COMPOSITE NONLINEAR FEEDBACK CONTROL FOR A CLASS OF NONLINEAR SYSTEMS WITH INPUT SATURATION

Weiyao Lan Ben M. Chen Yingjie He

Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117576

Abstract: This paper studies the technique of the composite nonlinear feedback (CNF) control for a class of cascade nonlinear systems with input saturation. In particular, the class of systems under consideration consists of two parts, a linear portion and a nonlinear portion with the output of the linear part connecting to the input of the nonlinear part and with the input of the given system being saturated. The objective of this paper is to design a composite nonlinear feedback control law based on the linear portion such that the output of the system tracks a step input rapidly with small overshoot and at the same time maintains the stability of the whole cascade system. The result has been successfully demonstrated by a numerical example.

Keywords: Composite nonlinear feedback control, nonlinear systems, input saturation, tracking control, control applications.

1. INTRODUCTION

The issue of tracking performance of control systems is practically important to many control applications. We consider in this paper a nonlinear control technique that would yield a better tracking performance for a class of partially linear composite systems with input saturation. The class of systems under consideration consists of two parts, a linear portion and a nonlinear portion with the output of the linear part connecting to the input of the nonlinear part and with the input of the given system being saturated. Many nonlinear systems can be transformed into partially linear composite systems via a state-space diffeomorphism and/or a preliminary feedback transformations (see, for example, Isidori (1995)). In recent two decades, the stabilization problems for partially linear composite systems have been extensively studied by many researchers such as Byrnes and Isidori (1991), Sussmann and Kokotovic (1991), Lin and Saberi (1992), Lin and Saberi (1993), Teel (1992), Jiang and Mareels (2001) and Jiang *et al.* (2003), to name just a few. In particular, it was shown in Sussmann and Kokotovic (1991) that a nonlinear system which is zero input globally asymptotically stable (GAS) will preserve its GAS property if its input decreases to zero with a very fast exponential rate. It is not difficult to make the output of the linear part, which is the input of the nonlinear part, to converge to zero with some exponential rate. However, the peaking phenomenon in linear systems may destroy the stability of the nonlinear systems before the output rapidly decays to zero.

In this paper, we consider a tracking problem (or an equivalent output regulation) for partially linear composite systems with input saturation. Particular attention is paid to improve the transient performance of the closed-loop system by using a so-called composite nonlinear feedback (CNF) control technique. The research on nonlinear output regulation problems has made great progress since 1990s. Related results have been extensively reported in the literature (see, for example, Isidori and Byrnes (1990), Byrnes *et al.* (1997), Huang (2001) and Huang and Chen (2002)). However, the transient performance is not considered in most of these works. We consider in this work a tracking control problem with a constant (or step) reference. To improve the tracking performance, Lin *et al.* proposed the CNF control technique in their pioneer work Lin *et al.* (1998) for a class of second order linear systems. Turner *et al.* (2000) later extended the results of Lin *et al.* (1998) to higher order and multiple input systems under a restrictive assumption on the system. However, both Lin *et al.* (1998) and Turner *et al.* (2000) considered only the state feedback case. Recently, Chen *et al.* (2003) have developed a CNF control to a more general class of systems with measurement feedback, and successfully applied the technique to solve a hard disk drive servo problem. The CNF control consists of a linear feedback law and a nonlinear feedback law without any switching element. The linear feedback part is designed to yield a closed-loop system with a small damping ratio for a quick response, while at the same time not exceeding the actuator limits for desired command input levels. The nonlinear feedback law is used to increase the damping ratio of the closed-loop system as the system output approaches the target reference to reduce the overshoot caused by the linear part. This paper aims to design a CNF control law for partially linear composite systems with input saturation based on the linear part of the composite system such that the closed-loop system has desired performances, e.g., quick response and small overshoot, and the tracking error decays to zero with sufficiently large exponential rate to guarantee the stability of the whole system.

