda_{\sigma_v}}{dR} = -\frac{\partial H_v}{\partial\sigma_v} \tag{28}$$
After trivial mathematical derivations, we can have
$$C_1 = \frac{1}{D(F_v, R_v)}\left\{\frac{1}{R_v}\left[e^{-F_{rv}}(F_{rv}+1) - e^{-F_{\hat{r}v}}(F_{\hat{r}v}+1)\right]\right. \\ \left. \sin\delta_v - F_v(e^{-F_{\hat{r}v}} - e^{-F_{rv}})\sin\sigma_v\right\} \tag{29a}$$
$$C_2 = \frac{1}{D(F_v, R)}\left\{-\frac{1}{R}\left[e^{F_{rv}}(F_{rv}-1) - e^{F_{\hat{r}v}}(F_{\hat{r}v}-1)\right]\right. \\ \left. \sin\delta_v + F_v(e^{F_{\hat{r}v}} - e^{F_{rv}})\sin\sigma_v\right\} \tag{29b}$$
where
$$D(F_v, R) = -\frac{F_v}{R}\left[e^{F_{\Delta rv}}(F_{\Delta rv} - 2) - e^{-F_{\Delta rv}}(F_{\Delta rv} + 2) + 4\right]$$
with $F_{\Delta rv} = F_v(R - R_{lock})$. Therefore, the optimal curvature κ in (38) is given by
$$\kappa = \frac{-F_v^2}{R_v D(F_v, R)}\left[-e^{F_{\Delta rv}}(F_{\hat{r}v}+1) - e^{-F_{\Delta rv}}(F_{\hat{r}v}-1) + 2F_v R\right]\sin\delta_v \\ + \frac{F_v^3}{D(F_v, R)}\left(e^{F_{\Delta rv}} + e^{-F_{\Delta rv}} - 2\right)\sin\sigma_v \tag{30}$$

Comparing (30) to (22), the resulting optimal control gains K_1 and K_2 are obtained as

$$K_{1v} = \frac{F_{rv}\left[e^{F_{\Delta rv}}(F_{\tilde{r}v}+1) + e^{-F_{\Delta rv}}(F_{\tilde{r}v}-1) - 2F_{rv}\right]}{e^{F_{\Delta rv}}(F_{\Delta rv}-2) - e^{-F_{\Delta rv}}(F_{\Delta rv}+2)+4} \quad (31a)$$

$$K_{2v} = -\frac{F_{rv}^2(e^{F_{\Delta rv}} + e^{-F_{\Delta rv}} - 2)}{e^{F_{\Delta rv}}(F_{\Delta rv}-2) - e^{-F_{\Delta rv}}(F_{\Delta rv}+2)+4} \quad (31b)$$

After the pitching acceleration command has been obtained, the control variable α_m is related to a_{mp} by

$$\alpha_m = \frac{2a_{mp}}{\rho v_m^2 s_m C_{mL\alpha}}. \quad (32)$$

Shaping phase: PN shaping guidance. In this phase, minimizing the position error for satisfactory accuracy becomes more important. To this end, we set the guidance gain $K_{1v} = 0$. An appropriate form of the gain K_{2v} is then constructed such that the acceleration command issued from the midcourse and terminal phases are linked smoothly.

The PN acceleration command can be stated as

$$a_{PN} = N v_m \dot{\theta}_v \quad (33)$$

where N is the navigation ratio and $\dot{\theta}_v$ indicates the line-of-sight angle rate. In tactical radar homing missiles using PN guidance, the seeker provides an effective measurement of the line-of-sight angle rate. To relate the PN to the midcourse guidance discussed previously, (33) can be rewritten as

$$\begin{aligned} a_{PN} &= N v_m \frac{d}{dt}\sin^{-1}\left(\frac{R_{mth}}{R}\right) \\ &= N v_m \frac{1}{\cos\theta_v}\left(\frac{v_{mth}}{R} + \frac{\sin\theta_v}{t_{go}}\right) \end{aligned} \quad (34)$$

where $t_{go} = -R/V_c$.