2. PROBLEM DESCRIPTION AND PRELIMINARIES

Consider a partially linear composite systems with input saturation characterized by

$$\dot{\xi} = f(\xi, y), \quad \xi(0) = \xi_0 \tag{1}$$
$$\dot{x} = Ax + B\,\mathrm{sat}(u), \qquad x(0) = x_0 \tag{2}$$
$$y = Cx \tag{3}$$

where $(\xi, x) \in \mathbb{R}^m \times \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ the control input, and $y \in \mathbb{R}$ the output of the system, f is a smooth (i.e., C^∞) function, A, B and C are appropriate dimensional constant matrices, and $\mathrm{sat} : \mathbb{R} \to \mathbb{R}$ represents the actuator saturation defined as

$$\mathrm{sat}(u) = \mathrm{sgn}(u) \min\{u_{\max}, |u|\} \tag{4}$$

with $u_{\max}$ being the saturation level of the input. We aim to design a CNF control law for (1)–(3) such that the resulting closed-loop system is stable and the output of the closed-loop system will track a step reference input r rapidly without experiencing large overshoot. To this end, we assume that

A1: (A, B) is controllable;

A2: (A, B, C) is invertible and has no invariant zeros at $s = 0$; and

A3: There exists a C^1 positive definite function $V_\xi(\xi)$ and class K_∞ functions α_1 and α_2 such that

$$\alpha_1(\|\xi\|) \le V_\xi(\xi) \le \alpha_2(\|\xi\|), \tag{5}$$
$$\frac{\partial V_\xi(\xi)}{\partial \xi} f(\xi, r) < 0, \tag{6}$$

for all $\xi \in \mathbb{R}^m$.

Lemma 2.1. Consider the nonlinear control system of the form

$$\dot{\xi} = f(\xi, r + \eta(t)), \tag{7}$$

which satisfies Assumption A3. Given any $\gamma > 0$ and $\beta > 0$, there exists a scalar $a > 0$ such that for any

$$|\eta(t)| \le \beta e^{-at}, \quad t \ge 0, \tag{8}$$

the solution $\xi(t)$ of (7) exists and is bounded for all $t \ge 0$ provided that $\xi(0) \in \Omega_\gamma := \{\xi \,:\, \|\xi\| \le \gamma\}$. For such an a, we say that a is good for (γ, β). ∎

Proof. The proof of this Lemma follows the lines of reasoning as in Theorem 4.1 of Sussmann and Kokotovic (1991). Noting that $V_\xi(\xi)$ is a C^1 positive definite function, we let

$$c = \max\{V_\xi(\xi) \;:\; \xi \in \Omega_\gamma\} \tag{9}$$

for any given $\gamma > 0$. Since $V_\xi(\xi)$ is C^1 and $f(\xi, y)$ smooth, there exists a constant $h > 0$ such that, for all $\xi \in \Omega_\gamma$ and $|v| \le \beta$,

$$\left| \frac{\partial V_\xi(\xi)}{\partial \xi} f(\xi, r + v) \right| \le h. \tag{10}$$

Let $\tau = \frac{1}{h}$. Then for every solution $\xi(t)$ of (7) under any admissible input such that $|\eta(t)| \le \beta$ and $\xi(0) \in \Omega_\gamma$,

$$V_\xi(\xi(t)) \le c + 1, \;\; 0 \le t \le \tau. \tag{11}$$

By the continuousness of $\frac{\partial V_\xi(\xi)}{\partial \xi} f(\xi, r)$ and (6), there exists an $\alpha > 0$ such that

$$\frac{\partial V_\xi(\xi)}{\partial \xi} f(\xi, r + v) \le 0 \tag{12}$$

when $c \le V_\xi(\xi) \le c + 1$ and $|v| \le \alpha$.

Next, we choose a such that

$$\beta e^{-a\tau} \le \alpha. \tag{13}$$

If η is an input satisfying (8), and $\xi(t)$ is the solution of (7) with $\xi(0) \in \Omega_\gamma$, we claim that

$$V_\xi(\xi(t)) \le c + 1, \;\; t \ge 0. \tag{14}$$

In fact, we have proved that $V_\xi(\xi(t)) \leq c+1$ for $0 \leq t \leq \tau$. For $t > \tau$, (8) and (13) implies $|\eta(t)| < \alpha$, and then by (12), we have

$$\frac{\partial V_\xi(\xi)}{\partial \xi} f(\xi, r+\eta) \leq 0. \tag{15}$$

Thus,

$$V_\xi(\xi(t)) \leq V_\xi(\xi(\tau)) \leq c+1, \quad t > \tau. \tag{16}$$

Moreover, $\xi(t)$ is bounded by

$$\|\xi(t)\| \leq \alpha_1^{-1}(V_\xi(\xi)) \leq \alpha_1^{-1}(c+1). \tag{17}$$

This completes the proof of Lemma 2.1. ∎

Remark 2.1. Assumption A3 can be relaxed to be satisfied locally, e.g., in $\Omega_{\bar{\gamma}}$. In this case, it is clear that, from the proof of Lemma 2.1, by selecting $0 < \gamma < \bar{\gamma}$, and $\beta > 0$ such that