Setting $K_{1v} = 0$, then equating the optimal trajectory shaping guidance (15) with the proportional guidance command (34) gives

$$-\frac{K_{2v}}{R} v_m^2 \sin\sigma_v = N v_m \frac{1}{\cos\theta_v}\left(\frac{v_{th} - v_m \sin\gamma}{R} + \frac{v_c \sin\theta_v}{R}\right)$$

From which we have the guidance gain

$$K_{2v} = -\frac{N}{\sin\sigma_v \cos\theta_v}\left(\frac{v_{rh}}{v_m} + v\sin\theta_v\right) \quad (35)$$

where $v = V_c/V_m$ and the guidance law is

$$a_{mp} = -\frac{N v_m}{R\cos\theta_v}(v_{rh} + v_c \sin\theta_v) \quad (36)$$

The control variable is obtained via

$$\alpha_{PN} = \frac{2a_{mp}}{\rho v_m^2 s_m C_{mL\alpha}} \quad (37)$$

Terminal phase; Combination of PN shaping and neural net-based terminal guidance. We seek to determine a multilayer feedforward neural network $NN(\vec{y};W,V)$, $\vec{y} = [\sigma_v \quad \dot{\sigma}_v]^T$ such that the following instantaneous normed-square error is minimized with respect to the set of parameter matrices $W = \{W^1, W^2, \ldots, W^N\}$ and $V = \{V^1, V^2, \ldots, V^N\}$:

$$E = \frac{1}{2}[(\gamma(k) - \gamma_f(k))^2 + \xi(\gamma(k) - \theta(k))^2] \quad (38)$$

where $\xi \geq 0$ is used to weight the heading error angle. During the terminal phase, $\gamma_f(k)$ is determined by $\gamma_f = \tan^{-1}\left(\frac{h_f - h_m}{x_f - x_m}\right)$ where $x_f = x_t + v_{tx}t_{go}$, $\theta_v = \tan^{-1}\left(\frac{R_{mth}}{R_{mtx}}\right)$. If the seeker fails to track the target, t_{go} could be estimated via $t_{go} = R^2 / R_{mtx}v_{rx} + R_{mty}v_{ry} + R_{mth}v_{rh}$.

In the (N+1)-layer network, the input to the network is $z^0 = \vec{y}$. The input and output are related by the recursive relationship

$$\begin{aligned} net^j &= W^j z^{j-1} + V^j, \quad j = 1, \cdots, N-1 \\ z^j &= f_j(net^j) \end{aligned} \quad (39)$$

and

$$\begin{aligned} net^N &= W^N z^{N-1} + V^N \\ z^N &= net^N \end{aligned} \quad (40)$$

The output is

$$\alpha_{NN} = z^N f_o(R)$$

where

$$f_o(R) = \frac{1}{1 + e^{(R-R_{c1})/R_{c2}}}$$

with R_{c1} and R_{c2} being the appropriate range constants, $f_o(R)$ is used to prevent large transient resulting from the neural network when missiles enter the terminal phase guidance. The weights W^j and V^j are of the appropriate dimension. V^j is the connection weight vector to the bias node. The activation function vectors $f_j(\cdot)$, $j = 1,2,\ldots,N-1$, are usually chosen to be some kind of sigmoids but they may possibly be simple identity gains for some cases. The neural network can thus be succinctly expressed as

$$\begin{aligned} NN(y;W,V) = f_N(W^N f_{N-1}(W^{N-1} f_{N-2}(\ldots W^2 f_1(W^1 y + V^1) \\ + V^2) + \cdots + V^{N-1}) + V^N) \end{aligned}$$

As the back propagation algorithm, the gradient descent method is developed to train the network weights. For the hidden layers, the weight adaptation algorithm is given by

$$\begin{aligned} \Delta W^j(k) &= -\eta \nabla_{W^j} E(k), \qquad j = 1, \ldots, N \\ \Delta V^j(k) &= -\eta \nabla_{V^j} E(k), \end{aligned} \quad (41)$$

where the gradient vector $\nabla_{W^j} E(k) = \partial E(k) / \partial W^j(k)$.