$$\{\xi \ : \ V_\xi(\xi) \leq c+1\} \subset \Omega_{\bar{\gamma}}, \tag{18}$$

then there exists an $a > 0$ which is good for (γ, β). ∎

3. DESIGN OF THE COMPOSITE NONLINEAR FEEDBACK CONTROL LAW

In this section, we proceed to design a CNF control law for the system (1)–(3). We assume that the given system (1)–(3) satisfies Assumptions A1 to A3, and all the states of the linear system (2) are available for feedback. The CNF control law can be constructed by the following step-by-step procedure.

STEP S.1. Select appropriate scalars $\gamma > 0$, $\beta > 0$ and $a > 0$ such that a is good for (γ, β).

STEP S.2. Design a linear feedback law

$$u_{\rm L} = Fx + Gr \tag{19}$$

where r is a step command input and F is chosen such that

1. $A + BF$ is Hurwitz and the output of the following system,

$$\dot{x} = (A+BF)x, \quad y = Cx, \tag{20}$$

has $\|y(t)\| \leq ke^{-at}$ for some $k > 0$; and
2. The closed-loop system $C(sI - A - BF)^{-1}B$ has certain desired properties, e.g., having a small damping ratio.

The existence of such an F is guaranteed by Assumption A1, i.e., (A, B) is controllable. In fact, it can be designed using methods such as the H_2 and H_∞ optimization approaches, as well as the robust and perfect tracking technique. G is a scalar given by

$$G = -[C(A+BF)^{-1}B]^{-1}. \tag{21}$$

Note that G is well defined since $A+BF$ is Hurwitz and the triple (A, B, C) is invertible and has no invariant zeros at $s = 0$. We also let

$$H := [1 - F(A+BF)^{-1}B]G \tag{22}$$

and

$$x_{\rm e} := G_{\rm e}r := -(A+BF)^{-1}BGr. \tag{23}$$

STEP S.3. Given a positive-define matrix $W \in \mathbb{R}^{n \times n}$, solve the Lyapunov equation

$$(A+BF)'P + P(A+BF) = -W \tag{24}$$

for $P > 0$. Note that such a P exists since $A+BF$ is asymptotically stable. Then, the nonlinear feedback control law $u_{\rm N}(t)$ is given by

$$u_{\rm N} = \rho(r, y)B'P(x - x_{\rm e}) \tag{25}$$

where $\rho(r, y)$ is any non-positive function locally Lipschitz in y. This nonlinear control law is used to change the system closed-loop damping ratio as the output approaches the step command input.

STEP S.4. The CNF control law is given by combining the linear and nonlinear feedback law derived in the previous steps,

$$\begin{aligned} u &= u_{\rm L} + u_{\rm N} \\ &= Fx + Gr + \rho(r, y)B'P(x - x_{\rm e}). \end{aligned} \tag{26}$$

Theorem 3.1. Consider the given system (1)–(3) satisfies Assumptions A1 to A3. Let scalars $\gamma > 0$, $\beta > 0$ and $a > 0$ be selected such that a is good for (γ, β), and let

$$\mathcal{N} := \left\{ x \in \mathbb{R}^n \ : \ \|x\| \leq \frac{\beta}{k}\sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}} \right\}. \tag{27}$$

For any $\delta \in (0, 1)$, let $c_\delta > 0$ be the largest positive scalar satisfying the following condition:

$$|Fx| \leq u_{\max}(1-\delta) \tag{28}$$

for all $x \in \mathbf{X}_\delta$, where

$$\mathbf{X}_\delta := \{x \ : \ x'Px \leq c_\delta, \ x \in \mathcal{N}\}. \tag{29}$$

Then for any non-positive function $\rho(r, y)$, locally Lipschitz in y, the state trajectory of the closed-loop system comprising the given system (1)–(3) and the CNF control law (26) is bounded for all $t \geq 0$, provided that the initial states ξ_0 and x_0, and amplitude of step input r satisfy $\xi_0 \in \Omega_\gamma$, and

$$\tilde{x}_0 := (x_0 - x_{\rm e}) \in \mathbf{X}_\delta, \quad |Hr| \leq u_{\max}. \tag{30}$$