For the output layer, the connection weights are adapted via

$$\Delta w_{rs}^N(k) = -\eta \Theta(k)\, z_s^{N-1}(k) \quad (42a)$$

$$\Delta v_r^N(k) = -\eta \Theta(k) \quad (42b)$$

The whole guidance command is the combination of PN shaping and neural net-based terminal guidance:

$$\alpha_t = \alpha_{NN} + \alpha_{PN}$$

Fig. 6 shows the schematic diagram of the resulting guidance system.

4.2 Horizontal Plane Guidance

The analytic optimal guidance gains K_{1h} and K_{2h} for the horizontal midcourse guidance law can be analogously derived the same as those shown in (31a) and (31b) by considering the horizontal equations of motion which are governed by the state variables v_m, ψ, x_m and y_m, and consider the side slip angle β as the control variable. The resulting lateral acceleration command can be directly obtained as follows

$$a_{my} = \left(-\frac{K_{1h}}{R} \sin \delta_h + \frac{K_{2h}}{R} \sin \sigma_h \right) v_m^2 \tag{43}$$

The control variable ϕ is then commanded as

$$\phi = \sin^{-1} \left(\frac{m a_{my}}{T \sin \alpha + L} \right) \tag{44}$$

5. SIMULATION RESULTS

The acceptable final miss distance is limited to be 25 m or less. Also, suppose that the missile's thrust vanishes at 7.5 seconds. The range constants R_{c1} and R_{c2} were set as 13000 and 1000, respectively. The tactical ballistic target with the incoming speed of 1800 m/s. For the target, related parameters were given as $\beta_t = 2440$ $\mathrm{kg/m^2}$, $C_{tD0} = 1.81$ and M_t =1200 kg. Suppose that the ground radar has detected the tactical ballistic target after reentry at a range of 50.0 km. The target evasive accelerations $a_{ty}(t)$ and $a_{th}(t)$ were generated by two first-order Gauss-Markov processes to statistically represent 5 g lacteal accelerations; for the axial direction, $a_{tx}(t) = 0$.

Fig. 7 shows the resulting defensible volume. The defensible volumes shrink at the highest and lowest altitudes. This is due to the fact that the missile's maneuverability reduces with the increasing height. On the other hand, if the interceptor engages the target at the lower altitude, the total flight time might not be enough for it to build up speed. For the interception point A ($(5562,5624,14672)$ (m)), the initial target position was (20000, 20000, 36000) (m) and the azimuth angle was 45 degrees. The final interception time is $16.92\,\mathrm{sec}$. and the MD is 0.56 m. The final missile velocity is $1021\,(\mathrm{m/s})$. With regard to the guidance performance, related tracking responses, control histories and missile velocity are shown in Fig. 8. It is found from the profiles of δ and σ that the missile successfully achieved the head-on condition while it entered the terminal phase. In addition, one can also observe that the missile didn't consume much control energy during the midcourse guidance phase. Fig. 9 illustrates the 3D trajectory of engagement.

6. CONCLUSION

An optimality-based missile guidance system designed to counter hypersonic target in the three-dimensional space is studied in which a mixed optimal trajectory shaping guidance and neural network correction guidance algorithm is developed. The optimal trajectory shaping guidance maximizes the final speed of the midcourse phase at lock-on point and offers a better counterattack condition. Neural networks incorporated with the PN guidance in the terminal phase are used to compensate for the tracking error suitable for real-time implementation. It is found that mixed guidance scheme can effectively extend the defensible volume. Simulation results confirm superiority of the proposed scheme over proportional navigation guidance and show its performance robustness.