Moreover, the system output y tracks asymptotically the step command input of amplitude r. ∎

Proof. The closed-loop system system comprising the given plant (1)-(3) and the CNF control law (26) is given by

$$\dot{\xi} = f(\xi, y) \tag{31}$$
$$\dot{x} = Ax + B\text{sat}(Fx + Gr + \rho B' P(x - x_\text{e})) \tag{32}$$
$$y = Cx \tag{33}$$

Let $\tilde{x} = x - x_\text{e}$. The closed-loop system system (31)-(33) can be expressed as

$$\dot{\xi} = f(\xi, r + C\tilde{x}) \tag{34}$$
$$\dot{\tilde{x}} = (A + BF)\tilde{x} + Bw \tag{35}$$

where

$$w = \text{sat}(F\tilde{x} + Hr + \rho B' P\tilde{x}) - F\tilde{x} - Hr. \tag{36}$$

Define a Lyapunov function $V(\tilde{x}) = \tilde{x}' P\tilde{x}$, then we have

$$\lambda_\text{min}(P)\|\tilde{x}\|^2 \leq V_{\tilde{x}}(\tilde{x}) \leq \lambda_\text{max}(P)\|\tilde{x}\| \tag{37}$$

where $\lambda_\text{min}(P)$ and $\lambda_\text{max}(P)$ are the minimal and maximal eigenvalues of P respectively. Then,

$$\begin{aligned}\dot{V}(\tilde{x}) &= \frac{\partial V(\tilde{x})}{\partial \tilde{x}}\left((A + BF)\tilde{x} + Bw\right) \\ &= -\tilde{x}' W\tilde{x} + \frac{\partial V(\tilde{x})}{\partial \tilde{x}} Bw.\end{aligned}$$

It have been shown in (Chen *et al.*, 2003) that,

$$\frac{\partial V(\tilde{x})}{\partial \tilde{x}} Bw = 2\tilde{x}' PBw \leq 0$$

for all $\tilde{x} \in \mathbf{X}_\delta$ and $|Hr| \leq \delta u_\text{max}$. Thus

$$\dot{V}(\tilde{x}) \leq -\tilde{x}' W\tilde{x}, \quad \tilde{x} \in \mathbf{X}_\delta \tag{38}$$

i.e., $\mathbf{X}_\delta$ is an invariant set of the system (35). Thus the solution of (35) exists and is bounded for all $t \geq 0$ and $\tilde{x}_0 \in \mathbf{X}_\delta$. Noting that $x = x_\text{e} + \tilde{x}$, x exists and is bounded for all $t \geq 0$ and x_0 satisfies (30).

To show the existence and boundedness of the solution ξ of (34), it suffices to show that $\|\tilde{y}\| := \|C\tilde{x}\| \leq \beta e^{-at}$. To this end, consider the solution of the following system

$$\dot{\tilde{x}} = (A + BF)\tilde{x} + Bw, \quad \tilde{x}(0) \in \mathbf{X}_\delta.$$

Since $\frac{\partial V(\tilde{x})}{\partial \tilde{x}} Bw \leq 0$, we have

$$\frac{\partial V(\tilde{x})}{\partial \tilde{x}}\left((A + BF)\tilde{x} + Bw\right) \leq \frac{\partial V(\tilde{x})}{\partial \tilde{x}}(A + BF)\tilde{x}.$$

Noting that F is chosen such that

$$\|Cz(t)\| \leq k\|z(0)\|e^{-at}.$$

where $z(t)$ is the solution of

$$\dot{z} = (A + BF)z, \qquad z(0) \in \mathbf{X}_\delta$$

it is clear that

$$\|\tilde{x}(t)\| \leq \left(\frac{\lambda_\text{max}(P)}{\lambda_\text{min}(P)}\right)^{1/2} \|z(t)\|$$

for $\tilde{x}(0) = z(0) \in \mathbf{X}_\delta$. Thus

$$\begin{aligned}\|\tilde{y}(t)\| = \|C\tilde{x}\| &\leq \left(\frac{\lambda_\text{max}(P)}{\lambda_\text{min}(P)}\right)^{1/2} \|Cz(t)\| \\ &\leq k\left(\frac{\lambda_\text{max}(P)}{\lambda_\text{min}(P)}\right)^{1/2} \|z(0)\|e^{-at} \\ &\leq \beta e^{-at}, \ \ \tilde{x}(0) = z(0) \in \mathbf{X}_\delta.\end{aligned}$$

By Lemma 2.1, the solution of (34) exists and is bounded for all $t \geq 0$ and $x_0 \in \Omega_\gamma$.

Moreover, noting that $W > 0$, all trajectories of (35) starting from $\mathbf{X}_\delta$ will converge to the origin. Thus,

$$\lim_{t\to\infty} x(t) = x_\text{e}$$

for all initial state x_0 and the step command input of amplitude r that satisfy (30). Therefore,

$$\lim_{t\to\infty} y(t) = Cx_\text{e} = -C(A + BF)^{-1}BGr = r.$$

This completes the proof of Theorem 3.1. ■

Remark 3.1. The CNF control law (26) is reduced to the linear feedback control law (19) when the function $\rho(r, y) = 0$. Thus, Theorem 3.1 shows that the additional nonlinear feedback control law (25) does not affect the ability of the closed-loop system to track the command input. Any command input that can be asymptotically tracked by the linear control law (19) can also be asymptotically tracked by the CNF control law (26). However, this additional term u_N in the CNF control law can be used to improve the performance of the overall closed-loop system. This is the key property of the control technique studied in this manuscript. ■

Remark 3.2. The main purpose of adding the nonlinear part to the CNF control law is to speed up the settling time, or equivalently to contribute a significant value to the control input when the tracking error, $r - y$, is small. The nonlinear part, in general, will be in action when the control signal is far away from its saturation level and, thus, it will not cause the control input to hit its limits. Under such a circumstance, it is straightforward to verify that the closed-loop system comprising (2) and (26) can be expressed as

$$\dot{\tilde{x}} = (A + BF)\tilde{x} + \rho(r, y)BB' P\tilde{x}. \tag{39}$$

It is clear that eigenvalues of the closed-loop system (39) can be changed by the function $\rho(r, y)$. In fact, define the auxiliary system $G_\text{aux}(s)$ as

$$\begin{aligned}G_\text{aux}(s) &:= C_\text{aux}(sI - A_\text{aux})^{-1}B_\text{aux} \\ &:= B' P(sI - A - BF)^{-1}B.\end{aligned}$$

Then, the system (39) can be expressed as Figure 1. Using the well-known classical root-locus theory. The poles of the closed-loop system (39) approach the location of the invariant zeros of $G_{\mathrm{aux}}(s)$ as $|\rho|$ becomes larger and larger. ∎

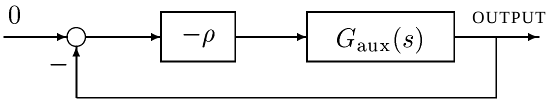


Fig. 1. Interpretation of the nonlinear function $\rho(r, y)$.

Remark 3.3. We usually choose $\rho(r, y)$ as a function of the tracking error $r - y$, which in most practical situations is known and available for feedback, such that $\rho(r, y)$ has the following two properties, 1) when the output y is far away from the final set point, $|\rho(r, y)|$ is small and thus the effect of the nonlinear part on the overall system is very limited; and 2) when the output approaches the set point, $|\rho(r, y)|$ becomes larger and larger, and the nonlinear control law will become effective. Of course, the choice of $\rho(r, y)$ is non-unique. The following choice is one of the suitable candidates,

$$\rho(r, y) = -\beta_{\mathrm{n}} \left| e^{-\alpha_{\mathrm{n}}|y(t)-r|} - e^{-\alpha_{\mathrm{n}}|y(0)-r|} \right|, \quad (40)$$

where $\beta_{\mathrm{n}} > 0$ and $\alpha_{\mathrm{n}} > 0$ are tuning parameters. ∎

4. AN ILLUSTRATIVE EXAMPLE

Consider a partially linear composite system (see Lin and Saberi (1993)) characterized by

$$\dot{\xi} = -\xi + \xi^2 y, \quad (41)$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \mathrm{sat}(u) \quad (42)$$

$$y = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} x \quad (43)$$

with $u_{\max} = 1$. We will design a CNF controller for the system (41)–(43) such that the output of the closed-loop system tracks a step reference $r = 0.5$. It is simple to verify that the triple (A, B, C) is controllable and has a relative degree of 1 and four invariant zeros at $\{j, -j, j, -j\}$. Thus, Assumptions A1 and A2 are satisfied. Let $\gamma = 1$ and $\beta = 1$, then it can be shown that any $a > 0$ is good for (γ, β). To design the CNF control law, we use the linear feedback control law $u = Fx$ with