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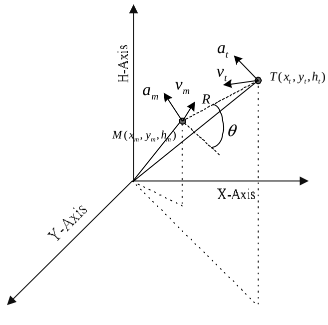

Fig. 1 Three-dimensional intercept geometry

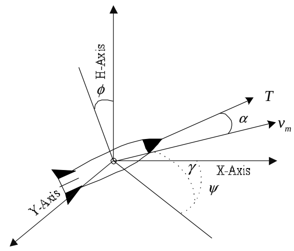

Fig. 2 Definitions of the angle-of-attack α and rolling angle ϕ

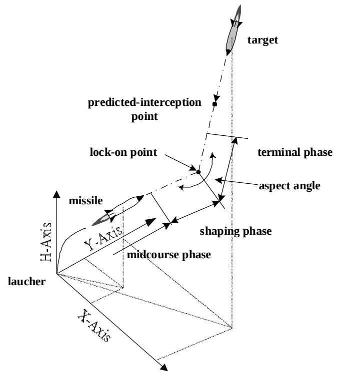

Fig. 3 Engagement scenario

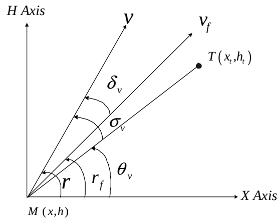

Fig. 4 Definitions of σ and δ on the vertical plane

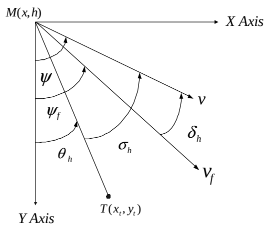

Fig. 5 Definitions of σ and δ on the horizontal plane

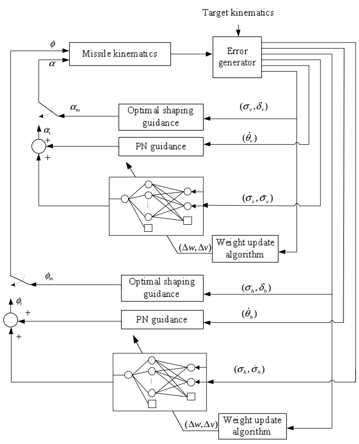

Fig. 6 Implementation of the complete guidance scheme

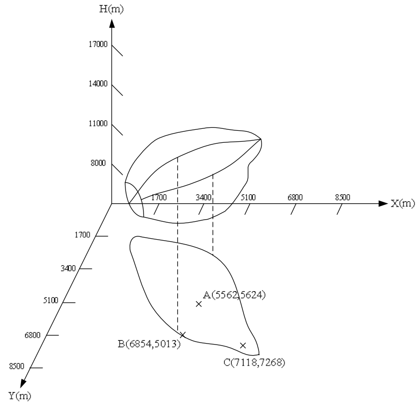

Fig. 7 Defensible volumes against the target

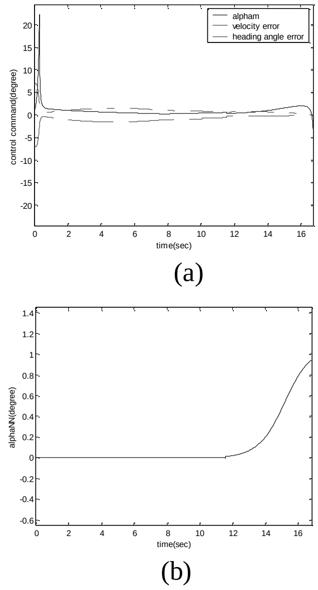

Fig. 8 Control histories corresponding to the defensible point A; (a) optimal control command of the vertical plane, (b) neural control command in the vertical plane

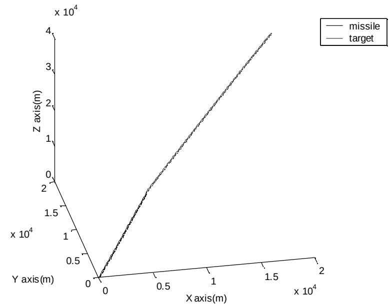

Fig. 9 3-D tracking trajectory w.r.t. the defensible point A