$$F = -\begin{bmatrix} -0.403 & 0.0001 & 0.204 & 4.06 & 10.4 \end{bmatrix} \quad (44)$$

as reported in Lin and Saberi (1993). Next, we select $W = I_5$ and solve the following Lyapunov equation

$$(A + BF)'P + P(A + BF) = -W, \quad (45)$$

which yields a solution

$$P = \begin{bmatrix} 12.74 & -0.50 & -8.29 & -25.89 & -2.48 \\ -0.50 & 12.82 & 26.68 & 4.49 & 0.19 \\ -8.29 & 26.68 & 75.50 & 26.78 & 1.93 \\ -25.89 & 4.49 & 26.78 & 70.77 & 6.72 \\ -2.48 & 0.19 & 1.93 & 6.72 & 0.69 \end{bmatrix} > 0.$$

The nonlinear function $\rho(r, y)$ is chosen as

$$\rho(r, y) = -25.5(e^{-0.8|y-r|} - e^{-0.8|y(0)-r|}). \quad (46)$$

Finally, the CNF control law is given by

$$u = Fx + Gr + \rho(r, y)B'P(x - x_{\mathrm{e}}) \quad (47)$$

where $G = 10.201$ and $x_{\mathrm{e}} = [0.5\ 0\ 0.5\ 0\ 0.5]'$. The simulation result is shown in Figure 2 where the transient performance is compared between the linear control law and the CNF control law under the same initial conditions $\xi(0) = -0.2$ and $x(0) = 0$. Clearly, the CNF control has outperformed the linear counterpart significantly. Comparing Figure 2.(a) and Figure 2.(b), we can see that all the states of the closed-loop system under the CNF control convergence to the steady state quickly, and their transient amplitudes are much smaller than the ones under the linear control law. Figure 2.(c) and Figure 2.(d) show the system output of the closed-loop system and the control inputs applied on the system under the linear control and the CNF control. The overshoot under the linear control is 19.19%, but under the CNF control, there is no overshoot at all.

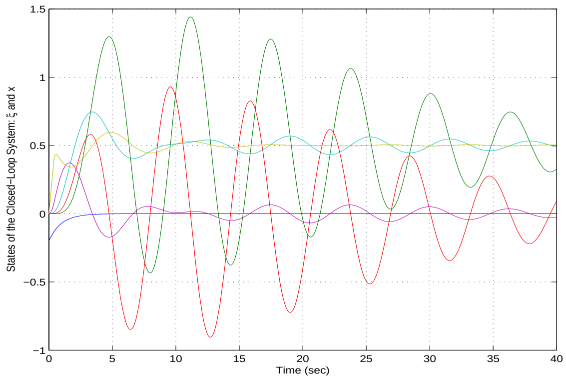
5. CONCLUSIONS

The composite nonlinear feedback control technique is extended to the partially linear composite system with input saturation. Simulation result shows that the nonlinear control law greatly improved the performance of the closed-loop system. It should be noted that, in this paper, we have assumed that the linear part of the composite system is SISO, and all the states of the linear part are available to feedback. It should not be too difficult to extend the result of this paper to MIMO systems with measurement feedback using the result reported in He *et al.* (2003).

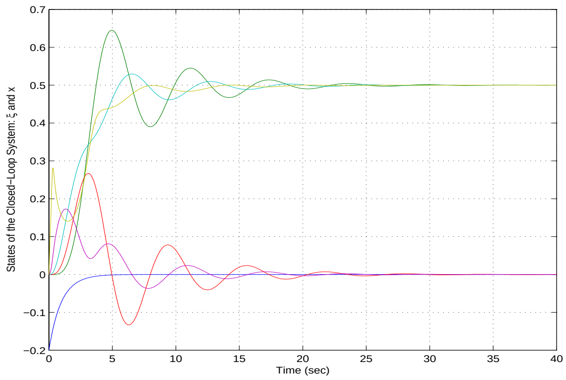
REFERENCES

Byrnes, C. I. and A. Isidori (1991). Asymptotic stabilization of minimum phase nonlinear systems. *IEEE Transactions on Automatic Control* **36**, 1122–1137.

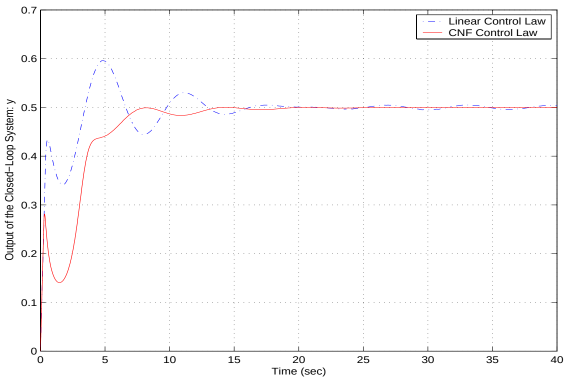
Byrnes, C. I., F. D. Priscoli, A. Isidori and W. Kang (1997). Structurally stable output regulation of nonlinear systems. *Automatica* **33**, 369–385.



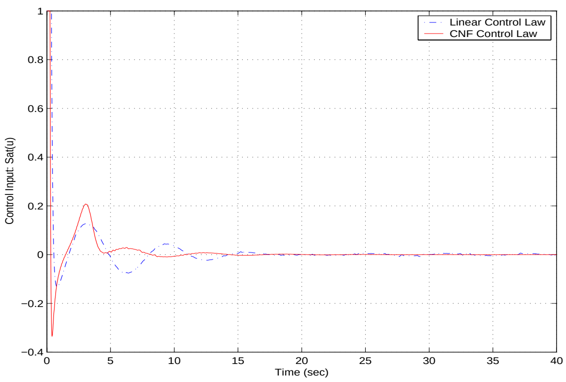
(a) State responses with the linear control law.



(b) State responses with the CNF control law.



(c) System output of the closed-loop system.



(d) Control signals.

Fig. 2. State responses and control signals of the closed-loop systems.

Chen, B. M., T. H. Lee, K. Peng and V. Venkataramanan (2003). Composite nonlinear feedback control for linear systems with input saturation: Theory and an application. *IEEE Transactions on Automatic Control* **48**, 427–439.

He, Y., B. M. Chen and C. Wu (2003). Composite nonlinear control with state and measurement feedback for general multivariable systems with input saturation. In: *Proceedings of the 42nd Conference on Decision and Control*. Vol. 5. Hawaii, USA. pp. 4469–4474. Also, to appear in *Systems & Control Letters*.

Huang, J. (2001). Remarks on the robust output regulation problem for nonlinear systems. *IEEE Transactions on Automatic Control* **46**(12), 2028–2031.

Huang, J. and Z. Chen (2002). A general framework for tackling output regulation problem. In: *Proceedings of the 2002 American Control Conference*. pp. 102–109.

Isidori, A. (1995). *Nonlinear Control Systems*. Communications and Control Engineering Series. Springer-Verlag. London.

Isidori, A. and C. I. Byrnes (1990). Output regulation of nonlinear systems. *IEEE Transactions on Automatic Control* **35**, 131–140.

Jiang, Z. P. and I. Mareels (2001). Robust nonlinear integral control. *IEEE Transactions on Automatic Control* **46**(8), 1336–1342.

Jiang, Z. P., I. Mareels, D. J. Hill and J. Huang (2003). A unifying framework for global regulation via nonlinear output feedback. In: *Proceedings of the 42nd IEEE Conference on Decision and Control*. Maui, Hawaii, USA. pp. 1047–1052.

Lin, Z. and A. Saberi (1992). Semi-global stabilization of partially linear composite systems via feedback of the state of the linear part. In: *Proceedings of the 31st Conference on Decision and Control*. Arizona, USA. pp. 3431–3433.

Lin, Z. and A. Saberi (1993). Semi-global stabilization of partially linear composite systems via linear dynamic state feedback. In: *Proceedings of the 32nd Conference on Decision and Control*. Texas, USA. pp. 2538–2543.

Lin, Z., M. Pachter and S. Banda (1998). Toward improvement of tracking performance–nonlinear feedback for linear systems. *International Journal of Control* **70**, 1–11.

Sussmann, H. J. and P. V. Kokotovic (1991). The peaking phenomenon and the global stabilization of nonlinear systems. *IEEE Transactions on Automatic Control* **36**(4), 424–439.

Teel, A. R. (1992). Semi-global stabilization of minimum phase nonlinear systems in special normal forms. *System & Control Letters* **19**, 187–192.

Turner, M. C., I. Postlethwaite and D. J. Walker (2000). Nonlinear tracking control for multivariable constrained input linear systems. *International Journal of Control* **73**, 1160–1172.